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Spontaneous Emission and Harmonic Generation in Free-electron Laser with Elliptically Polarised Electromagnetic Wiggler

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ABSTRACT

A study of the spontaneous emission and gain in radiation of a free-electron laser having an elliptically polarised electromagnetic wiggler at the fundamental Doppler upshifted wiggler frequency and its odd harmonics using single particle dynamics is presented.

Keywords: Free-electron laser, electromagnetic wiggler, wiggler frequency, spontaneous emission

1. INTRODUCTION

In a conventional free-electron laser (FEL), a relativistic electron beam travels along a wiggler magnet and amplifies copropagating electromagnetic radiation of frequency $w_r \approx 2\gamma_0^2 w_0$, where γ_0 is the relativistic factor and w_0 is the wiggler frequency. While keeping the electron energy $(\gamma_0 mc^2)$ constant, laser frequency may be increased by increasing the wiggler frequency. For this purpose, short period electromagnetic wigglers have been proposed[1]. The frequency of emission in this case increases to $w_r \approx 4\gamma_0^2 w_0$, where w_0 represents the frequency of the backward travelling electromagnetic wave which acts as a wiggler in the electron frame of reference. Two-stage[2,3] FEL has also been designed for increasing the radiation frequency. Generally in such devices, the first stage generates a large amplitude electromagnetic wave which is then used in the second stage as an electromagnetic wiggler. Several advantages of electromagnetic wigglers have been pointed out by many researchers[4-7]. One such advantage is that its polarisation can be changed without much difficulty.

Recently, it has been shown that elliptically polarised radiation can be made use of in various applications. Spontaneous radiation emitted in an FEL with planar wiggler and axial magnetic field is elliptically polarised and maximum gain is obtained, if external radiation is of similar polarisation[8]. Also, in a circularly polarised wiggler FEL configuration utilising imperfect electron beam injection, coupling with elliptically polarised radiation leads to optimum gain[9]. Intense elliptically polarised laser pulses coupled with magnetised plasma can generate large electric fields, which would be capable of accelerating electrons to high energies[10]. This motivates to study

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a new configuration of FEL comprising an elliptically polarised electromagnetic wiggler, as a source of high intensity elliptically polarised radiation.

In this paper, a detailed study of electron dynamics, spontaneous emission and gain in an FEL having elliptically polarised electromagnetic wiggler has been made. An electron beam to be perfectly injected[11] into an elliptically polarised electromagnetic wiggler FEL is considered.

2. ELECTRON DYNAMICS & SPONTANEOUS EMISSION

Consider an electron beam (of charge $-|e|$, mass m and energy $\gamma_0 mc^2$) travelling with velocity $c\beta_0$, along the $\hat{z}$-direction of an FEL in the presence of an elliptically polarised backtravelling electromagnetic wave (of amplitude $B_{x,y}$, wavelength $\lambda_0 (= 2\pi / k_0)$ and phase $\alpha = k_0 z + w_0 t + \psi$) represented by

$$\vec{B}_w = (B_x \cos\alpha, B_y \sin\alpha, 0)$$

$$\vec{E}_w = \left(\frac{w_0}{ck_0}\right)(-B_y \sin\alpha, B_x \cos\alpha, 0)$$

An elliptically polarised electromagnetic radiation [of frequency $w_r (= ck_r)$, amplitude $E_{x,y}$ and phase $\xi\ (= f k_r z - f w_r t + \phi)$, f labels the harmonics] given by

$$\vec{B}_r = (E_x \cos\xi, -E_y \sin\xi, 0)$$

$$\vec{E}_r = (-E_y \sin\xi, -E_x \cos\xi, 0)$$

copropagates with the electron beam within the interaction region.

The Lorentz force equations of electron motion in the combined electromagnetic wiggler and radiation fields are given by

$$\frac{d(\gamma\beta_x)}{dt} = U_{B_y}\left(\frac{w_0}{ck_0} + \beta_z\right)\sin\alpha + U_{E_y}(1-\beta_z)\sin\xi \qquad (1)$$

$$\frac{d(\gamma\beta_y)}{dt} = -U_{B_x}\left(\frac{w_0}{ck_0} + \beta_z\right)\cos\alpha + U_{E_x}(1-\beta_z)\cos\xi \qquad (2)$$

$$\frac{d(\gamma\beta_z)}{dt} = -(\beta_x U_{B_y}\sin\alpha + \beta_y U_{B_x}\cos\alpha) + (\beta_x U_{E_y}\sin\xi - \beta_y U_{E_x}\cos\xi) \qquad (3)$$

