Defence Science Journal, Vol. 52, No. 2, April 2002, pp. 211-219


# Modelling Effectiveness of Machine Gun Fire

D. Dutta and S. Sabharwal
*Institute for Systems Studies & Analyses, Delhi*–110 054

**ABSTRACT**

Machine gun is an effective infantry weapon which can cause heavy damage to enemy targets, if sited in a tactically favourable position. It can be engaged effectively against both static and moving targets. The paper deals with the determination of target vulnerability under effective machine gun fire considering relevant tactical parameters, eg, target aiming point, trajectory of fire, sweep angle, target frontage, posture, direction of attack, etc.

**Keywords:** Machine gun fire, modelling, system effectiveness, target engagement, target vulnerability, area fire, tracking fire mode, fixed line fire mode, searching fire mode, traversing fire mode

## 1. INTRODUCTION

A machine gun is an effective infantry weapon and can be engaged against both personnel and soft-skin targets like jeeps or trucks, etc. It may cause heavy damage to enemy targets, if sited in a tactically suitable position. The firing modes of a machine gun can be categorised[1] mainly into: (i) fixed line fire, (ii) searching fire, (iii) traversing fire, and (iv) area fire, i.e., combination of both searching fire and traversing fire. Besides these, it can also be used in tracking fire mode. In this mode of firing, if a firer detects a moving target, he tracks the target and continues firing. In fixed line firing mode, a machine gun continues to fire at a specified range and bearing. It may be engaged against a point target, eg, major weapon positions like RCL, RL, antitank missile or a trench, etc. or against an area target like an infantry column passing through a route.

Area firing takes place when the firer wants to cover a wider area, either to engage an area target or to cover intended enemy locations, as the fire distribution covers both width and depth. An area target is considered here as a group of point targets, eg, a deployed section position or a moving platoon, etc. In searching fire mode, firing is distributed in depth by successive shifting of elevation of the gun. In traverse fire mode, firing is distributed in width by successive shifting in the bearing of the gun, i.e., firing sweeps from one direction (minor arc of fire) to another (major arc), as shown in Fig.1.

Effectiveness of a system is a quantitative measure of the degree to which the system performs an assigned task under a specific set of conditions[2]. Various criteria can be used as measure of effectiveness, eg, fractional target damage, a total number of targets destroyed/killed or in terms of cost, etc. The objective of this study is to determine machine gun fire effectiveness considering a total number of target attritions inflicted due to machine gun fire. Because of the dynamic nature of combat, the attrition pattern may change with time and therefore time-dependent attrition rate is

Revised 28 August 2001

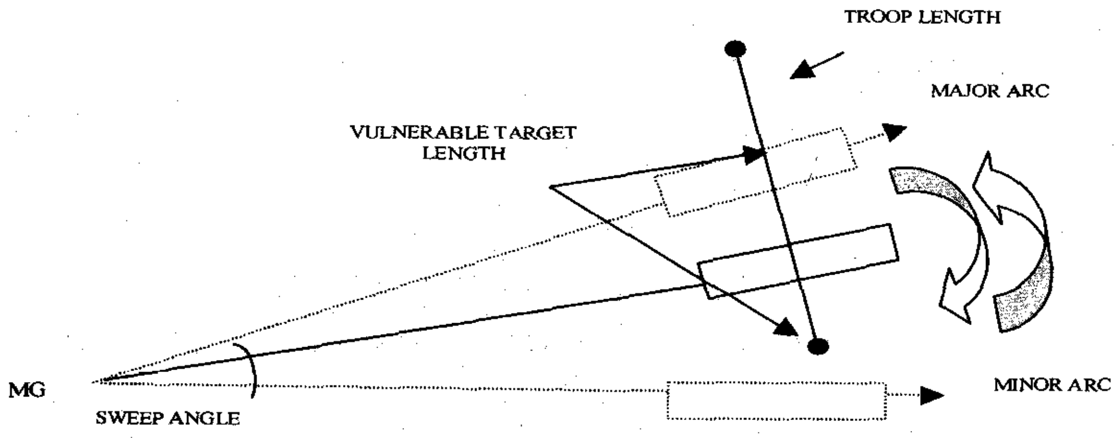


**Figure 1. Sweep fire of machine gun**

determined. Overall attrition of targets due to the machine gun fire at different time intervals can be obtained once the attrition rate is known.

