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Simulation of Particle Impact with a Wedge in Dilute Two-phase Flow

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ABSTRACT

Dilute solid-fluid flow over a wedge in a stationary channel is numerically solved using one-way coupling between fluid and solid particles. The two-dimensional, steady, laminar carrier-phase flow is determined by Galerkin finite-element method using Newton's iteration for primitive variables, pressure, and velocity. Velocity is interpolated biquadratically and pressure is interpolated linearly. Parameter continuation is used to compute solutions for relatively large values of flow Reynolds number. Individual particles are tracked from specified inlet positions by a fourth-order Runge-Kutta method applied to the equations of motion of the particle. Forces acting on the particle include drag, pressure, and inertia. Forces due to particle-particle interaction and Basset forces are neglected. Collisions with the wedge and the walls of the channel are modelled *via* assumed coefficients of restitution in both the normal and the tangential directions. The point of actual impact is determined by interpolation. Results are presented for various parameters, such as particle diameter, wedge angle, Reynolds number, particle density, etc.

Keywords: Fluid flow, solid-fluid flow, coupling, test rig, impact, one-way coupling, impact-wear test rig, two-phase flow, Reynolds number

1. INTRODUCTION

In solids handling equipment, two kinds of wear, sliding and impact[1] wear, are generally observed. In predicting total erosion rates in such equipment, experimentally determined specific energy coefficients for sliding and impact are necessary[1,2]. The present research has been applied to an impact-wear test rig, shown in Fig. 1. Solid-liquid flow through a channel strikes the impact wear specimen and causes erosion of the wear sample. By measuring the wear depth for a given test duration, the impact wear specific energy coefficient for a given slurry/wear material combination is calculated[3]. In general, the impact angle of the particle is assumed to be the same as the wedge angle. The impact wear specific energy coefficient depends on the angle of impact[4]. For certain parameters of the solid-liquid flow, the angle of impact may significantly differ from the wedge angle. The aim of the present study is to quantify the difference between the actual angle of impact and the wedge angle for a range of parameters.

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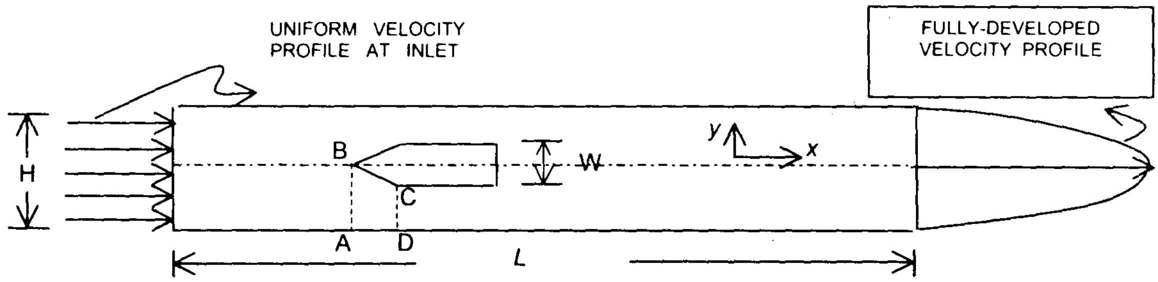

Figure 1. Flow domain-wedge placed along the channel axis

Dilute two-phase laminar flow has been considered in this work. The carrier-phase flow field is determined by Galerkin finite-element method with 9-node isoparametric elements. The velocity is interpolated biquadratically and the pressure bilinearly. One-way coupling is assumed, i.e., particle motion does not affect the flow field. Thus, once the carrier-phase flow field is determined, the individual monosize particles are tracked by the Lagrangain method. The Lagrangain approach allows explicit handling of particle-wall collisions. The flow is assumed to be in a horizontal plane, so that the influence of gravity is not significant.

