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SHORT COMMUNICATION

Two-Stage Conditional Repetitive Group Sampling Plan

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ABSTRACT

This paper enlarges the concept of conditional repetitive group sampling plan. The operating characteristic and average sample number functions of the generalised plan have been derived by graphical evaluation and review technique. Lastly, Poisson unity values have been tabulated to facilitate the operation and construction of the plan. The plan that provides a quick discrimination of good and bad quality lots, could be used for both the process control and goods acceptance or rejection in various stages of production in defence organisations.

Keywords: Conditional repetitive group sampling plan, Poisson unity value, graphical evaluation and review technique, repetitive group sampling plan

1. INTRODUCTION

The concept of repetitive group sampling (RGS) plan was introduced by Sherman[1] in which acceptance and rejection of the lot was based on the repeated sample results of the same lot. The detailed procedures and tables for construction and selection of RGS plan have been given by Soundararajan and Ramaswamy[2], and Singh[3], *et. al*. Later on, Shankar and Mohapatra[4] developed conditional RGS plan as an extension of the classical RGS plan. Mohapatra[5] compared the conditional RGS plan with the RGS plan and observed that the conditional RGS plan is better in sample size efficiency than the RGS plan. The purpose of present investigation is two-fold. Firstly, following Stephens and Dodge[6], the proposed plan uses different sample sizes in the normal and the tightened phases of inspection. Secondly, the dynamic characteristics of the proposed plan have been modelled and analysed through graphical evaluation and review technique[7-9] (GERT), which has been used by several authors to study quality control systems. A brief account of such studies has been given by Shankar[10]. The formula for operating characteristic and average sample number (ASN) functions of the plan has been derived by applying Mason's[11] rule on the GERT network representation of the inspection system. Lastly, Poisson unity values have been tabulated to facilitate the operation and construction of the plan.

Military standard plans are commonly used in defence establishments by the military, as a consumer. In application of the MIL-STD-105D system, it is intended that a switch to tightened inspection, in case of poor quality, provides a psychological and economic incentive for the producer to improve the level of the product quality submitted. The proposed two-stage plan based on normal and tightened inspection may be useful as an addition to MIL-STD-105D system for quick discrimination of good and bad quality lots. The present study may also be used when output from a continuous process is inspected

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and/or sampled continuously to determine the quality of goods. Thus, the proposed procedure could be used for both the process control and goods acceptance or rejection in various stages of production in defence organisations.

2. TWO-STAGE CONDITIONAL REPETITIVE GROUP SAMPLING PLAN

Following the notations and concepts similar to those of Sherman, and Shankar and Mohapatra, the proposed two-stage conditional RGS plan is carried out through the following steps:

(a) Draw a random sample of size n_1 from the lot for normal inspection and determine the number of defectives (d) found therein.

(b) Accept the lot, if $d \le c_1$

Reject the lot, if $d > c_2$

(c) If $c_1 < d \le c_2$, then

- Switch to tightened inspection by taking a sample of size n_2 ($> n_1$) from the lot and determine the number of defectives (d) found therein.
- Accept the lot, if $d \le c_1$
- Reject the lot, if $d > c_2$.
- If $c_1 < d \le c_2$, then repeat the above steps provided previous i lots are accepted under normal inspection.
- Otherwise reject the lot.

Here, it may be noted that a lot with number of defectives $\le c_1$ is accepted at both the normal and tightened inspection states. Furthermore, the process is automatically switched to normal inspection after acceptance/rejection of the current lot.

The proposed plan is characterised by five parameters, namely n_1, n_2, c_1, c_2 and i. For $i = 0$, the resulting plan is two-phase inspection RGS plan due to Shankar[12]. Moreover, for $i = 0$ and $k = n_2/n_1 = 1$ (i.e. $n_1 = n_2$), one has RGS plan due to Sherman[1].

3. GRAPHICAL EVALUATION & REVIEW TECHNIQUE – ANALYSIS OF PLAN

The possible stages of the inspection system can be defined as follows:

S_0 Initial stage of the plan

S_1 Stage in which tightened inspection is performed for the current lot.

$S_2(j)$ Stage in which j^{th} ($j = 1, 2, 3, \ldots i$) preceding lot is accepted under normal inspection.

S_A Stage in which current lot is accepted.

S_R Stage in which current lot is rejected.

