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# Segmentation of Colour Images by Modified Mountain Clustering

Devendra Jha

*Scientific Analysis Group, Delhi* – 110 054

and

M. Hanmandlu

*Indian Institute of Technology, Delhi* – 110 016

**ABSTRACT**

Segmentation of colour images is an important issue in various machine vision and image processing applications. Though clustering techniques have been in vogue for many years, these have not been very effective because of problems like selection of the number of clusters. This problem has been tackled by having a validity measure coupled with the new clustering technique. This method treats each point in the dataset, which is the map of all possible colour combinations in the given image, as a potential cluster centre and estimates its potential wrt other data elements. First, the point with the maximum value of potential is considered to be a cluster centre and then its effect is removed from other points of the dataset. This procedure is repeated to determine different cluster centres. At the same time, the compactness and the minimum separation is computed amongst all the cluster centres, and also the validity function as the ratio of these quantities. The validity function can be used to choose the number of clusters. This technique has been compared to the fuzzy C-means technique and the results have been shown for a sample colour image.

**Keywords:** Fuzzy C-means clusters, computer vision, fuzzy Gaussian function, fuzzy image processing, membership function, validity criteria, mountain clustering

## 1. INTRODUCTION

Segmentation of images is an important task required in many fields. For example, segmentation of satellite images is useful for geographical information system (GIS) as this helps in planning the activities in the development of resources, study of changing environment, and observing the impact of disasters. The basis of segmentation may be on the image properties, such as colour or texture[1], or both. Mostly, colour can be used in the segmentation of natural scenes and textural features have been proved to be powerful in the segmentation of tissues in the biomedical imaging area. The perfect segmentation has eluded the researchers, still forcing them to try alternate approaches.

Segmentation of gray images has been widely dealt with in the literature involving stochastic methods[2]. However, an attempt has been made to use the colour property in this study. In view of wide acceptability and facility of fuzzy approach[3], attention has been mainly on these approaches for the segmentation of colour images[4]. Some of the important contributions are the fuzzy C-means

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approach[5] and robust clustering[6]. However, the mountain clustering of Yager and Filev[7] has been followed but the same has been for increased efficiency and adaptability to the colour imagery in the lines of Azeem[8,9], *et al.*

## 2. MODIFIED MOUNTAIN CLUSTERING

The purpose of clustering is to do natural groupings of a large set of data, producing a concise representation of system's behaviour. Yager and Filev proposed a simple and easy to implement, mountain-clustering algorithm for estimating the number and location of cluster centres. Their method is a grid-based, three-step procedure. In the first step, the hyperspace is discretised with a certain resolution in each dimension so that grid points are obtained. The second step uses the dataset to construct the mountain function around all grid points. The third step generates the cluster centres by an iterative destruction of mountain function. Though this method is simple, the computation grows exponentially with the dimension of hyperspace. In the $n$ dimensional hyperspace with $m$ number of grid lines in each dimension, the number of grid points that must be evaluated are $m^n$.

A modified form of Yager and Filev's method, as reported by Azeem[8,9], *et al.* is presented. An image matrix $X$ with arbitrary $l$ colour levels, is converted into a vector $X$. Also, each component of the image vector $X$ is represented by the R, G, B values of the pixel, the value of hue corresponding to each colour and the frequency of occurrence of these colours. It has been assumed that each data point (pixel), which in this case is represented by five dimensions each, has potential to become a cluster centre instead of grid points.