## 2. VULNERABILITY OF TARGET

When a machine gun fires burst of rounds, vibration of the gun, variation of ammunition and changes occurring in the atmospheric conditions cause dispersion of rounds both along and across the lines of fire. Each round traces a trajectory differing from that of the others and the resultant group of trajectories when strike the ground forms a long elliptical pattern known as beaten zone. In a beaten zone, rounds are concentrated around the mean point of impact and the density gradually decreases towards the boundary of this zone. Only that part of the beaten zone where 85 per cent of the rounds fall is considered effective and is known as effective beaten zone[1]. For calculation purposes, effective beaten zone has been considered as total beaten zone and it has been further assumed that this area is rectangular.

Depending upon the nature of the trajectories, there may appear three zones of interest around the target aiming point, namely: (i) beaten zone, (ii) danger zone, and (iii) safety zone (Fig. 2). The danger zone is the area where the trajectory height is less than that of the target. A target is vulnerable in both beaten zone and danger zone. A safety zone is the area between the beaten zone and the danger zone where trajectories of the bullets are above the target height and hence, the targets are

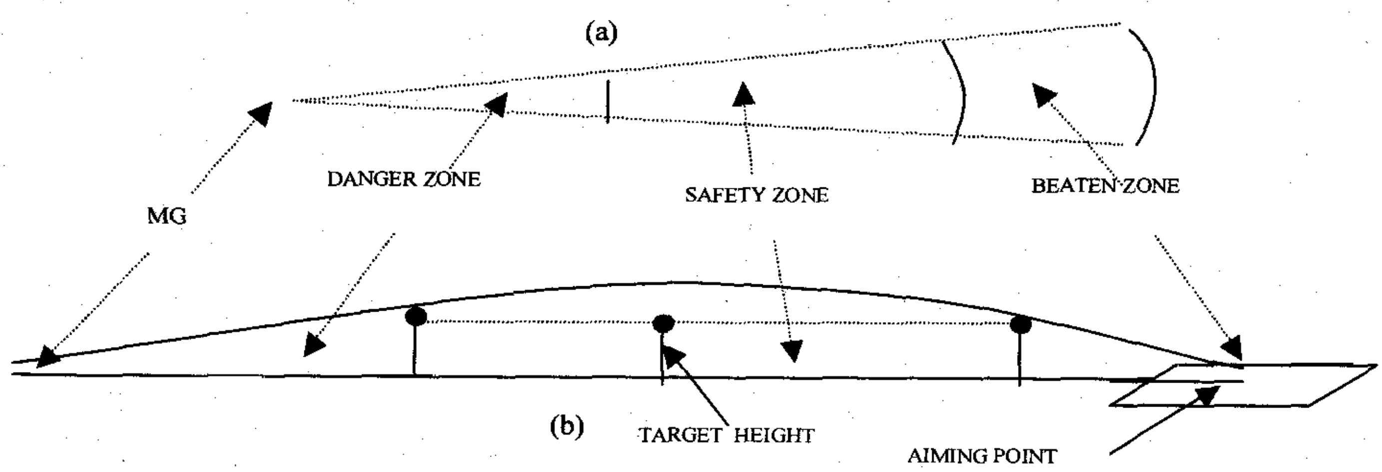


**Figure 2. Cross-sectional diagram of beaten zone, (a) horizontal view, and (b) vertical view**

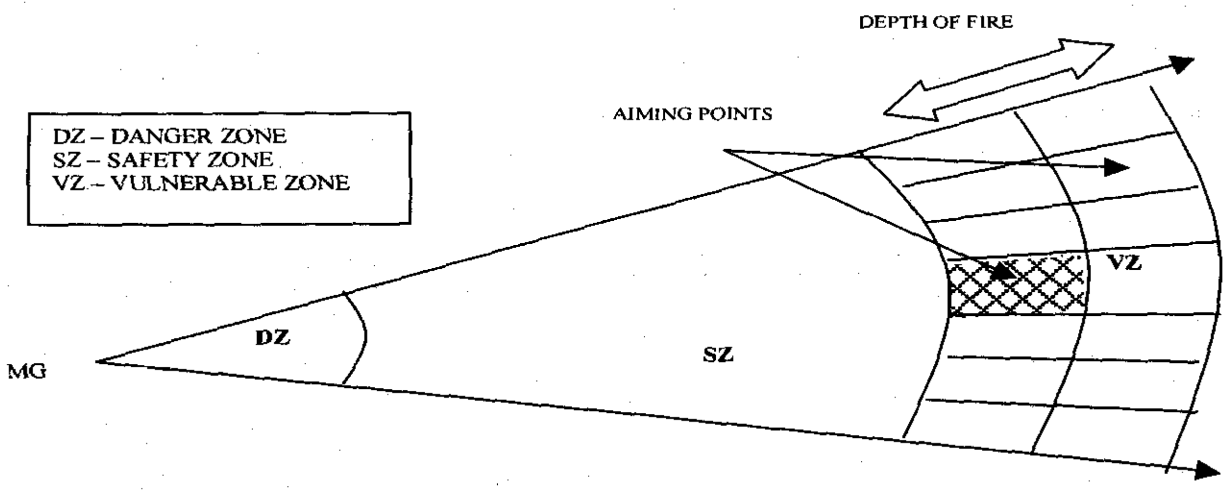


**Figure 3. Fire distribution covering a wider area**

not vulnerable in this zone. For a shorter aiming range, the trajectory remains almost flat and hence, there is no safety zone in that case.

For modelling purposes, area firing mode is considered as a general case as all other cases can be derived from it as special cases. The aggregation of the beaten zones corresponding to each of the aiming points constitutes the vulnerable zone for area fire (Fig. 3.).

## 3. ASSUMPTIONS

The following assumptions have been considered in the model:

(a) Rounds are falling uniformly within the beaten zone area

(b) Rate of machine gun fire is constant

