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SHORT COMMUNICATION

Oscillating Plate Temperature Effects on Mixed Convection Flow Past a Semi-infinite Vertical Porous Plate

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ABSTRACT

The effects on mixed convection flow past a semi-infinite vertical porous plate have been studied when the plate temperature oscillates about a non-zero mean. Only out-of-phase component of unsteady part of the temperature is shown graphically. The results show that there is always a phase-lead in the rate of heat transfer at small values of ω.

Keywords: Semi-infinite verticle plate, oscillating plate, steady mixed convection flow, fluid flow

1. INTRODUCTION

Steady mixed convection flow of a viscous incompressible fluid past a semi-infinite vertical plate has been studied by many researchers. The plate temperature was assumed to be either constant or varied as some power of the distance from the leading edge. Another important situation was to study the effects of oscillating plate temperature on the mixed convection flow past a semi-infinite vertical plate which was studied for a non-porous semi-infinite vertical plate by Soundalgekar and Vighnesam[1]. They now propose to study this problem for a semi-infinite vertical porous plate when the plate temperature is oscillating about a non-zero plate temperature.

2. MATHEMATICAL ANALYSIS

The flow of a viscous incompressible fluid past a semi-infinite vertical porous plate, with the x-axis along the plate and the y-axis normal to the plate is assumed. Let U_0 be the uniform velocity of the fluid, and v_w be the suction or injection velocity at the plate. The temperature is assumed to be represented by an unsteady component $\theta_1(x, y)$ to be superimposed on the steady temperature $\theta_s(x, y)$ as

$$\theta = \frac{T - T_\infty}{T_w - T_\infty} = \theta_s(x,y) + \in e^{i\omega t}\ \theta_1(x,y) \tag{1}$$

where θ_s is the steady non-zero mean temperature,

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$\in<1$ is the amplitude of the oscillation, ω is the frequency, and t is the time. Substituting Eqn (1) into unsteady Navier-Stokes equations, equating the harmonic and non-harmonic terms, one gets:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+g\beta(T_w-T_\infty)\theta_s \quad (2)$$

$$u\frac{\partial \theta_s}{\partial x}+v\frac{\partial \theta_s}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 \theta_s}{\partial y^2}+\frac{\nu}{c_p(T_w-T_\infty)}\left(\frac{\partial u}{\partial y}\right)^2 \quad (3)$$

$$i\omega\theta_1+u\frac{\partial \theta_1}{\partial x}+v\frac{\partial \theta_1}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 \theta_i}{\partial y^2} \quad (4)$$

with the following boundary conditions:

$$\begin{array}{llll} u=0, & v=v_w, \ \theta_s=1, \ \theta_1=1 & \text{at} & y=0 \\ u=U_0, & \theta_s=0, \ \theta_1=0 & \text{as} & y=\infty \end{array} \quad (5)$$

Assuming low-frequency condition, the unsteady part of the temperature, θ_1 in terms of in-phase and out-of-phase components can be expressed as

$$\theta_1 = \theta_r + i\ \theta_i \quad (6)$$

Substituting Eqn (6) in Eqns (4) and (5), and equating the real and imaginary parts, one gets:

$$-\omega\theta_i+u\frac{\partial \theta_r}{\partial x}+v\frac{\partial \theta_r}{\partial y}=\frac{K}{\rho c_p}\frac{\partial^2 \theta_r}{\partial y^2} \quad (7)$$

$$\omega\theta_r+u\frac{\partial \theta_i}{\partial x}+v\frac{\partial \theta_i}{\partial y}=\frac{K}{\rho c_p}\frac{\partial^2 \theta_i}{\partial y^2} \quad (8)$$

with the following boundary conditions:

$$\begin{array}{ll} \theta_r(0)=1, & \theta_i(0)=0 \\ \theta_r(\infty)=0, & \theta_i(\infty)=0 \end{array} \quad (9)$$

