Optimal Decision Procedure to Declare Human Fatigue and Prediction of 3-Mean Repair Times with One Repairman

Dr V.S. Srinivasan

Ex-Scientist, Naval Physical & Oceanographic Laboratory, Kochi - 682 004

ABSTRACT

This paper gives an optimum procedure to decide whether fatigue is present in a repairman and if it is true prediction of the 3-mean repair times of the three failed systems.

Keywords: Mean repair times, optimal decision procedure, human fatigue

NOMENCLATURE

- MRT Mean repair time
- A, B, C Three support systems for which MRTs are needed.
- η_1, η_2 Multiplying constant for t_3

 α Type I error

 β Power function

 t_1, t_2, t_3 3-MRTs of support systems A, B, C, respectively

 Z_b A random variable which takes values between 1 or 0, depending on whether fatigue of repairman exists or not.

$$S = \sum_{1}^{M} Z_{b}$$

S

M Total number of MRTs for which repair completions are over, i.e., M = 3

 H_0 Null hypothesis

 H_1 Alternate hypothesis

1. INTRODUCTION

Consider a multisystem model (N + 3), where N is the number of systems, in the major system which has three additional systems called support systems. These support systems are denoted by A, B and C. The repair of A, B and C is taken by a single repairman in the order A, B, C. The mean repair completion times for the three support systems A, B and C (3-MRTs) are denoted by symbols t_1 , t_2 and t_3 . The human fatigue problems enter the 3-MRT process since there is one repairman only. In such cases, it is easy to think and formulate the whole problem as problems in the decision-making¹ and decision procedures.

2. DECISION-MAKING PROBLEM

The decision-making problem is formulated as

- H_0 No fatigue exists in the repairman
- H_1 Fatigue exists in the repairman, or in other words $t_1 < t_2 < t_3$

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2.1 Decision Procedure

- Step 0 Choose α (type I error)
- Step 1 If $t_1 < t_2 < t_3$, then $Z_b = 1$, otherwise $Z_b = 0$

Step 2 Compute
$$S = \sum_{b=1}^{M} Z_b$$

- Step 3 Put M = 1, 2, 3 in summation S successively
- Step 4 Determine k, such that $binf(k;1/6,3) = \alpha$
- Step 5 Reject H_0 iff S = 2 and if $S \neq 1$.

2.2 Analysis

Define
$$h(t_1) = \int_{t_1}^{\infty} dt_2 \int_{t_2}^{\infty} dt_3 \exp f(t) \exp f(t)$$
 (1)

It is to be remembered that Eqn (1) is true regardless of H_0 and H_1 .

It can be easily shown that

$$P_{0} = Pr \{Z_{b} = 1; \text{ regardless of } H_{0}, H_{1}\}$$
$$= E \{h(t_{1})\}$$
(2)

where E is the expected value.

Now

$$Pr\left\{S=r; H_0 \text{ is true}\right\} = binm(r; 1/6, 3) \tag{3}$$

$$Pr\left\{S=r;H_{1} \text{ is true}\right\} = binm(r;P_{1},3)$$
(4)

It can easily be proved that $P_1 < 1/6$ under H_1 .

Hence, from Neyman-Pearson lemma, decision procedure [Eqns (1) to (4)] is optimum. The performance of decision rule [Eqn (3)] is evaluated for two illustrations. The power function is:

$$\beta = binf(3; P_1; 3) \tag{5}$$

The Eqn (5) can be illustrated for two cases by putting the expression for

$$F(t_1) = F(t_3 - 2\Delta)$$

$$F(t_2) = F(t_3 + \Delta)$$

$$F(t_3) = \exp f(t)$$

The second illustration can be given for

$$F(t_{1}) = F(\eta_{1} t_{3}), \ \eta_{1} < 1$$

$$F(t_{2}) = F(\eta_{2} t_{3}), \ \eta_{2} < 1$$

$$F(t_{3}) = \exp f(t)$$

3. CONCLUSION

Two problems have been analysed. The first problem is a decision procedure to decide whether human fatigue is present in the repairman, when M = 3. By setting probabilities P_0 , P_1 and P_2 equal to t_1 , t_2 and t_3 , respectively, one can easily predict t_1 , t_2 and t_3 (3-MTRs) after deciding that there is human fatigue in the repairman.

Type I error and M as $\alpha = 0.05$ and M = 3, respectively can be chosen. The value of k in the relation $S \le k$ should be chosen as one among any values for k = 2 or k = 3.

REFERENCES

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