(c) The formation length of the moving unit is constant and the targets within it are distributed uniformly

(d) Shifting of machine gun aiming from one point to another is instantaneous.

## 4. INPUT PARAMETERS

The following input parameters have been considered in the model for the machine gun and target:

- **Machine Gun**

(a) Rate of fire

(b) Beaten zone area

(c) Location of machine gun

(d) Major and minor arcs of fire

(e) Target aiming point

(f) Depth of fire

(g) Maximum effective range of fire.

- **Target**

*For Point Target*

(h) Location

(i) Exposed area of an individual target (height × width).

*For Area Target*

(j) Centre location

(k) Target frontage

(l) Target depth

(m) Force strength, i.e., number of point targets within the area

(n) Exposed area of an individual target (height × width).

*For Moving Target*

(o) Centre location

(p) Force strength, i.e., number of point targets within the area

(q) Exposed area of an individual target (height × width)

(r) Formation type (line/assault)

(s) Length of the target

(t) Speed of movement

(u) Direction of move.

## 5. METHODOLOGY

The effectiveness of a machine gun fire is represented here in terms of attrition rate, which is the number of casualties caused by a machine gun per unit of time. When the target is outside the maximum effective range of fire or outside the arc of fire, the attrition rate is zero. For a moving troop, with the given troop location and direction of movement, coordinates of the front and rear edges of the troop formation length are determined. It is then checked whether the troop is within the vulnerable area fully or partially, and then the expected vulnerable length of the troop is determined. Vulnerability of the static area target depends upon the intersection of the target area and the vulnerable area of machine gun fire which is proportional to the sweep angle and the depth of fire. The single-shot hit probability of a round is determined considering expected location of the target and trajectory of rounds. Cumulative kill probability of rounds fired by the machine gun is then calculated considering total rounds of fire.The attrition rate of a target $\alpha(t)$ at time $t$ due to machine gun fire is expressed (*Appendix A*) as