If β_1 is the phase-shift between the temperature changes within the boundary layer and at the plate, it can be represented by

$$\beta_1 = \tan^{-1}(\theta_i / \theta_r) \quad (10)$$

For low frequency, this shift is very small and hence, $\theta_i \approx \theta_r$. So for small frequency, $-\omega\theta_i$ is very small and hence, this term can be neglected in Eqn (7), which is then reduced to:

$$u\frac{\partial \theta_r}{\partial x}+v\frac{\partial \theta_r}{\partial y}=\frac{K}{\rho c_p}\frac{\partial^2 \theta_r}{\partial y^2} \quad (11)$$

The following transformations have been introduced in Eqns (2), (3), (8) and (11)

$$\left.\begin{array}{c} \eta=y\sqrt{\dfrac{U_0}{\nu x}},\ \psi=\sqrt{U_0\nu x}\,.f(\eta),\ u=U_0 f'(\eta) \\ v=-\dfrac{1}{2}\sqrt{\dfrac{U_0\nu}{x}}\,.(f-\eta f'),\ \theta_s=\varphi(\eta) \\ \theta_r=\psi_r(\eta),\ \theta_i=\left(\dfrac{U_0 x}{\nu}\right)\psi_i(\eta) \end{array}\right\} \quad (12)$$

and taking into account the equation of continuity, one gets:

$$f'''+\frac{1}{2}ff''+(Gr/Re^2)\phi=0 \quad (13)$$

$$\phi''+\frac{1}{2}Pr\,f\phi'+Pr\,Ec\,f''^2=0 \quad (14)$$

$$\psi_r''+\frac{1}{2}Pr\,f\psi_r'=0 \quad (15)$$

$$\psi_i''+\frac{1}{2}Pr\,f\psi_i'-Pr\,f'\psi_i-\beta\,Pr\psi_r=0 \quad (16)$$

With the following boundary conditions:

$$\left.\begin{array}{l} f(0)=f_w, f(0)=0, \phi(0)=1, \psi_r(0)=1, \psi_i(0)=0 \\ f'(\infty)=1, \phi(\infty)=0, \psi_r(\infty)=0, \psi_i(\infty)=0 \end{array}\right\} \quad (17)$$

Here the non-dimensional quantities are defined as

$$\left.\begin{aligned} & Gr=\frac{g\beta(T_w-T_\infty)x^3}{\nu^2},\; Re=\frac{U_0 x}{\nu},\; Pr=\frac{\mu c_p}{K} \\ & Ec=\frac{U_0^2}{c_p(T_w-T_\infty)},\; \beta=\frac{\omega\nu}{U_0^2},\; f_w=-2\nu\sqrt{\frac{x}{U_0\nu}} \end{aligned}\right\} \quad (18)$$

Here, Gr is the Grashof number, Re is the Reynolds number, Pr is the Prandtl number, Ec is the Eckert number and β is the frequency parameter. As Gr and Re are functions of x, Eqns (13) to (16) are local non-similar equations. These equations are solved numerically on a computer for Pr = 0.71 (air) and for different values of Gr/Re^2, f_w, Ec and β. Here $f_w > 0$ corresponds to suction and $f_w < 0$ corresponds to injection. The function ψ_i is shown in Fig. 1.