This modification makes the computation complexity independent of the dimension, because the number of grid points is equal to the number of data points. The second advantage of this modification is that it eliminates the need to specify a grid resolution, in which a compromise between accuracy and computational complexity must be struck. The procedure of the modified method is as follows:

Let the $j^{th}$ data in $X_p$ hyperspace be defined as

$$x_{jp} \equiv \{x_j\} = \{x_1(j),\ x_2(j),..,\ x_n(j)\}$$
$$\forall j = 1,....., M; \quad p = 1,\ 2,....n \qquad (1)$$

Without loss of generality, each dimension of the hyperspace has been normalised, so that data points are bounded by hypercube. The normalised data points are defined as

$$\bar{x}_{jp} \equiv \left\langle x_{jp} - \left(x_{jp}\right)_{\min}\right\rangle . / \left\langle \left(x_{jp}\right)_{\max} - \left(x_{jp}\right)_{\min}\right\rangle$$
$$\forall j = 1,\ldots,M \qquad (2)$$

where

$$\left(x_{jp}\right)_{\min} = \left\{\min_{j=1}^{M} x_1,\ \min_{j=1}^{M} x_2,\ \ldots,\ \min_{j=1}^{M} x_n\right\} \qquad (3)$$

and

$$\left(x_{jp}\right)_{\max} = \left\{\max_{j=1}^{M} x_1,\ \max_{j=1}^{M} x_2,\ \ldots,\ \max_{j=1}^{M} x_n\right\} \qquad (4)$$

Treating each data point as a cluster centre, a measure of potential is defined, which is a mountain function, of data point $\bar{x}_{rp}$ as a function of distance $d^2\left(\bar{x}_{rp},\bar{x}_{jp}\right) = \left(\bar{x}_{rp} - \bar{x}_{jp}\right)Q\left(\bar{x}_{rp} - \bar{x}_{jp}\right)'$ between $\bar{x}_{rp}$ and all other data points given as

$$P_{r,1} = \sum_{j=1}^{M} \exp\left[-\left(\frac{d^2\left(\bar{x}_{rp},\bar{x}_{jp}\right)}{d_1^2}\right)\right] \qquad \forall r = 1,2,\ldots,M \ (5)$$

where, $Q$ is a $(n + 1) \times (n + 1)$ positive definite matrix and $d_1$ is the positive constant defining the neighbourhood of data point. Data points outside radial distance $d_1$ have a little influence on the potential value. It is evident from the mountain function that the potential value of datum is an approximation of the density of data point (cardinality) in the vicinity of datum. The higher the potential value of each data point in hypercube, the higher the chances the data point of being a cluster centre. The first cluster centre is selected with the highest value of $P_{r,1}$ as follows:

$$\bar{c}_{1p} = \bar{x}^*_{1p} \Leftarrow P^*_1 = \max_{r=1}^{M}(P_{r,1}) \tag{6}$$

For the selection of second cluster centre, the potential value of each data point is revised in order to deduce the effect of mountain function around the first cluster centre as follows:

$$P_{r,2} = P_{r,1} - P^*_1 \exp\left[-\left(\frac{d^2(\bar{x}^*_{rp}, \bar{c}_{1p})}{d_2^2}\right)\right] \quad \forall r = 1, 2, \ldots, M \tag{7}$$

where, $d_2$ is the positive constant defining the neighbourhood of cluster centre. It is evident from Eqn (5) that the data points near the first cluster centre have greatly reduced potential value and are unlikely to be selected as the next cluster centre. After revision of potential value of each data point, second cluster centre is selected with the highest value of $P_{r,2}$ as

$$\bar{c}_{2p} = \bar{x}^*_{2p} \Leftarrow P^*_2 = \max_{r=1}^{M}(P_{r,2}) \tag{8}$$

Similarly, for the selection of $k^{th}$ cluster centre, revision of potential value for each data point is done as

$$P_{r,k} = P_{r,k-1} - P^*_{k-1} \exp\left[-\left(\frac{d^2(\bar{x}_{rp}, \bar{c}_{(k-1)p})}{d_2^2}\right)\right] \quad \forall r = 1, 2, \ldots, M \tag{9}$$

and $k^{th}$ cluster centre is selected with the highest value of $P_{r,k}$ as

$$\bar{c}_{kp} = \bar{x}^*_{kp} \Leftarrow P^*_k = \max_{r=1}^{M}(P_{r,k}) \tag{10}$$