2. GOVERNING EQUATIONS

The governing equations in nondimensional form for the carrier-phase laminar flow in the domain of Fig. 1 are:

$$\frac{\partial U}{\partial x}+\frac{\partial V}{\partial y}=0 \tag{1}$$

$$U\frac{\partial U}{\partial x}+V\frac{\partial U}{\partial y}=-\frac{\partial p}{\partial x}+\frac{1}{Re}\left(\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}\right) \tag{2}$$

$$U\frac{\partial V}{\partial x}+V\frac{\partial V}{\partial y}=-\frac{\partial p}{\partial y}+\frac{1}{Re}\left(\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}\right) \tag{3}$$

where U and V are the velocity components in the x and y directions, respectively; p is the pressure, Re is the Reynolds number $= \rho U_0 L / \mu$, ρ being the carrier density; μ is its viscosity, and L is the length of the channel. The velocity components are nondimensionalised wrt the inlet velocity, U_0 all-length measurement wrt to L, and pressure wrt ρU_0^2. Due to symmetry about the channel centreline, only one-half of the channel is modelled. The non-dimensional boundary conditions are as follows:

(i) At the inlet, $U = 1$, $V = 0$

(ii) At the centreline of the channel, $V = 0$, $\frac{\partial U}{\partial y}=0$

(iii) At the wedge surface and channel wall, $U = 0$, $V = 0$

(iv) At the channel exit, $P = 0$, $\frac{\partial U}{\partial x}=0$, $\frac{\partial V}{\partial x}=0$.

The nondimensional scalar forms of the equations governing particle motion are given as

$$\frac{dU_p}{dt}=-\frac{1}{(S+C_v)}\frac{\partial p}{\partial x}+\frac{3C_D}{4d_p(S+C_v)}\left|U-U_p\right|\left(U-U_p\right)+\frac{C_v}{S+C_v}\left(U\frac{\partial U}{\partial x}+V\frac{\partial U}{\partial y}\right) \tag{4}$$

$$\frac{dV_p}{dt}=-\frac{1}{(S+C_v)}\frac{\partial p}{\partial y}+\frac{3C_D}{4d_p(S+C_v)}\left|U-U_p\right|\left(V-V_p\right)+\frac{C_v}{S+C_v}\left(U\frac{\partial V}{\partial x}+V\frac{\partial V}{\partial y}\right) \tag{5}$$

where S is the relative density of particle, and C_v is the added mass coefficient. In these equations, time is nondimensionalised wrt L/U_0. The drag coefficient is calculated[2] as

$$C_D = \begin{cases} 0.44 & Re_p > 1000 \\ (24/Re_p)(1+0.14\,Re_p^{0.7}) & Re_p \le 1000 \end{cases} \qquad (6)$$

where Re_p, the particle Reynolds number, is defined as

$$Re_p = \frac{|U - U_p| d_p}{\upsilon}$$

Particles are treated as point masses during collisions with the walls or wedge, and as spheres of definite diameter while solving Eqns (4) and (5). A coefficient of restitution may be introduced in both the normal and the tangential directions. For certain combinations of parameters, the particles may slide along the wall, in which case a frictional force must also be included in the equations of motion.

3. NUMERICAL METHOD

The governing equations for the carrier phase are transformed into weak form by Galerkin finite-element method. The velocity is interpolated biquadratically and the pressure bilinearly. The residual equations obtained by this process are:

$$R_{p_i} = \int_{\Omega^{(e)}} N_{p_i}\left(\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y}\right) d\Omega^{(e)} \qquad (7)$$

$$\begin{aligned} R_{u_i} = &\int_{\Omega^{(e)}} N_i\left(U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y}\right) d\Omega^{(e)} \\ &+ \int_{\Omega^{(e)}} \left\{\frac{\partial N_i}{\partial x}\left(\frac{2}{Re}\frac{\partial U}{\partial x} - P\right) + \frac{\partial N_i}{\partial y}\frac{1}{Re}\left(\frac{\partial U}{\partial y} + \frac{\partial V}{\partial x}\right)\right\} d\Omega^{(e)} \\ &- \int_{\Gamma^{(e)}} N_i\left\{\left(\frac{2}{Re}\frac{\partial U}{\partial x} - P\right)n_x + \frac{1}{Re}\left(\frac{\partial U}{\partial y} + \frac{\partial V}{\partial x}\right)n_y\right\} d\Gamma^{(e)} \end{aligned} \qquad (8)$$