$$\alpha(t) = [1.0 - \{1.0 - p_{hit}(r)\}^{v}].\phi(t).\psi(t).S(t)$$

where

| | |
|---|---|
| $p_{hit}(r)$ | Single-shot hit probability wrt range $r(t)$ |
| $v$ | Rate of fire |
| $\phi(t)$ | Fraction of vulnerable zone effective at time $t$ |
| $\psi(t)$ | Fraction of total area occupied by targets and is vulnerable at time $t$ |
| $S(t)$ | Number of point targets within the group surviving at time $t$. |

### 5.1 Derivation of $\phi(t)$

As stated before, several beaten zones form a vulnerable zone. But at one instance, a machine gun is effective at only one beaten zone (shaded area in Fig. 3) within the vulnerable zone. Therefore, the fraction of vulnerable zone effective at time $t$ is given as

$$\phi(t) = \phi_1(t) \times \phi_2(t)$$

where

| | |
|---|---|
| $\phi_1(t)$ | $\theta(t)/(\varphi_2 - \varphi_1)$, i.e., fraction of sweep angle effective at time $t$ |
| $\phi_2(t)$ | Fraction of depth effective at time $t$ (= beaten zone length/depth of fire) |
| $\theta(t)$ | Angle subtended by machine gun at beaten zone width at range $r(t)$ |
| $\varphi_2$ | Major arc of fire |
| $\varphi_1$ | Minor arc of fire. |

Here, the angles $\theta$, $\varphi_1$ and $\varphi_2$ are considered in the horizontal plane. In the area fire mode model, if the sweep angle, i.e.,$(\varphi_2 - \varphi_1)$ is taken to be equal to $\theta$, then one has $\phi_1(t) = 1$ and it reduces to the model relating to searching fire mode. On the other hand, for fixed elevation of fire, i.e., no coverage of depth, $\phi_2(t) = 1$ and the model reduces to one relating to traversing fire mode. If both $\phi_1(t) = 1$ and $\phi_2(t) = 1$, i.e., $\phi(t) = 1$, then it becomes a fixed line firing mode model. Fixed line firing mode with changing aiming points converts the model further to tracking firing mode. It is to be noted that $\phi(t)$ can further be stated as the probability that a target inside the vulnerable zone

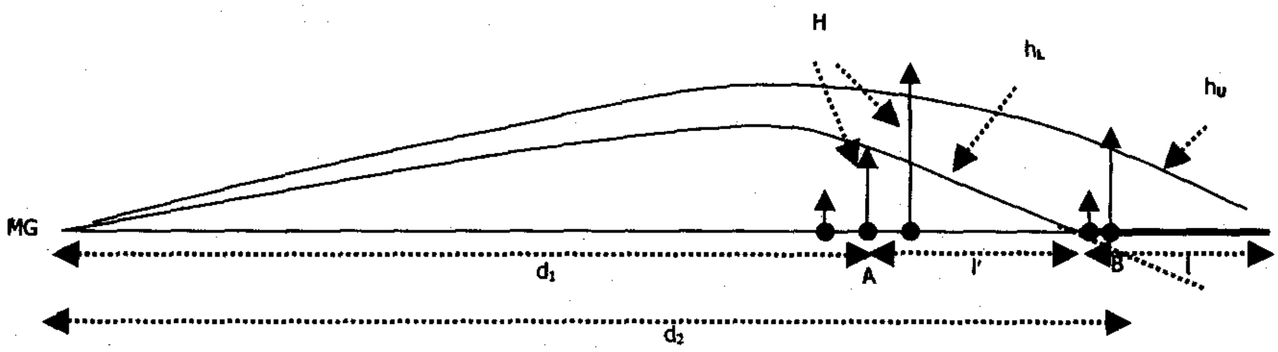


**Figure 4. Trajectory of bullets**

lies within the beaten zone (i.e., a target actually subjected to machine gun fire) at time $t$.