To stop this procedure, Yager and Filev have used the criterion $P_k^*/P_1^* < \delta$ ($\delta$ is a small fraction). The choice of $\delta$ affects the results. Small value of $\delta$ results in a large number of cluster centres and the large value of $\delta$ results in less number of cluster centres. It is difficult to establish a single value for $\delta$ that works well for all data. To overcome this difficulty, a gray region of $\delta$ value bounded by two limits, $\delta_u$ and $\delta_l$ is used. The upper limit $\delta_u$ is the threshold for absolute acceptance of cluster centre and the lower limit $\delta_l$ is the threshold for complete rejection and the end of clustering process. In the gray region, a good trade-off between reasonable potential value and sufficient distance from the existing cluster centre is used to accept a data point as a cluster centre as

$$\frac{P^*_k}{P^*_1} + \frac{d_{\min}}{d_1} \geq 1 \tag{11}$$

where, $d_{\min}$ = Minimum distance between $\bar{c}_{kp}$ and all cluster centres previously selected.

The optimum number of clusters is decided by the validity functions which is the ratio of compactness and separation[10] and is represented as

$$S = \frac{\sum_{k=1}^{m}\sum_{r=1}^{M} \mu^2_{r,k} \left\|\bar{x}_{rp} - \bar{c}_{kp}\right\|^2}{M \min_{i \neq j} \left\|\bar{c}_{ip} - \bar{c}_{jp}\right\|^2} \tag{12}$$

where the membership function $\mu_{r,k}$ represents the degree of association of $r^{th}$ data to the $k^{th}$ cluster centre and is defined as

$$\mu_{r,k} = \exp\left[-\left(\frac{d^2(\bar{x}_{rp} - \bar{c}_{kp})}{d_2^2}\right)\right] \tag{13}$$