and

$$\frac{d(\gamma)}{dt} = \left(\frac{w_0}{ck_0}\right)(\beta_x U_{B_y}\sin\alpha - \beta_y U_{B_x}\cos\alpha) + (\beta_x U_{E_y}\sin\xi + \beta_y U_{E_x}\cos\xi) \qquad (4)$$

where

$$U_{B_{x,y}} = |e|_{B_{x,y}} / mc, \quad U_{E_{x,y}} = |e|_{E_{x,y}} / mc$$

Equations (1) and (2) can be solved to yield the electron trajectory in the presence of the electromagnetic wiggler alone. Assuming that there is no energy exchange between the electron and the wiggler field[12], the solutions are given by

$$\beta_x = -\frac{K_y}{\gamma}\cos\alpha \qquad (5)$$

$$\beta_y = -\frac{K_x}{\gamma}\sin\alpha \qquad (6)$$

where $K_{x,y} = U_{B_{x,y}} / w_0$ and constants of integration are taken to be zero for perfect electron beam injection. Equations (5) and (6) show that the electron traces a perfect elliptical orbit about the $\hat{z}$-axis as it traverses the FEL cavity in the presence of the wiggler wave. The eccentricity of the elliptical orbit traced by the electron is given by

$$e_t = \sqrt{1 - K_x^2 / K_y^2} = \sqrt{1 - R^2} \qquad (7)$$

where

$$R = K_x / K_y$$

This eccentricity is the same as that of the elliptically polarised electromagnetic wiggler field.

Using β_x and β_y from Eqns (5) and (6) in the relation $\gamma_0^{-2} = 1 - \beta_\perp^2 - \beta_z^2$, one gets the square of the axial velocity as

$$\beta_z^2 = 1 - \frac{1}{\gamma_0^2}\left(1 + \frac{K_x^2 + K_y^2}{2}\right) - \frac{1}{2\gamma_0^2} \times (K_y^2 - K_x^2)\cos 2\alpha \tag{8}$$

For $\gamma_0 >> 1$, a binomial expansion of Eqn (8) leads to

$$\beta_z = 1 - \frac{1}{2\gamma_0^2}\left(1 + \frac{K_x^2 + K_y^2}{2}\right) - \frac{1}{4\gamma_0^2} \times (K_y^2 - K_x^2)\cos 2\alpha \tag{9}$$

The axial velocity is seen to be the superposition of a slow drift term and a fast oscillatory term, respectively is given by the following expressions:

$$\bar{\beta}_z = 1 - \frac{1}{2\gamma_0^2}\left(1 + \frac{K_x^2 + K_y^2}{2}\right) \tag{10}$$

and

$$\Delta\beta_z = -\frac{1}{4\gamma_0^2}(K_y^2 - K_x^2)\cos 2\alpha \tag{11}$$

Integrating Eqn (10) and (11) wrt time, yields:

$$\bar{z} = \bar{\beta}_z ct \tag{12}$$

$$\Delta z = -\frac{(K_y^2 - K_x^2)c}{8\gamma_0^2}\,\frac{\sin(2k_0\bar{\beta}_z ct + 2w_0 t + 2\psi)}{(k_0\bar{\beta}_z c + w_0)} \tag{13}$$

Equation (13) shows that axial oscillations are introduced in the electron motion due to the elliptical nature of electromagnetic wiggler field. The frequency of oscillation is twice the frequency of the electromagnetic wiggler field. These axial oscillations are absent for circularly polarised $(K_x = K_y)$ electromagnetic wiggler field.

The accelerated electron emits spontaneous radiation. The energy radiated per unit solid angle, $d\Omega$, per unit frequency interval, dwr, in the forward direction[13] is given by

$$\frac{d^2 I}{dw d\Omega} = 2|A(w_r)|^2 \tag{14}$$

where

$$\vec{A(w_r)} = \left(\frac{e^2 w_r^2}{8\pi^2 c}\right)^{1/2}\left(\frac{e^{-if\psi}}{2\gamma_0}\right)\left[(\hat{x}K_y J_a + i\hat{y}K_x J_b)\right] \times \frac{2\sin[w_r(1-\bar{\beta}_z) - f(ck_0\bar{\beta}_z + w_0)]N\pi / w_0\bar{\beta}_z}{[w_r(1-\bar{\beta}_z) - f(ck_0\bar{\beta}_z + w_0)]} \tag{15}$$

is the vector potential and

$$J_a = J_{-(f-1)/2}(f\chi) + J_{-(f+1)/2}(f\chi) \tag{16}$$

$$J_b = J_{-(f-1)/2}(f\chi) - J_{-(f+1)/2}(f\chi) \tag{17}$$

$$f\chi = \frac{k_r c(K_y^2 - K_x^2)}{8\gamma_0^2(k_0\bar{\beta}_z c + w_0)} \tag{18}$$