### 5.2 Derivation of $\psi(t)$

For a moving target, the fraction of troop length vulnerable to the machine gun fire, $\psi(t)$ is expressed as

$$\psi(t) = VL(t)/L$$

where, $VL(t)$ is the troop length inside the vulnerable area at time $t$ out of the total length of the troop $(L)$. For a static area target, vulnerability depends upon the intersection of the target area and the vulnerable area. Therefore

$$\psi(t) = A_v / A_{bz}$$

where, $A_v$ is the total area of target vulnerable to machine gun fire at time $t$ out of the total target area $A$. It is obvious that the values of $VL(t)$ and $A_v$ are zero in case the target lies outside the vulnerable zone and thus $\psi(t)$ becomes zero, which further makes the attrition rate zero.

In the case of a point target, i.e., $S(t) = 1$, one considers $\psi(t) = 1$, if the target lies within the vulnerable zone and zero, otherwise.

### 5.3 Hit Probability Calculation

5.3.1 *Static Area Target*

If $a_v$ is the exposed area of a target in deployed position and $A_{bz}$ is the beaten zone area, then for an area target, the probability of hit of an individual target can be calculated as

$$p_{hit} = a_v / A_{bz}$$

5.3.2 *Moving Target*

5.3.2.1 *Hit Probability Calculation within Beaten Zone*

The single-shot hit probability $[p_{hit}(d)]$ of a target corresponding to a given range $d$ is expressed as

$$p_{hit}(d) = p_x(d)\, p_y(d)$$

where, $p_x(d)$ and $p_y(d)$ are the hit probabilities along the line of fire and across the line of fire at a range $d$, respectively.

A burst of fire forms a cone of trajectories and any portion of the target which intersects the cone, is vulnerable. Let $h_L(d)$ and $h_U(d)$ be the heights of lower and upper trajectories, respectively at a range $d$, whereas $H$ and $W$ is the height and width of an individual target, respectively. As shown in Fig. 4, at point $A$, it has been found that outside the beaten zone, an individual target at a distance $d_1$ from the machine gun location is safe along the line of fire, if $H < h_L(d_1)$. The target is vulnerable, if $h_L(d_1) < H$. The heights of the trajectories at different ranges can be calculated using trajectory equations[3] as shown in *Appendix B*. For a beaten zone with width $b$, the hit probability for a target located at $A$ is thus calculated for the range $d = d_1$ as

$$p_{hit}(d)=\begin{cases}0 & ,H \le h_L(d)\\ \left[\dfrac{H-h_L(d)}{h_U(d)-h_L(d)}\right]\left[\dfrac{W}{b}\right] & ,h_L(d) < H < h_U(d)\\ \dfrac{W}{b} & ,H \ge h_U(d)\end{cases}$$

At point $B$ in Fig. 4, the target is shown inside the beaten zone. In this case, if the lower trajectory is extended further, one gets $h_L(d_2)$ as depth of the lower trajectory and $h_U(d_2) + h_L(d_2)$ the vertical cross-sectional height of the cone of fire at $d_2$. The hit probability for situation $B$ is thus calculated for the range $d = d_2$ as

$$p_{hit}(d)=\begin{cases}\left(\dfrac{h_U(d)}{h_U(d)+h_L(d)}\right)\left(\dfrac{W}{b}\right) & ,H \ge h_U(d)\\ \left(\dfrac{H}{h_U(d)+h_L(d)}\right)\left(\dfrac{W}{b}\right) & ,H < h_U(d)\end{cases}$$

From Fig. 4, it was found that $l + l'$ becomes the vulnerable length of the beaten zone, where $l$ is the length of the beaten zone and $l'$ is the distance from the inward edge of the beaten zone to the point where the target height touches the lower trajectory.