and

$$\begin{aligned} R_{v_i} = &\int_{\Omega^{(e)}} N_i\left(U\frac{\partial V}{\partial x} + V\frac{\partial V}{\partial y}\right) d\Omega^{(e)} \\ &+ \int_{\Omega^{(e)}} \left\{\frac{\partial N_i}{\partial x}\frac{1}{Re}\left(\frac{\partial U}{\partial y} + \frac{\partial V}{\partial x}\right) + \frac{\partial N_i}{\partial y}\left(\frac{2}{Re}\frac{\partial V}{\partial y} - P\right)\right\} d\Omega^{(e)} \\ &- \int_{\Gamma^{(e)}} N_i\left\{\frac{1}{Re}\left(\frac{\partial U}{\partial y} + \frac{\partial V}{\partial x}\right)n_x + \left(\frac{2}{Re}\frac{\partial V}{\partial y} - P\right)n_y\right\} d\Gamma^{(e)} \end{aligned} \qquad (9)$$

where $\Omega^{(e)}$ is the element domain, and $\Gamma^{(e)}$ the element boundary that coincides with the domain boundary of Neumann type. In the present problem, Neumann boundary conditions are applicable only at the exit of the channel. Note that $n_x = 1$ and $n_y = 0$ (for the exit) are the direction cosines of the outward-directed normal. The residuals represent a system of nonlinear equations in the nodal variables U_i, V_i, and P_i. These may be solved either by Newton's method or Picard's method.

Particle tracking requires the specification of initial position and velocity. Particles are assumed to enter with the uniform flow velocity, U_0 at specified locations on the inlet. The particle equations are solved by a fourth-order Runge-Kutta method. The point of collision with the wall is determined by interpolation.

4. RESULTS & DISCUSSION

Mesh refinement studies were conducted using 1800, 2075, and 2400 elements and the velocity profiles at the exit were found to exactly match the theoretical laminar flow parabolic profile, with a centreline nondimensioanl velocity of 1.5 (Fig. 2). Mass conservation was satisfied to within 2 per cent. Mesh-independent velocity profiles were also obtained near the wedge. Mesh independence for particle trajectories is shown in Fig. 3.

The parameters that significantly influence particle motion are carrier-phase fluid, Reynolds number, angle of wedge, particle size, particle density, and coefficient of restitution. Other important effects, such as particle rotation, properties of wall, particle shape, and surface roughness are not considered in the present work. The studies are conducted for half angles of 27° and 45° of the wedge. The