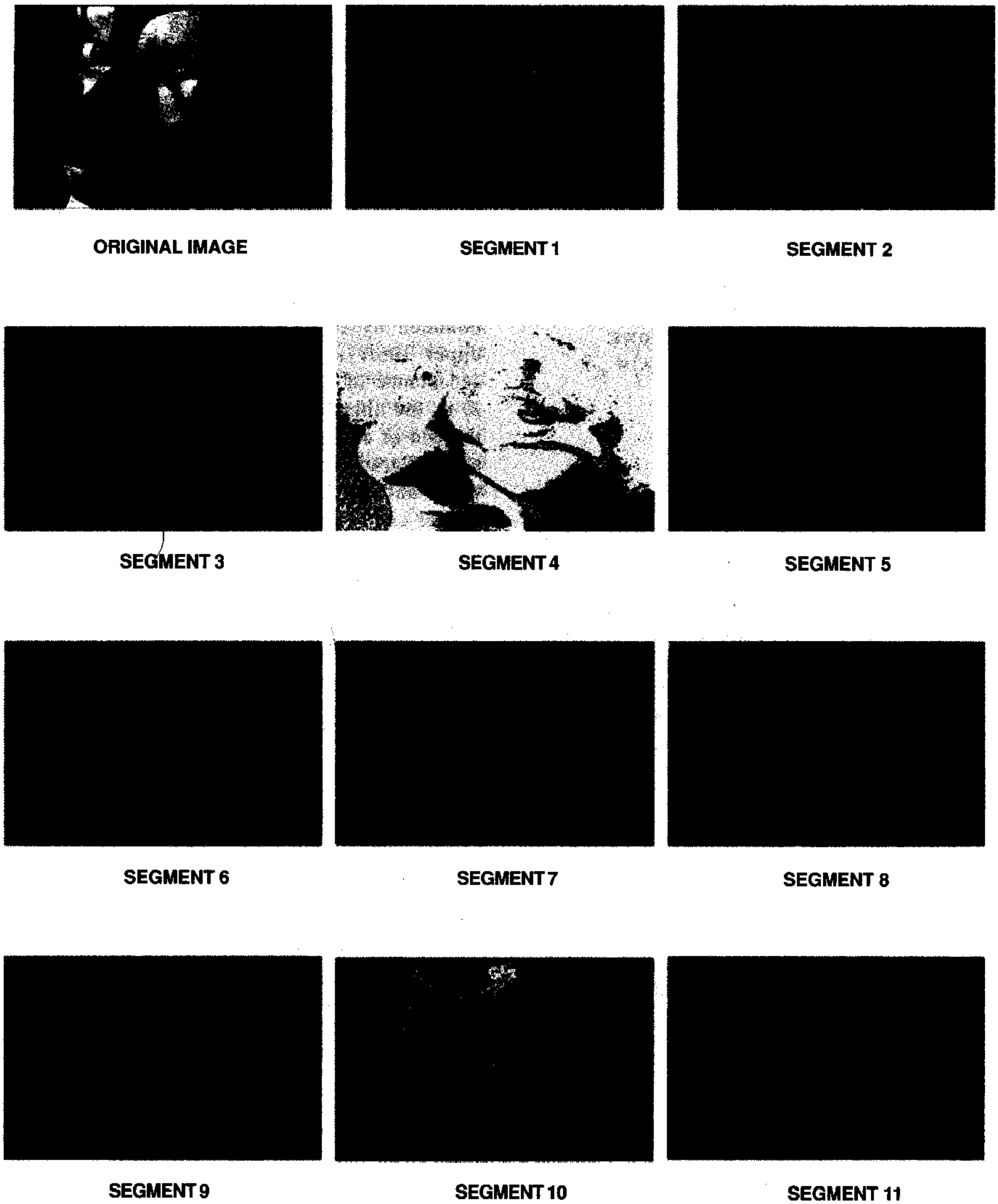


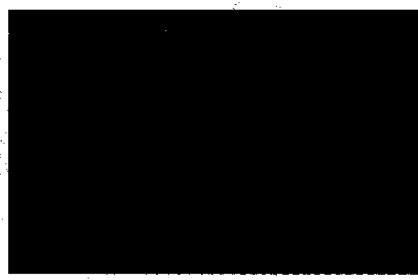

**Figure 1(a). Results using modified mountain clustering**

SEGMENT 1

SEGMENT 2

SEGMENT 3

SEGMENT 4



SEGMENT 5

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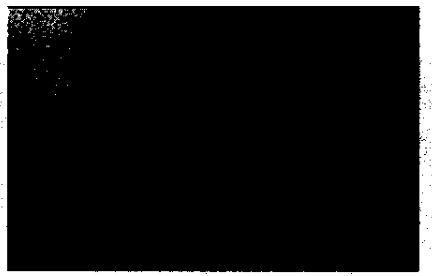

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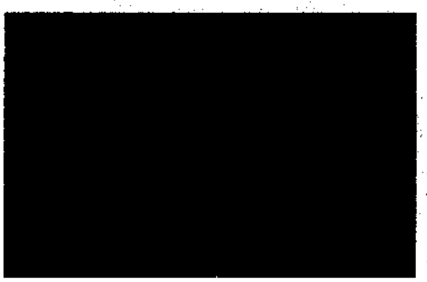

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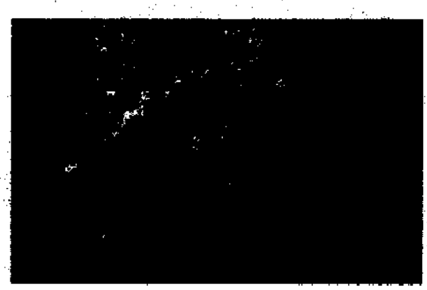