Equations (14) to (18) show that the vector potential representing the spontaneous emission is a superposition of two oscillations (one along the $\hat{x}$ and other along the $\hat{y}$ direction) having different amplitudes. Thus, spontaneous emission is elliptically polarised with eccentricity as

$$e_{fs} = \sqrt{1 - \alpha_{fs}^2} \tag{19}$$

where

$$\alpha_{fs} = \frac{J_b K_x}{J_a K_y} \tag{20}$$

This eccentricity is different from the eccentricity of the electron trajectory by a fraction J_b / J_a.

Substitution of Eqn (15) into Eqn (14) yields:

$$\frac{d^2I}{dwd\Omega}=\frac{4(e\gamma_0N)^2}{c}\sum_f\frac{c^2f^2}{L^2}\left(\frac{w_0}{ck_0}+\bar{\beta}_z\right)^2$$
$$\times\frac{K_y^2}{[1+(K_y^2/2)(1+R^2)]^2}K_{fs}^2\times$$
$$\times\frac{\sin^2[w_r(1-\bar{\beta}_z)-f(ck_0\bar{\beta}_z+w_0)]N\pi/w_0\bar{\beta}_z}{[w_r(1-\bar{\beta}_z)-f(ck_0\bar{\beta}_z+w_0)]^2} \tag{21}$$

where

$$K_{fs}=[(1+R^2)\{J^2_{-(f+1)/2}(f\chi)+J^2_{-(f-1)/2}(f\chi)\}$$
$$+2(1-R^2)\{J_{-(f+1)/2}(f\chi)J_{-(f-1)/2}(f\chi)\}]^{1/2} \tag{22}$$

is the coupling factor between electron motion and spontaneous emission. This emission occurs at the resonant frequency as

$$w_r=\frac{2\gamma_0^2f(w_0+ck_0\bar{\beta}_z)}{1+\frac{K_y^2}{2}(1+R^2)}$$

Equations (21) and (22) may be used to obtain the spontaneous emission intensity for an elliptically polarised magnetostatic wiggler as well, by substituting the limit $\frac{w_0}{ck_0}\to 0$.

Figure (1) shows a plot for spontaneously emitted intensity [in units of $\Delta'=\frac{4(e\gamma_0N)^2}{c}\frac{c^2}{L^2}\left(\frac{w_0}{ck_0}+\bar{\beta}_z\right)^2$] versus R (= K_x/K_y) for K_y = 1.0 and f = 1,3,5---. It may be seen that spontaneous radiation is emitted at the fundamental frequency and its odd harmonics. The emitted intensity at the fundamental frequency increases with an increase in R, that is, at the fundamental frequency, larger spontaneous emission is obtained for elliptic wiggler having smaller eccentricity $(=\sqrt{1-R^2})$. This is due to increased coupling between the electron motion and spontaneous radiation. For higher harmonics, spontaneous intensity reduces with an increase in R due to a reduction in coupling between electron motion and spontaneous radiation. Thus at each harmonic frequency, an elliptic wiggler with larger eccentricity would tend to emit higher intensity.

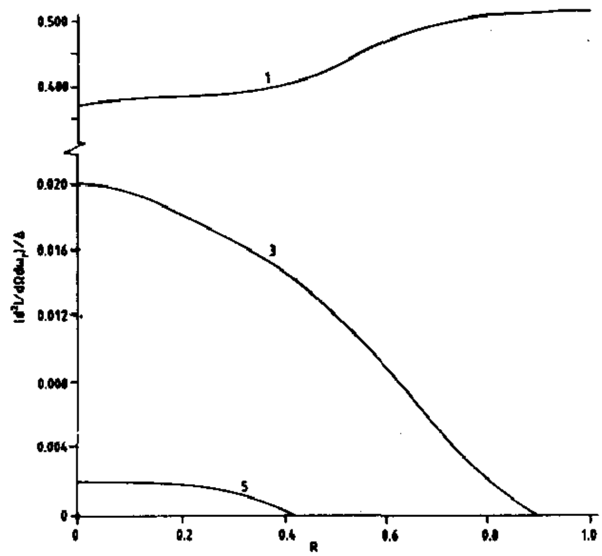

Figure 1. Spontaneous emission intensity (in units of $\Delta'=\frac{4(e\gamma_0N)^2}{c}\frac{c^2}{L^2}\left(\frac{w_0}{ck_0}+\bar{\beta}_z\right)^2$) versus R (= K_x/K_y) for K_y = 1.0 and f = 1, 3 and 5.