The above stages enable one to construct GERT network representation of the inspection system as shown in Fig. (1). The moment generating function of sample size n is:

$$M_n(\theta) = \exp n(\theta)$$

because n is constant. Now, applying Mason's[11] rule on the representation in Fig. (1), the W-function[7-9] from the initial node S_0 to the terminal nodes S_A and S_R, respectively is given by

$$W_A(\theta) = \left[P_{a_1} e^{n_1\theta} \left\{ 1 - P_{c_2} e^{n_2\theta} \left(P_{a_1} e^{n_1\theta} \right)^i \right\} + P_{c_1} P_{a_2} e^{(n_1+n_2)\theta} \right] \Big/ H$$

$$W_R(\theta) = \left[P_{r_1} e^{n_1\theta} \left\{ 1 - P_{c_2} e^{n_2\theta} \left(P_{a_1} e^{n_1\theta} \right)^i \right\} + P_{c_1} P_{r_2} e^{(n_1+n_2)\theta} + G \right] \Big/ H \quad (1)$$

where

$$H = 1 - P_{c_2} e^{n_1\theta} \left(P_{a_1} e^{n_1\theta} \right)^i$$

$$J = P_{c_1} P_{c_2} (1 - P_{a_1}) e^{(2n_1+n_2)\theta} \left\{ 1 - (P_{a_1} e^{n_1\theta})^i \big/ (1 - P_{a_1} e^{n_1\theta}) \right\}$$

$$P_{a_1} = L(p, n_1, c_1), \qquad P_{a_2} = L(p, n_2, c_1)$$

$$P_{r_1} = 1 - L(p, n_1, c_2), \quad P_{r_2} = 1 - L(p, n_2, c_2)$$

$$P_{c_1} = 1 - (P_{a_1} + P_{r_1}), \qquad P_{c_2} = 1 - (P_{a_2} + P_{r_2})$$

and

$L(p, n, c)$ is the probability of getting c or less defectives with product quality p and sample size n.

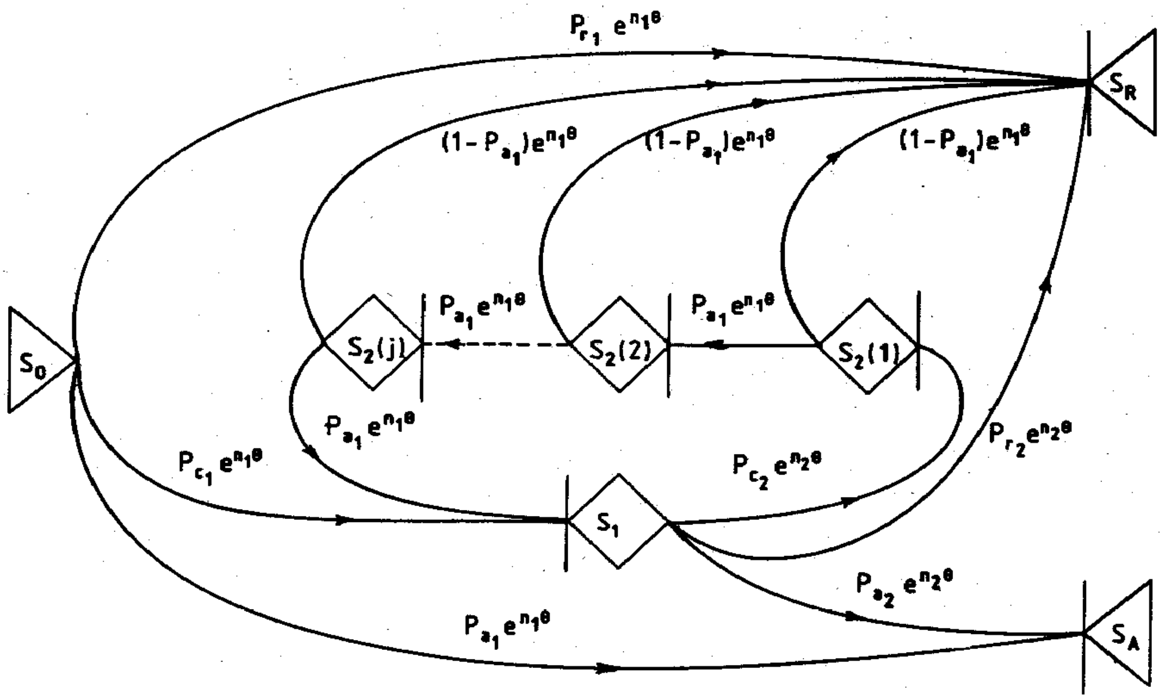

Figure 1. GERT network of two-stage conditional RGS plan

Now, from the definition of W-function, one gets:

$$P_A = \left[P_{a_1}\left(1-P_{c_2}P_{a_1}^{\,i}\right)+P_{c_1}P_{a_2}\right] \Big/ \left(1-P_{c_2}P_{a_1}^{\,i}\right) \tag{2}$$

$$P_R = \left[P_{r_1}\left(1-P_{c_2}P_{a_1}^{\,i}\right)+P_{c_1}P_{r_2}+P_{c_1}P_{c_2}\left(1-P_{a_1}^{\,i}\right)\right] \Big/ \left(1-P_{c_2}P_{a_1}^{\,i}\right)$$

Thus, $P_R = 1 - P_A$, where P_A and P_R stand for the probability of acceptance and rejection of the lot, respectively.