SEGMENT 11

**Figure 1(b). Results for fuzzy C-mean clustering**

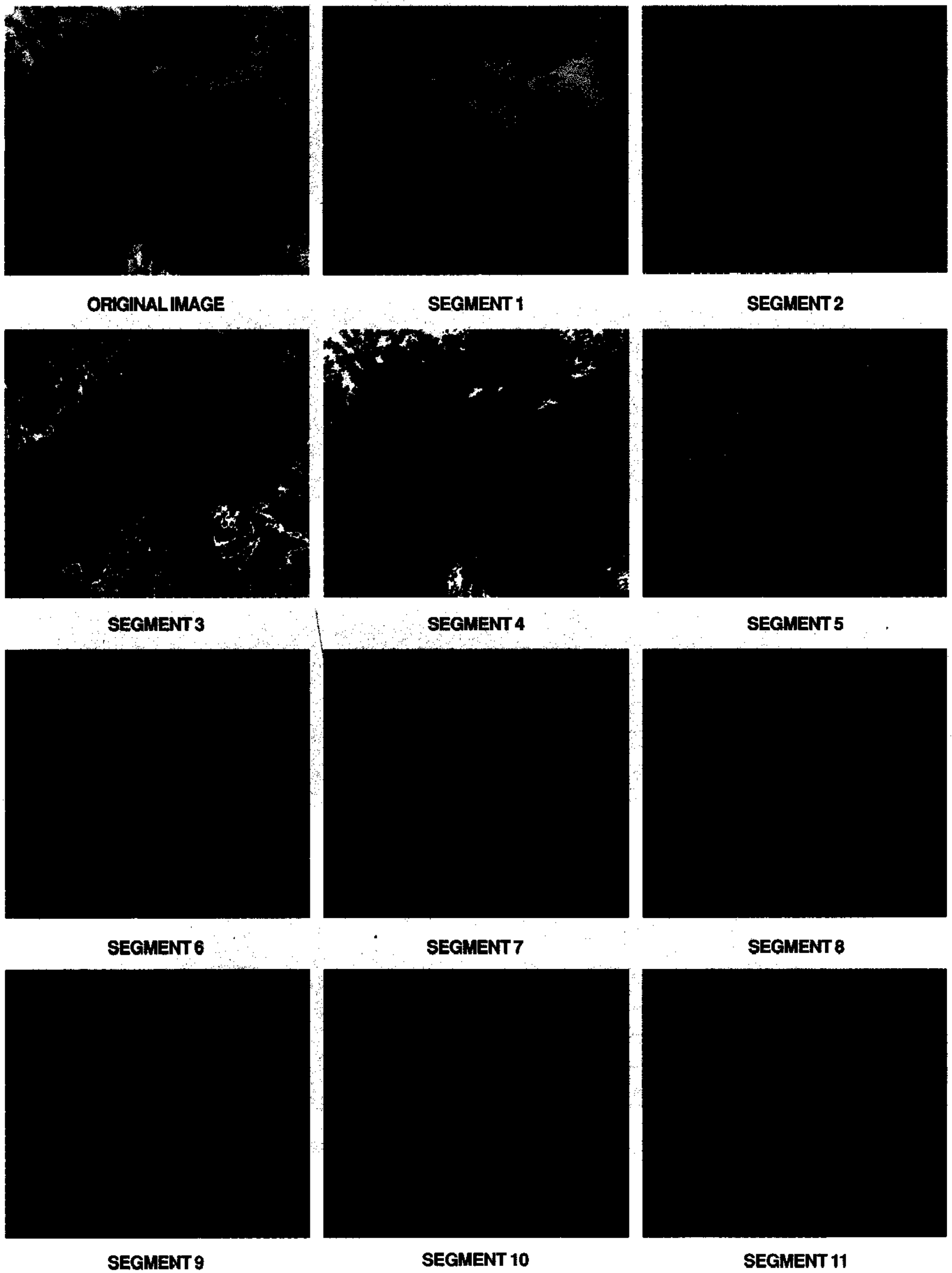


**Figure 2(a). Results using modified mountain clustering**

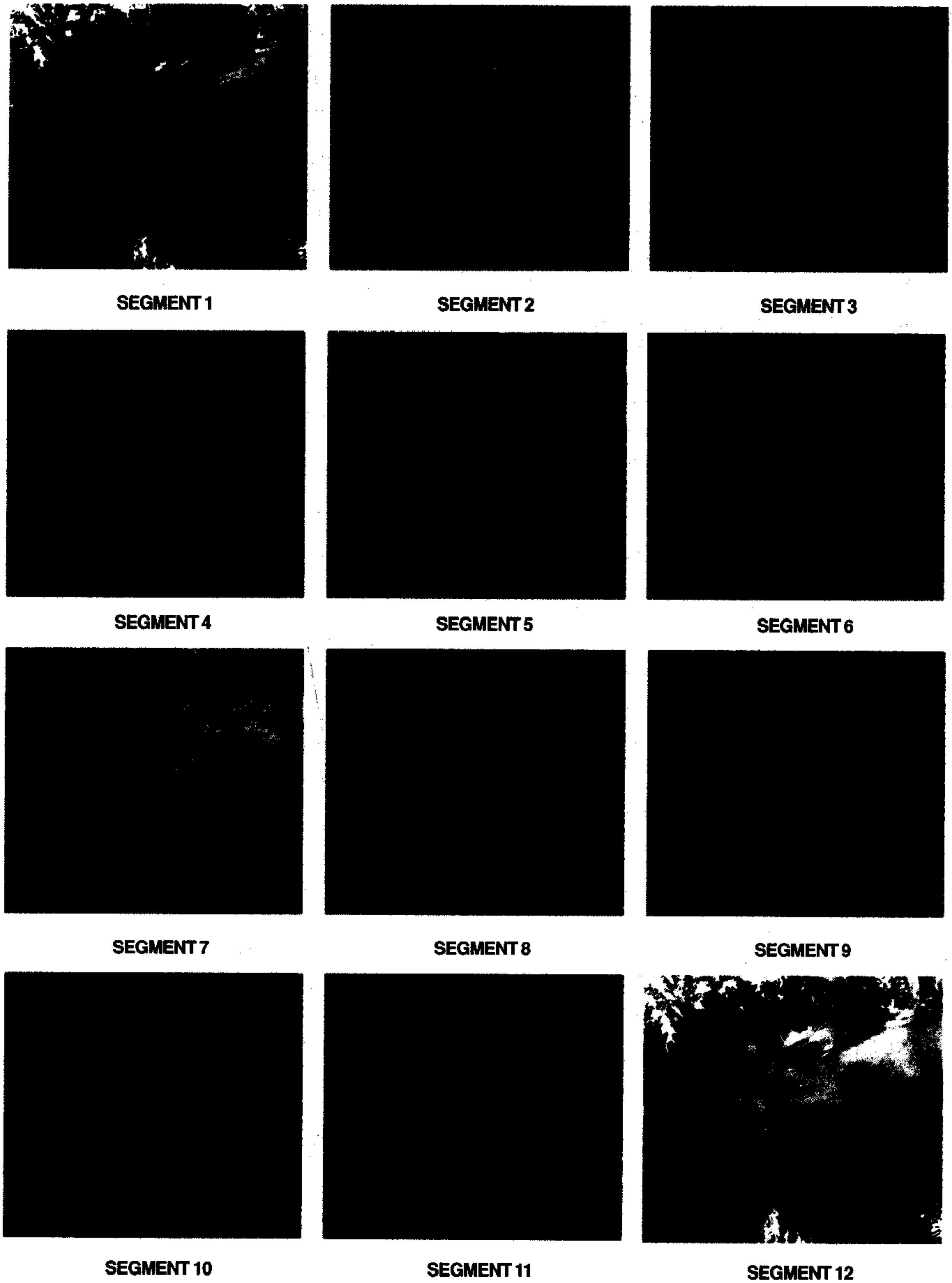


**Figure 2(b). Results using fuzzy C-means clustering**

Let $\Omega_c$ denotes the optimal candidate at any value of $c$; then the solution of

$$\min_{c_{\min} \le c \le c_{\max}} \left( \min_{\Omega_c} S \right) \quad (14)$$

is assumed to yield the most valid fuzzy clustering of the dataset. The validity function $S$ has a tendency to decrease eventually when the number of cluster centres is very large. So, the value of $S$ is meaningless when the number of cluster centres gets close to $M$. Since in practice, the feasible number of clusters is much smaller than the number of data points $M$, there is a reason to select $d_{1,\min} = 0.2 d_{1,\max}$.