The rate of heat transfer is given by

$$(U_0/2\nu x)^{-1/2}.\, q=\phi'(0)+\in|Q|\cos(\omega t+\alpha) \quad (19)$$

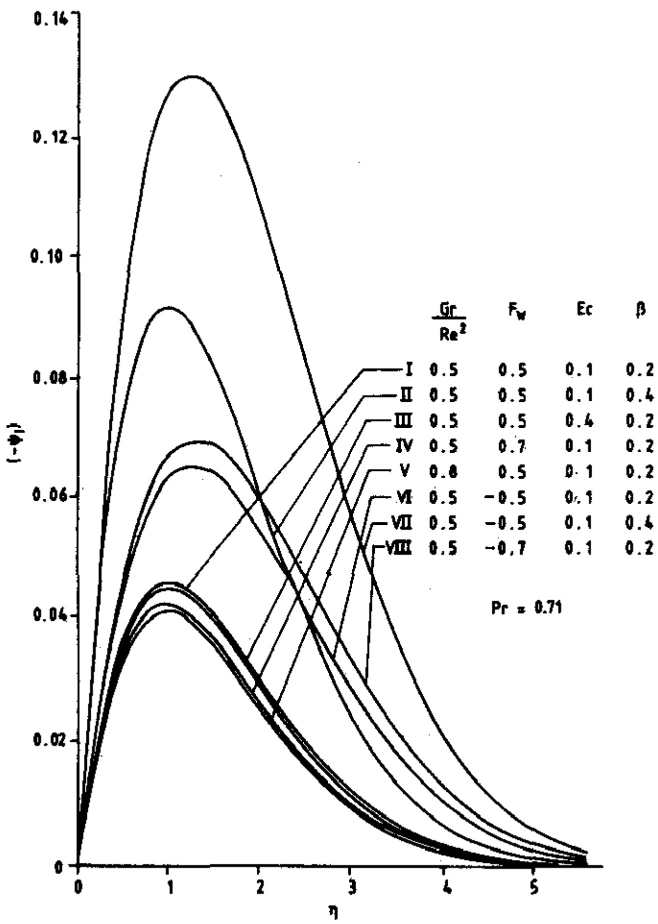

Figure 1. Function ψ_i

Table 1. Values of $[-\phi'(0)]$, $[-\psi'_r(0)]$, $[-\psi'_i(0)]$ at Pr = 0.71

Gr/Re^2	f_w	Ec	β	$\{-\phi'(0)\}$	$\{-\psi'_r(0)\}$	$\{-\psi'_i(0)\}$
0.5	0.5	0.1	0.2	0.4433	0.4827	0.1084
0.5	0.5	0.1	0.4	-	-	0.2169
0.5	0.5	0.4	0.2	0.3190	0.4851	0.1078
0.5	0.7	0.1	0.2	0.4921	0.5332	0.1061
0.8	− 0.5	0.1	0.2	0.4499	0.5043	0.1028
0.5	− 0.5	0.1	0.2	0.2322	0.2634	0.1174
0.5	− 0.5	0.1	0.4	-	-	0.2349
0.5	− 0.5	0.4	0.2	0.1371	0.2667	0.1163
0.8	− 0.5	0.1	0.2	0.2442	0.2924	0.1085
0.5	− 0.7	0.1	0.2	0.1979	0.2273	0.1185
0.5	− 0.7	0.1	0.4	-	-	0.2370
0.5	− 0.7	0.4	0.2	0.1083	0.2308	0.1172
0.8	− 0.7	0.1	0.2	0.2105	0.2571	0.1090

where

$$Q=\left\{\psi_r^2(0)\right\}^2+\left\{\frac{U_0 x}{\nu}.\psi_i'(0)\right\}^2 \quad (20)$$

and

$$\tan\alpha=\frac{U_0 x}{\nu}\,\frac{\psi_i'(0)}{\psi_r'(0)}$$

Table 1 shows the numerical values of $\{-\phi'(0)\},\{-\psi_r'(0)\},\{-\psi_i'(0)\}$.

At high frequency, Eqn (4) reduces to

$$i\omega\theta_1=\frac{K}{\rho c_p}\frac{\partial^2\theta_1}{\partial y^2}$$

which has a shear wave-type solution.

3. CONCLUSIONS

The results drawn are:

- An increase in Gr/Re^2 or β or Ec leads to an increase in the amplitude of the rate of heat transfer at small vaues of ω. But an increase in suction velocity leads to an increase in the amplitude of the rate of heat transfer, whereas an increase in injection velocity leads to a decrease in the amplitude of the rate of heat transfer.

- There is always a phase-lead in case of the rate of heat transfer at small values of ω.
- At large values of frequency ω, the mean temperature is superimposed by a shear wave-type solution which is not affected by ω or suction-injection velocity.

REFERENCES

1. Soundalgekar, V.M. & Vighnesam, N.V. Oscillating plate temperature effects on combined convection flow past a semi-infinite vertical plate. *Int. J. Heat Tech*, 1997, **15**, 11-15.