For further characterisation of the plan, ASN function is defined as follows:

$$\text{ASN} = P_A\left[\frac{d}{d\theta}M_A(\theta)\right]_{\theta=0} + P_R\left[\frac{d}{d\theta}M_R(\theta)\right]_{\theta=0}$$

$$= \left[n_1\left\{1-P_{c_2}P_{a_1}^{i}+P_{c_1}P_{c_2}\left(1-P_{a_1}\right)^{-1}\left(1-P_{a_1}^{i}\right)\right\}+n_2 P_{c_1}\right] \Big/ \left(1-P_{c_2}P_{a_1}^{i}\right) \tag{3}$$

where

$$M_A(\theta) = W_A(\theta)/W_A(0)$$

and

$$M_R(\theta) = W_R(\theta)/W_R(0)$$

Here, it may be observed that for $i = 0$, one has:

$$P_A = P_{a_1} + P_{c_1}P_{a_2}/(1-P_{c_2})$$

and

$$\text{ASN} = n_1 + n_2 P_{c_1}/(1-P_{c_2}) \tag{4}$$

These results agree with Shankar[12]. Furthermore, for k = 1 i.e. $n_1 = n_2$

$$P_{a_1} = P_{a_2},\ P_{r_1} = P_{r_2},\ \text{and}\ P_{c_1} = P_{c_2}$$

Therefore

$$P_A = P_{a_1}/(1-P_{c_1})$$

and

Table 1. Values of $R = p_2/p_1$ and n_1p_2 for two-stage conditional RGS plan, where $\alpha = 0.05$, $\beta = 0.10$, $c_1 = 0$, $c_2 = 2$

	k = 1.00		k = 1.25		k = 1.50		k = 1.75		k = 2.00	
i	R	n_1p_2	R	n_1p_2	R	n_1p_2	R	n_1p_2	R	n_1p_2
1	06.894	2.6697	07.109	2.5175	07.402	2.4365	07.776	2.3829	08.072	2.3496
2	08.014	2.6615	08.276	2.5218	08.549	2.4307	08.885	2.3800	09.241	2.3478
3	08.653	2.6605	09.033	2.5216	09.296	2.4281	09.673	2.3783	10.065	2.3464
4	09.158	2.6601	09.544	2.5216	09.906	2.4260	10.331	2.3769	10.745	2.3453
5	09.518	2.6599	09.918	2.5216	10.327	2.4342	10.727	2.3760	11.153	2.3446
6	09.784	2.6597	10.201	2.5217	10.626	2.4343	11.036	2.3753	11.388	2.3442
7	09.987	2.6595	10.422	2.5217	10.862	2.4343	11.282	2.3748	11.718	2.3436
8	10.143	2 6594	10.596	2.5217	11.051	2.4343	11.370	2.3746	12.017	2.3431
9	10.265	2.6593	10.734	2.5217	11.203	2.4344	11.577	2.3741	12.248	2.3426
10	10.360	2.6593	10.845	2.5217	11.327	2.4344	11.868	2.3735	12.249	2.3426

$$\mathrm{ASN} = n_1/(1-P_{c_1})$$

These results coincide with Sherman[1]

4. CONSTRUCTION OF TABLES

The expression for the probability of acceptance (P_A) and ASN of the two-stage conditional RGS plan under Poisson model are given by Eqns (2) and (3), respectively with the following expressions:

$$P_{a_1} = \sum_{x=0}^{c_1} (n_1 p)^x \exp(-n_1 p)/x$$

$$P_{a_2} = \sum_{x=0}^{c_1} (kn_1 p)^x \exp(-kn_1 p)/x$$

$$P_{r_1} = 1 - \sum_{x=0}^{c_2} (n_1 p)^x \exp(-n_1 p)/x$$

$$P_{r_2} = 1 - \sum_{x=0}^{c_2} (kn_1 p)^x \exp(-kn_1 p)/x$$

where

$k = n_2/n_1$

Let $\quad P_a(p') = L(p', n, c)$

Now, for given p_1, p_2, α and β, i.e. the two points $(p_1, 1-\alpha)$ and (p_2, β) on the operating characteristic curve, one may write the following expressions:

$$P_A(p_1) = 1-\alpha \tag{5}$$

and

$$P_A(p_2) = \beta \tag{6}$$

For given c_1, c_2, k, α and i, Eqn (5) is a function of n_1p_1 only. Therefore, unity values n_1p_1 have been obtained from Eqn (5) by Newton's method of successive approximation. Similarly, the values of n_1p_2 are also obtained from Eqn (6) by Newton's method of successive approximation. The operating ratio $R = n_1p_2/n_1p_1 = p_2/p_1$. Only the extract table showing relevant entries for $\alpha = 0.05$, $\beta = 0.10$ and k = 1.00, 1.25, 1.50, 1.75 and 2.00 are presented.