**Table 1. Beaten zone of machine gun fire at different aiming ranges**

| Aiming range (m) | Beaten zone (length × width) (m) |
|---|---|
| 600 | 100 × 1 |
| 1000 | 80 × 2 |
| 1400 | 55 × 3 |
| 1800 | 50 × 4 |

Table 1 displays beaten zone areas at different aiming ranges for a typical machine gun fire. Table 2 displays trajectory heights wrt minimum and maximum angles of projection ( $\theta \pm \theta_e$, where $\theta_e$ is the range of angular dispersion ) and corresponding hit probabilities at different ranges wrt an aiming distance of 1000 m. From Table 2, it is observed

**Table 2. Trajectory heights at different ranges corresponding to 1000 m aiming ranges (target height × target width is 1.5 m × 0.3 m)**

| Distance of the target from MG location ($d$) (m) | Height of lower trajectory $h_L(d)$ | Height of upper trajectory $h_U(d)$ | Probability of hit $p_{hit}(d)$ |
|---|---|---|---|
| 900.0 | 1.439 | 3.470 | 0.00453 |
| 910.0 | 1.225 | 3.279 | 0.02000 |
| 920.0 | 1.003 | 3.080 | 0.03590 |
| 930.0 | 0.772 | 2.871 | 0.05199 |
| 940.0 | 0.532 | 2.654 | 0.06840 |
| 950.0 | 0.283 | 2.428 | 0.08510 |
| 960.0 | 0.025 | 2.192 | 0.10209 |
| 962.5 | –0.041 | 2.132 | 0.10209 |
| 970.0 | –0.497 | 1.947 | 0.09210 |
| 980.0 | –0.777 | 1.692 | 0.09110 |
| 990.0 | –1.067 | 1.428 | 0.08589 |
| 1000.0 | –1.366 | 1.153 | 0.06860 |
| 1010.0 | –1.677 | 0.868 | 0.05120 |
| 1020.0 | –1.997 | 0.572 | 0.03339 |
| 1030.0 | –2.329 | 0.266 | 0.01539 |
| 1040.0 | –2.584 | 0.029 | 0.00170 |

that though the beaten zone actually starts from 960 m to 1040 m (the beaten zone length at an aiming range of 1000 m is 80 m) the target is also vulnerable at 900 m range. At 900 m, the target is likely to be hit by the lower trajectory as the height of this trajectory is less than the height of the target (taken to be 1.5 m). Further, as the target moves to the right, i.e., from 900 m towards 960 m, its vulnerability increases due to increased exposed area against the cone of fire. In Table 2, the negative value indicates that, if the lower trajectory is extended further the ground level, it would hit the vertical line passing through the target below. In that case, the round hits the ground short of the target and the target is unaffected by this trajectory.

**Table 3. Average hit probabilities of a machine gun fire at different aiming ranges**

| Range (m) | Exposed target area | |
|---|---|---|
| | 1.5 m x 0.3 m ($m^2$) | 1.2 m x 0.3 m ($m^2$) |
| 600 | 0.246 | 0.147 |
| 1000 | 0.056 | 0.049 |
| 1400 | 0.024 | 0.021 |
| 1800 | 0.012 | 0.010 |

Using the above probabilities, the average hit probability of the target in the vulnerable zone corresponding to the given aiming range can be estimated. The average hit probability of a target within the vulnerable zone formed at an aiming range 1000 m is calculated as 0.056. For different postures, exposed areas of a target are different and hence hit probabilities. The average hit probabilities of a target with different aiming ranges and exposed areas are given in Table 3.

5.3.2.2 *Hit Probability Calculation at Danger Zone*

In a danger zone, since the trajectory of a bullet is less than the height of the target, the probability of hit of a round along the line of fire is considered to be unity, i.e., $p_x(d) = 1$ and $p_y(d) = (W/b)$.


## ACKNOWLEDGEMENTS

The authors are thankful to Shri V.V.N. Rao, Ex-Sci E, Institute for Systems Studies & Analyses (ISSA), Delhi.