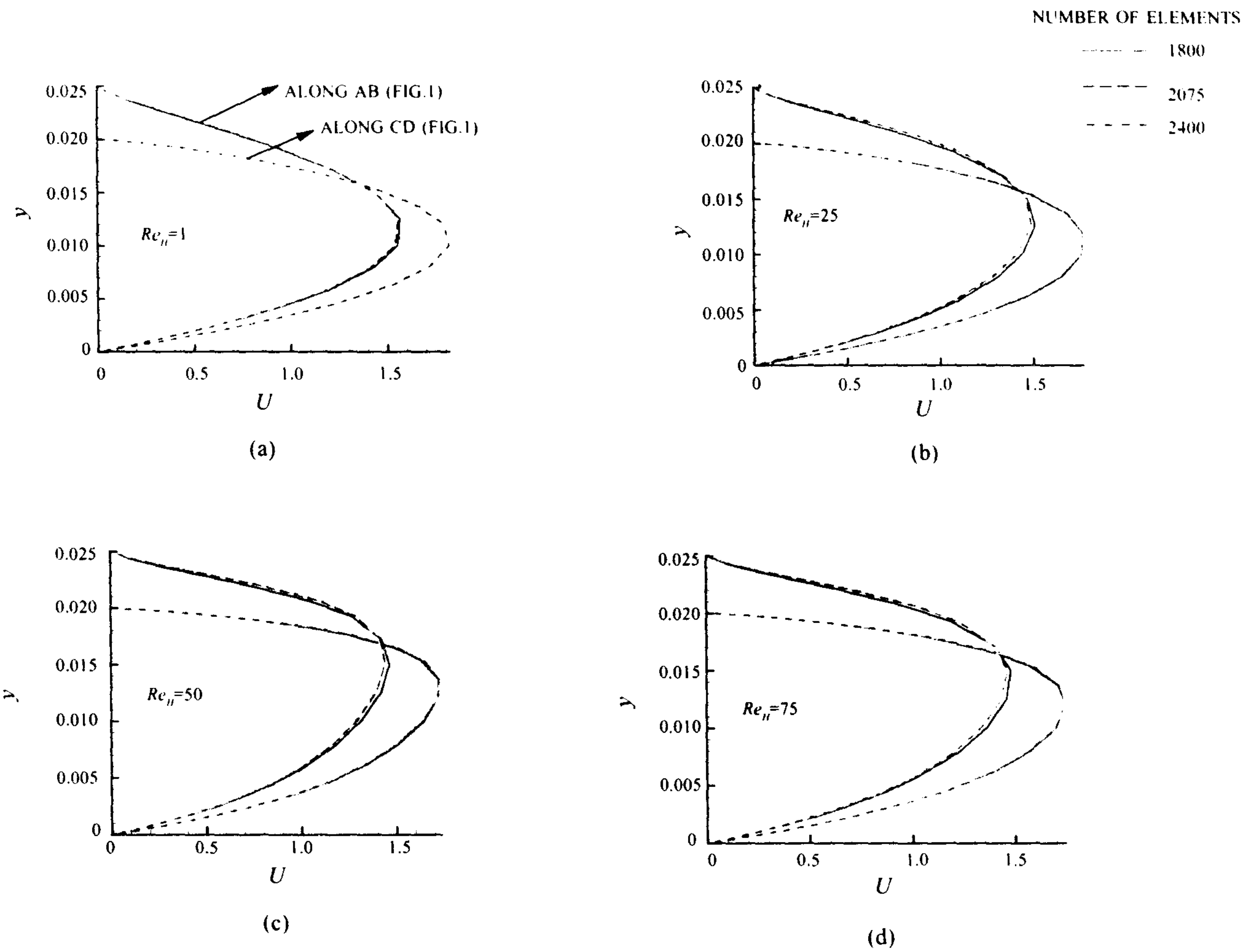

Figure 2. Mesh refinement for velocity profiles near the wedges for different Reynolds numbers. H/L = 0.05, β = 0.30, W = 0.20 H