3. PENDULUM EQUATION & GAIN

The transverse motion of the electron in the presence of combined electromagnetic wiggler and radiation field may be obtained from Eqns (1) and (2). The second term on the right hand side of these equations represent the transverse force acting on the electron due to the external electromagnetic radiation and may be neglected in comparison to the first representing the transverse force due to the electromagnetic wiggler field. That is

$$U_E(1-\beta_z)<<U_B\left(\frac{w_0}{ck_0}+\beta_z\right)$$

Thus the trajectory followed by the electron is the same as given in Eqns (5) to (9).

Substituting Eqns (5) and (6) in Eqn (4), one gets:

$$\frac{d\gamma}{dt} = -\frac{|e|^2}{2m^2c^2ck_0\gamma}\left(\frac{w_0}{ck_0}\right)(B_y^2 - B_x^2)\sin 2\alpha$$

$$-\frac{|e|}{mc\gamma}K_yE_y\sin\xi\cos\alpha - \frac{|e|}{mc\gamma}K_xE_x\sin\alpha\cos\xi \quad (23)$$

Neglecting the fast oscillatory term in Eqn (23) and using Eqns (12) and (13), one gets:

$$\frac{d\gamma}{dt} = -\frac{|e|}{2mc\gamma}(K_yE_yJ_a + K_xE_xJ_b) \times \sin[\zeta(t) + \phi'] \quad (24)$$

where

$\phi' = \phi + f\psi$ and $\zeta_f(t) = f(w_0 + ck_0\bar{\beta}_z)t - fw_r(1-\bar{\beta}_z)t$ represents the dimensionless phase of interaction.

Differentiating $\zeta(t)$ twice wrt t, and using Eqn (24) along with the time derivative of Eqn (10), one gets:

$$\ddot{\xi}_f(t) = \frac{-f(w_r + w_0)|e|}{2mc\gamma^{(4)}}\left(1+\frac{K_x^2+K_y^2}{2}\right) \times (K_yE_yJ_a + K_xE_xJ_b)\sin(\zeta_f(t)+\phi') \quad (25)$$

This is the self-consistent pendulum equation describing the interaction between the electron motion and the radiation.

Following a procedure similar to that given by Jha, *et al.*[14], the gain in radiation averaged over initial phase of radiation is given by

$$G = \frac{8\pi\rho_e mc^2}{(E_x^2 + E_y^2)}\int_0^T <\gamma>_\phi dt \quad (26)$$

where ρ_e is the electron density and T is the transit time of the electron through FEL interaction region of length L. To evaluate the gain, Eqn (24) is solved simultaneously with Eqn (25) up to the second order in E. The value of γ thus obtained is averaged over the initial phase of radiation and substituted in Eqn (26) to give the lowest order, nonvanishing gain as

$$G^2 = aF(x)\frac{K_y^2J_a^2}{(1+S^2)}\{1+S\alpha_{fs}\}^2 \quad (27)$$

where

$$a = \frac{4\pi^2N|e|^2\rho_e f}{\gamma_0^3 mc^2}\left(\frac{w_0}{ck_0}+\bar{\beta}_z\right)L^2$$

$$F(x) = \frac{1}{x^3}(2 - 2\cos x - x\sin x)$$

$x = f\Delta wt$ and $S = E_x / E_y$

Maximum gain at frequency

$$fw_r \approx \frac{2\gamma_0^2}{[1+(K_y^2/2)(1+R^2)]} \times\left\{f(w_0 + ck_0\bar{\beta}_z) - \frac{2.6\bar{\beta}_zc}{Lf}\right\}$$

is found to occur slightly off resonance, at $x = 2.6$. Frequency varies with eccentricity of elliptic wiggler, besides other FEL parameters (γ_0, w_0 L, f. It may be noted that gain occurs at the fundamental Doppler upshifted wiggler frequency and its odd harmonics ($f = 1,3,5$......).