## REFERENCES


1. Basic field manual: Browing machine gun. War Department, US Army, 1940. Field Manual No. FM-23-55. pp.182-86.
2. Engineering Design Handbook. Army Weapon System Analysis, Part-I, DARCOM-P706-101. US Army Material Development & Readiness Command, November 1977. pp. 2-5.
3. Sellier, Karl G. & Kneuebuehl, Beat P. *In* Wound ballistics and the scientific background. Elsevier Science B.V., 1994. pp.114-16.


**Contributors**


**Mr D Dutta** obtained his MSc and MPhil in Operational Research from the University of Delhi in 1982 and 1983, respectively. He joined Institute for Systems Studies & Analyses (ISSA), Delhi, in 1984 as Scientist and has been working in the field of modelling and simulation. Since 1991, he has been involved in the development of combat modelling in computer wargame packages, *Shatranj* and *Sangram*. He received training at the British Aerospace, UK, in software development of weapon systems effectiveness. His area of research includes simulation modelling of warfare and optimisation.

**Dr(Mrs) S Sabharwal** obtained her MA and PhD in Operations Research from the University of Delhi in 1966 and 1975, respectively. She joined Defence Institute of Psychological Research (DIPR), Delhi, in 1968 as Scientist and later on transferred to ISSA in 1971. She had been working in the field of modelling and simulation, development of combat modelling for computer wargame software packages, *Shatranj* and *Sangram*. Her area of research includes simulation modelling of warfare and optimisation.

## Computation of Expected Casualties

If $p$ be the single-shot kill probability of a target and if $n$ rounds are fired, then considering independence between the rounds, the cumulative kill probability of the target, $P$ can be written as

$$P = 1.0 - (1.0 - p)^n$$

Now, if a homogeneous force $X$ is receiving area fire from a machine gun, then the expected number of survivors of the force $x(t)$ at time $t$ can be computed as

$$x(t) = (1.0 - P) \times x_0, \quad \text{with initial force strength } x_0 = x\ (0)$$

Therefore, expected number of casualties after time $t$ is

$$x_0 - x\ (t) = P \times x_0$$

It may be possible that all the targets within the force may not be under fire at any one instance. Therefore, if $\phi$ is the fraction of targets vulnerable to fire, then

$$x_0 - x\ (t) = P \times \phi \times x_0$$

Because of the dynamic nature of the battlefield conditions, either the state of the targets and the mode of machine gun fire or both may undergo change, therefore, the values of $P$ and $\phi$ may vary with time. Thus, the expected casualties for the time duration $\Delta t$ can be expressed as

$$x(t + \Delta t) - x(t) = [1.0 - (1.0 - p)^{\Delta n}] \times \phi\ (t) \times x\ (t)$$

where $\Delta n$ is the number of rounds fired and $\Delta t$ is the computational time.

**Trajectory Equations (Based on Karl & Beat)**

Let

| | |
|---|---|
| $d$ | Horizontal distance of a target position from machine gun location |
| $\theta_0$ | Angle of fire wrt aiming range $r$ |
| $v_0$ | Muzzle velocity of the machine gun |
| $R_0$ | Retardation coefficient |
| $g$ | Gravitational constant |
| $z$ | $2R_0 d$ |

The angle of hit $\theta$, on a vertical target at a distance $d$ from the machine gun location and the height of the trajectory $h(d)$ at $d$ is calculated by the following equations:

$$\theta = \tan^{-1}\left( \tan(\theta_0) - \frac{g \times d}{v_0^2 \times \cos^2(\theta_0)} \times F_1(2R_0 d) \right)$$

$$h(d) = d \times \tan(\theta) - \frac{g \times d^2}{2 \times v_0^2 \times \cos^2(\theta_0)} \times F_2(2R_0 d)$$

where

$F_1(z) = (e^z - 1)/z$

$F_2(z) = 2\ (e^z - z - 1)/z^2$

Using the above set of equations, trajectory height at distance $d$ corresponding to aiming range $r$ can be calculated.