Optimal $S$ candidate corresponds to $\bar{c}^*_{kp}$ and $d^*_2$. The de-normalised value of $\bar{c}^*_{kp}$ is

$$c^*_{kp} = \left[ \bar{c}^*_{kp} \cdot * \left\{ \left(x_{jp}\right)_{\max} - \left(x_{jp}\right)_{\min} \right\} \right] + \left(x_{jp}\right)_{\min} \quad (15)$$

where

$$c_{kp} = \left\{ c^k_p \right\} = \left\{ c^k_{1,p}, c^k_{2,p}, \ldots, c^k_{n,p} \right\}$$

The initial width of each membership function on each dimension of hyperspace is determined from the value of $d^*_2$ as

$$\sigma^*_{kp} = \left[ d^*_2 \cdot \left\{ \left(x_{jp}\right)_{\max} - \left(x_{jp}\right)_{\min} \right\} \right] \quad (16)$$

where,

$$\sigma_{kp} = \left\{ \sigma^k_p \right\} = \left\{ \sigma^k_{1,p}, \sigma^k_{2,p}, \ldots, \sigma^k_{n,p} \right\}$$

## 3. RESULTS & DISCUSSION

The algorithm described has been applied on two images, a 200 × 320 pixel clown image with 81 colour levels, shown in Figs 1(a) and 1(b) and a 340 × 320 pixel pseudocoloured satellite image with 256 colour levels, shown in Figs 2(a) and 2(b) are used as test images. During the application, matrix $Q$ in Eqn (5) has been considered to be an identity matrix, making the distance $d^2\left(\bar{x}_{rp}, \bar{x}_{jp}\right)$ Euclidean in nature. The value of $d_1$ was taken as 0.15 and $d_2$ as 0.23. Though the effect of $d_1$ is on mountain function which makes the two potential values closer or farther numerically, the effect of $d_2$ is noticed in terms of number of significant clusters. A large value of $d_2$ makes membership larger, hence more pixels are grouped into one cluster. Eventually, the number of clusters decrease with increase in the value of $d_2$.

While grouping the colour levels into various clusters, only a quarter of the cluster centres initially calculated were considered based on their corresponding potential values considered along with the user-defined validity criteria. By this, the number of clusters are found to be 11, which are sufficient for the reconstruction of the original image, almost entirely. These are shown as segments 1–11.

It is observed that when the modified mountain clustering technique is applied to test images, one obtains the most acceptable results for images in which colours are visibly more distinct. The results of modified mountain clustering have been compared with those obtained by fuzzy C-means clustering. Some clusters are found identical to fuzzy C-means clustering methods, when numbers of clusters are taken as 11. The validity measure for the clusters of satellite image are given in Tables 1 and 2. Also, the normalised cluster centres are listed in the tables.

It is observed that in modified mountain clustering, clusters are identified one after another. Some of the segments in modified mountain clustering are more prominent in colour. In fuzzy C-means clustering, one also has similar clusters, but these are dim. This is because replicas of the same clusters with less intensity also occur. This fact can be used to assert that with the modified mountain clustering, one can choose the distinct number of clusters. If one wants to have more number of clusters, then inclusion of one or more new clusters does not pose any problem in the proposed approach, whereas in fuzzy C-means clustering, the clustering has to be done all-over again. For small images, the fuzzy C-means (with *a priory* known number of clusters) is better.