5. RESULTS & DISCUSSION

Suppose a two-stage conditional RGS plan is desired having 95 per cent probability of acceptance at $p_1 = 0.01$ and 10 per cent probability of acceptance at $p_2 = 0.0855$ with $c_1 = 0$, $c_2 = 2$ and $k = 1.5$.

The operating ratio, $R = p_2/p_1 = 8.55$, where R corresponds to $k = 1.5$ (Table 1) providing $i = 2$.

The sample size is:

Table 2. The effect of different choices of k on operating characteristic and average sample number

p	P_A	Average sample number (ASN)				
		k = 1	k = 1.25	k = 1.50	k = 1.75	k = 2.00
p_1 = 0.01	0.99	27.1167	26.0582	25.2366	24.5728	24.0270
	0.95	58.8096	57.4323	56.4059	55.6001	54.9485
p* = 0.05	0.50	51.5196	51.4251	51.4213	51.4800	51.5887
p_2 = 0.10	0.10	43.7175	44.2184	45.2071	46.6148	48.3789
	0.05	47.1490	47.8127	49.1597	51.0117	53.2199

$$n_1 = n_1 p_2 / p_2 = 2.4307/0.0855 = 28.429 \simeq 28$$

and

$$n_2 = k \times n_1 = 1.5 \times 28.429 = 42.643 \simeq 43 .$$

Thus, the desired plan consists of $c_1 = 0$, $c_2 = 2$, $n_1 = 28$, $n_2 = 43$ and $i = 2$.

Table 2 shows the effect of different choices of k on operating characteristic and ASN for some selected values of p, say p_1, p_2 and p^* with $c_1 = 0$, $c_2 = 2$ and $i = 1$, where $P_A(p^*) = 0.5$.

It is observed from this table that for the lots of good quality, the ASN of the plan decreases with increasing k. However, the situation is other way for poor quality lots, i.e. for $p = p_2$. Thus, the proposed plan is suitable to the lots of good quality for its quick disposal.

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REFERENCES

1. Sherman, R.E. Design and evaluation of repetitive group sampling plan. *Technometrics*, 1965, **7**, 11-21.

2. Soundararajan, V. & Ramaswamy, M.M. Designing repetitive group sampling (RGS) plan indexed by AQL and LQL. *IAPQR Transactions*, 1984, **9**(1), 9-14.

3. Singh, H.R.; Shankar, G. & Mohapatra, B.N. GERT analysis of repetitive group sampling plan. *Metron*, 1989, **47**(1-4), 339-49.

4. Shankar, G. & Mohapatra, B.N. GERT analysis of conditional repetitive group sampling plan. *IJQRM*, 1993, **10**(2), 50-62.

5. Mohapatra, B.N. Certain application of GERT network analysis in statistical quality control, Chapter III. Pt. Ravishankar Shukla University, Raipur, 1992. PhD Thesis.

6. Stephens, K.S. & Dodge, H.F. Two-stage chain sampling inspection plans with different sample sizes in the two stages. *J. Qual. Technol.*, 1976, **8**(4), 209-24.

7. Pritsker, A.A.B. & Happ, W.W. Graphical evaluation and review technique, Part I: Fundamentals. *J. Ind. Engg.*, 1966, **17**, 267-74.

8. Pritsker, A.A.B. & Whitehouse, G.E. Graphical evaluation and review technique, Part II: probabilistic and industrial engineering applications. *J. Ind. Engg.*, 1966, **17**, 293-01.

9. Whitehouse, G.E. & Pritsker, A.A.B. Graphical evaluation and review technique, Part III: Further statistical results: Counters, renewal times and correlations. *AIIE Transactions*, 1969, **1**, 45-50.

10. Shankar, G. GERT methods used in quality control. *Eco. Qual. Contr.*, 1993, **8**(4), 209-24.

11. Mason, S.J. Some properties of signal flow graphs. *Proceedings IRE*, 1953, **41**(9), 1144-156.

12. Shankar, G. GERT analysis of repetitive group sampling plan : Two-phase inspection. *Sankhya*, Series B, 1996, **58**(3), 431-38.

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