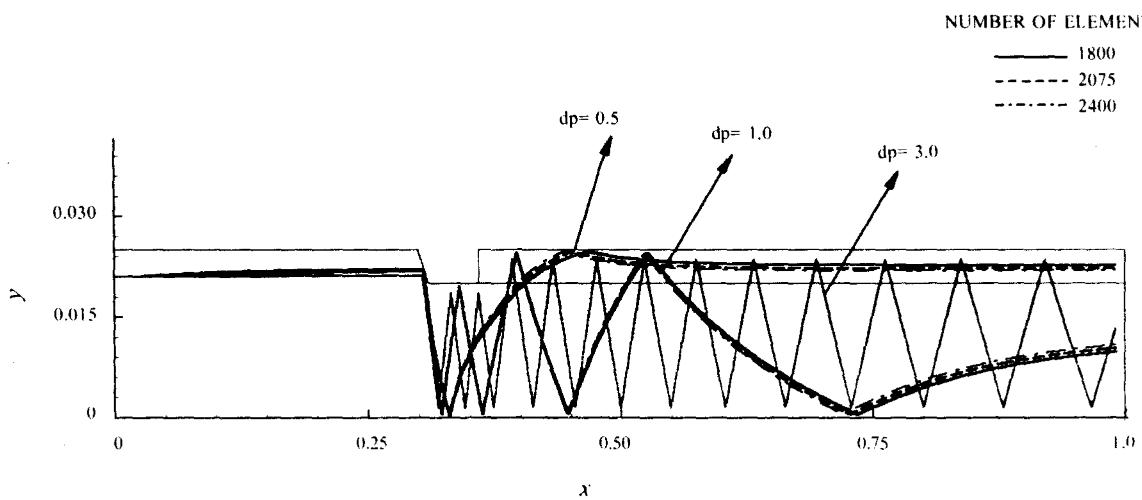

Figure 3. Mesh independence for particle trajectories. H/L = 0.05, β = 0.30, Re_H = 75, W = 0.20 H, ρ = 1.23 kg/m^3, ρ_p = 2700 kg/m^3

Table 1. Effect of carrier-phase fluid viscosity on the deviation of impact angle from the wedge angle*

Case No.	υ (× 10^{-6}) (m^2/s)	d_p (mm)	ρ (kg/m^3)	θ (% deviation) (rad)	Impact angle (rad)
1	15.1	0.5	1.23	0.000352 (0.07469)	0.470887
2	1.0	0.5	1000	0.006425 (1.36360)	0.464813
3	15.1	3.0	1.23	0.000473 (0.10037)	0.470766
4	1.0	3.0	1000	0.041623 (8.83200)	0.429615

*Re_H = 75, β = 0.3, ρ_p = 2700 kg/m^3

pressure gradient parameter, β is related to the half-angle α as $\alpha = \beta\pi/2$.

The effect of carrier-phase fluid viscosity is shown in Table 1. As expected, when air is the carrier ($\upsilon = 15.1\times10^{-6}$ m^2/s), the deviation (θ) of actual impact angle from the wedge angle is small compared to that for water. Similar results are also shown in Table 2 for the effect of particle size, d_p. For the same Re_H, being the height based Reynolds number, the deviation in impact angle increases with particle size.

Table 2. Effect of particle size on deviation of impact angle for the wedge angle*

Case No.	d_p (mm)	θ (% deviation) (rad)	Impact angle (rad)
1	0.5	0.000352 (0.07469)	0.470887
2	1.0	0.001535 (0.32573)	0.469704
3	1.5	0.001383 (0.29348)	0.469856
4	2.0	0.001016 (0.21560)	0.470223

*Re_H = 75, β = 0.3, υ (× 10^{-6}) = 15.1 m^2/s, ρ = 1.23 kg/m^3, ρ_p = 2700 kg/m^3

5. CONCLUSIONS

- Dilute two-phase flow in a channel with impact wedge is simulated using Galerkin finite-element method for the carrier-phase and Lagrangian particle tracking.
- Mesh refinement is used to obtain mesh-independent solutions for both the flow-field and particle behaviour.
- It has been found that the actual impact angle of the particle deviates from the wedge angle. The extent of deviation depends on various parameters, such as flow, Reynolds number, particle size, and carrier-phase fluid. These effects are quantitatively predicted by computer simulations. The deviations in air are small. For water, large particles tend to deviate relatively more.

REFERENCES

1. Krishnan, P.V. & Helmly, F.W. Applications of materials wear testing to solids transport via centrifugal slurry pumps. *In* Wear Testing of Advanced Materials, edited by R. Divakar and P.J. Blau. ASTM-STP-1167, 1992. pp. 114-26.
2. Tuzson, J.J. & Clarke, H. Mcl. Slurry erosion process in the coriolis wear tester. Proceedings of Conference, Fluids Engineering Division of the American Society of Mechanical Engineers, Washington DC. Paper No. FEDSM 98-5144, **245**, 1998.
3. Gupta, P.K. Numerical prediction of dilute two-phase flow in rotating channel. Indian Institute of Technology Delhi, New Delhi. MS Thesis, 2001.
4. Roco, M.C.; Nair, P.; Addie, G.R. & Dennis, J. Erosion of concentrated slurry in turbulent flow. Proceedings of Conference, Fluids Engineering Division of the American Society of Mechanical Engineers, 1984, **13**, 69-77.

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