Differentiation of Eqn (27) wrt S shows that maximum gain occurs when $S = \alpha_{fs}$. In otherwords, maximum gain will be obtained when the eccentricity ($e_c = \sqrt{1-S^2}$) of the external radiation matches with the eccentricity (e_{fs}) of spontaneous emission. Gain in radiation (in units of

$$\Delta = \frac{4\pi^2N|e|^2\rho_e}{\gamma_0^3mc^2}\left(\frac{w_0}{ck_0}+\bar{\beta}_z\right)L^2 \times 0.135\,)$$

versus S (= E_x / E_y) for two different values of R (= 0.2 and 0.5, respectively) with $K_y = 1.0$ and $f = 1,3,5$ is plotted in Fig. 2. It is seen that the fundamental frequency and its odd harmonics appear. As S increases [eccentricity (e_c) decreases] the gain at the fundamental as well as its odd harmonic frequencies increases to a maximum and then falls. Maximum gain is obtained when $S = \alpha_{fs}$ (or $e_c = e_{fs}$). For example at the fundamental frequency $\alpha_{fs} = R$ [Eqn (20)].

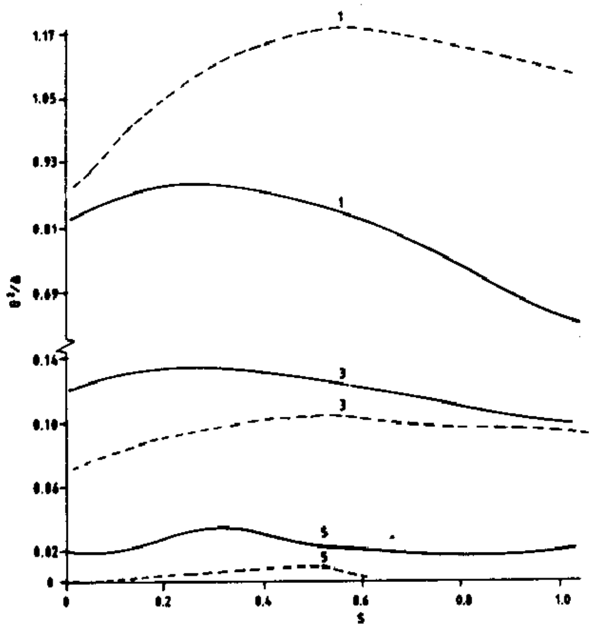

Figure 2. Gain (in units of $\Delta = \frac{4\pi^2 N|e|^2 \rho_e}{\gamma_o^3 mc^2}\left(\frac{w_o}{ck_o}+\bar{\beta}_z\right)L^2 \times 0.135$) versus S for $R = 0.2$ (——) and 0.5 (-----) with $K_y = 1.0$ and $f = 1, 3$ and 5.

maximum gain at the fundamental frequency will be obtained for $S = R$. It may also be seen that as R changes from 0.2 to 0.5, the peak gain at the fundamental frequency increases, whereas the peak gain at harmonics correspondingly decreases. By reducing the eccentricity of the wiggler field, gain at harmonics may be suppressed, thereby enhancing the gain at the fundamental frequency. Conversely, gain at harmonics can be increased by increasing the eccentricity of the wiggler field.

4. CONCLUSIONS

Study of an FEL with an elliptically polarised electromagnetic wiggler field leads to a more generalised analysis of an FEL system and can be reduced to either circularly polarised or linearly polarised electromagnetic wiggler configuration under certain conditions. Gain at harmonics (which is not obtainable with circular electro-magnetic wiggler using perfect electron beam injection) is obtainable even with perfect electron beam injection for the elliptically polarised electro-magnetic wiggler system.

It has been shown that for FEL with elliptically polarised electromagnetic wiggler field, a perfectly injected electron beam traces an elliptical orbit with an eccentricity equal to that of the elliptically polarised electromagnetic wiggler field. Spontaneously emitted radiation is also elliptically polarised but has slightly different eccentricity from that of the electron orbit. Spontaneous and coherent emissions occur at the fundamental Doppler upshifted frequency and its odd harmonics. Along with other FEL parameters, fundamental frequency varies with eccentricity of the elliptically polarised wiggler. For a given eccentricity of the wiggler field, maximum gain is obtained when the eccentricity of the external radiation is equal to the eccentricity of the spontaneous emission. Also, appropriate variation of wiggler eccentricity will lead to the enhancement in gain at either the fundamental or harmonic frequencies.

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