**Table 1. Normalised cluster centres and validity with modified mountain clustering for satellite image**

| Segment No. | R | G | B | Freq. of intensity | Validity ($\times 10^{-5}$) |
|---|---|---|---|---|---|
| 1 | 0.7031 | 0.2891 | 0.1250 | 0.0010 | 0.49 |
| 2 | 0.8672 | 0.5781 | 0.2891 | 0.0008 | 0.53 |
| 3 | 0.5781 | 0.4141 | 0.2891 | 0.0017 | 1.83 |
| 4 | 0.8672 | 0.4141 | 0.1250 | 0.0015 | 1.97 |
| 5 | 0.4141 | 0.1250 | 0.0004 | 0.0025 | 1.51 |
| 6 | 0.2891 | 0.1250 | 0.1250 | 0.0025 | 2.10 |
| 7 | 0.7031 | 0.5781 | 0.2891 | 0.0002 | 2.06 |
| 8 | 0.7031 | 0.1250 | 0.0006 | 0.0005 | 2.03 |
| 9 | 0.4141 | 0.2891 | 0.2891 | 0.0002 | 2.43 |
| 10 | 0.8672 | 0.7031 | 0.4141 | 0.0003 | 2.55 |
| 11 | 0.5781 | 0.2891 | 0.1250 | 0.0017 | 2.81 |

**Table 2. Normalised cluster centres and validity with fuzzy C-means clustering for satellite image**

| Segment No. | R | G | B | Freq. of intensity | Validity ($\times 10^{-5}$) |
|---|---|---|---|---|---|
| 1 | 0.9710 | 0.9300 | 0.8495 | 0.0006 | 0.10 |
| 2. | 0.1226 | 0.0473 | 0.0487 | 0.0045 | 0.43 |
| 3 | 0.3621 | 0.1104 | 0.0549 | 0.0021 | 2.61 |
| 4 | 0.6770 | 0.1093 | 0.0380 | 0.0006 | 4.27 |
| 5 | 0.6999 | 0.4025 | 0.2722 | 0.0007 | 4.51 |
| 6 | 0.4150 | 0.3326 | 0.3088 | 0.0003 | 3.21 |
| 7 | 0.9259 | 0.5535 | 0.0958 | 0.0007 | 3.64 |
| 8 | 0.6688 | 0.5811 | 0.4230 | 0.0006 | 2.23 |
| 9 | 0.9221 | 0.5907 | 0.3519 | 0.0005 | 2.38 |
| 10 | 0.6974 | 0.3231 | 0.0963 | 0.0012 | 5.75 |
| 11 | 0.8836 | 0.7365 | 0.6225 | 0.0008 | 3.47 |

However, for clustering of a large image without any knowledge of clusters, the modified mountain clustering method is a better option.

## 4. CONCLUSIONS

This paper presents a new clustering approach called modified mountain clustering for the segmentation of images. The main concept is that a mountain function is defined for each element of the dataset, i.e., a set of all possible colours in the given image, which forms a potential cluster, and the strength of this function is calculated as a function of distance of neighbouring elements. On the basis of the strength it is declared as a cluster and the effect of this is removed on all other data elements. Next, another element is chosen as next potential cluster centre. This procedure is repeated until a validity criterion comprising a ratio of compactness of the clusters to the separation among the clusters is violated. The results are comparable to the results of fuzzy C-means technique but are computationally much more efficient.

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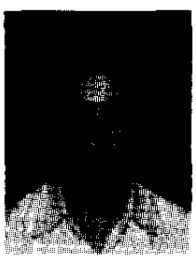
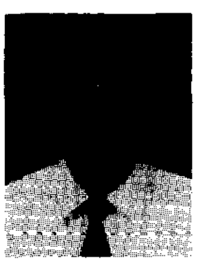
**Contributors**



**Mr Devendra Jha** received his ME from the Delhi College of Engineering in 1997. Presently, he is working as Sci D at the Scientific Analysis Group (SAG), Delhi. Currently, he is looking after the simulation lab activities pertaining to the analysis of cryptosystems. His research interests include: System analysis and simulation, fuzzy signal processing, and fuzzy clustering for signal analysis.



**Dr M Hanmandlu** received his MTech from the Regional Engineering College, Warangal, Jawaharlal Technological University, in 1976 and PhD from the Indian Institute of Technology (IIT), New Delhi, in 1981. Presently, he is working as Professor in the Faculty of Engineering, Multimedia University, Cyberjaya, Malaysia. His research interests include: Computer vision, fuzzy logic applications in image processing and document processing, fuzzy modelling, and control of dynamic systems.