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Nonlinear Seismic Response of Articulated Offshore Tower

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ABSTRACT

The seismic response of articulated offshore tower has been investigated by the spectral analysis method which is based on the principle of random vibration, where seismic excitation is assumed to be a broadband stationary process. The nonlinear dynamic equation of motion is derived using Lagrangian approach and the solution is obtained by Newmark's β integration scheme. The present study includes nonlinearities associated due to variable submergence, drag force, Coulomb damping, variable buoyancy, and added mass along with the geometrical nonlinearities of the system. The study includes the joint occurrence of waves and seismic forces together with the current under random sea state. A parametric study has been conducted to investigate the relative importance of the seismic response in comparison to the response due to wave forces.

Keywords: Articulated offshore tower, earthquake loading, seismic response, random vibration, base shear, drag, nonlinear dynamics, equation of motion

NOMENCLATURE

$[Me]$	$[M] + [Ma]$
$[Ma]$	Added mass matrix = $(Cm - 1)\ \rho V$
$[M]$	Structural mass matrix
V	Lumped volume of the tower at a node
ρ	Mass density
Cm	Inertia coefficient
C_D	Drag coefficient
$x,\ \dot{x},\ \ddot{x}$	Tower's linear displacement, linear velocity, and acceleration, respectively
$u,\ \dot{u},\ \ddot{u}$	Water particles linear displacement, linear velocity, and linear acceleration, respectively
$\dot{x}g,\ \ddot{x}g$	Seismic ground velocity and ground acceleration, respectively.

1. INTRODUCTION

An articulated tower is one of the compliant offshore structures economically attractive under deep sea conditions and has applications in a large number of operations, such as drilling, production, flaring of waste gases, tanker mooring, field controlling and loading/off-loading terminals[1]. It essentially

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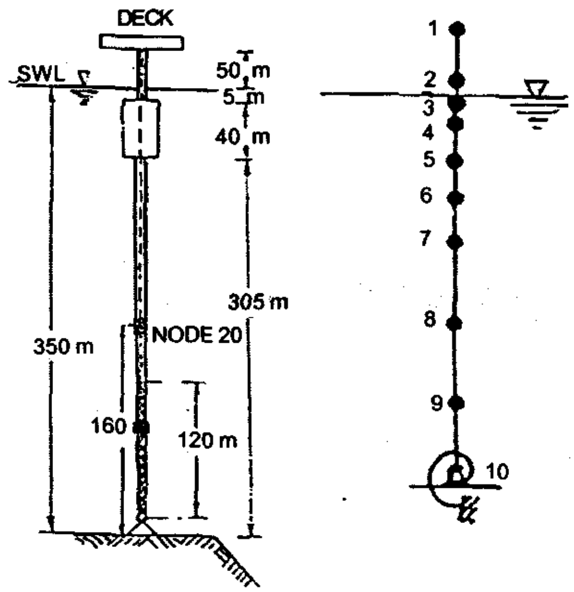


Figure 1. Model of single-hinged articulated tower

consists of a buoyant shaft connected to sea bed through a universal joint (Fig. 1). The compliance of the articulated tower avoids the concentration of high overturning moments and the resulting stresses.

Ocean beds are often found to be seismically quite active. The multipurpose applications of articulated tower, sometimes leave no option but to install it on these sea beds. Foo and Thompson[2] have carried out the case study of a single-point mooring (SPM) under seismic excitations, using a finite-element software. A stick model in which the hinge is modelled as a spring with a dashpot and the masses of the shafts are lumped at the nodes. In this study, the emphasis has been on the methodology of solution of equation of motion for model responses rather than the relative significance of seismic and hydrodynamic loadings and their joint effect on the articuated tower behaviour. Forces on hinge due to seismic forces alone are presented. The behaviour of the fluid surrounded by the tower changes due to tower shaking under earthquake loading, as mentioned by Kokkinowrachos and Thanos[3]. The combined effect of seismic and hydrodynamic loadings on another compliant structure (guyed tower) has been presented by Ryu and Yun [4]. In this study, the effect of hydrodynamic damping due to water has also been incorporated, while the hydrodynamic drag force has been linearised.

The present approach gives valuable insight into the structural behaviour under specified ground motion through a simple deterministic dynamic model of articulated tower shown in Fig.1. It presents a simplified but effective approach of seismic analysis of articulated tower. It enables the response study under joint occurrence of waves and seismic forces together with the current under random sea state, so as to model the overall forcing function in a realistic manner. The influence of major nonlinearities, such as variable submergence, drag force, Coulomb damping, variable buoyancy, added mass along with the geometrical nonlinearities as well as the nonlinearities associated with the forcing function have been studied.

2. MATHEMATICAL IDEALISATION & EQUATION OF MOTION

The articulated tower is idealised as a stick model with masses lumped at the nodes. The universal joint is represented by a massless rotational spring of zero stiffness[4]. The frictional resistance of the hinge is taken as negligible. Assuming earthquake as a broadband random stationary process, response of the articulated tower is carried out by random vibration analysis using the ground acceleration time history (El – Centro accelogram, shown in Fig. 2) as input.

The nonlinear equation of motion is derived by Lagrangian approach[5] which relates the kinetic and potential energies of the system in terms of rotational degree-of-freedom (DOF) as follows:

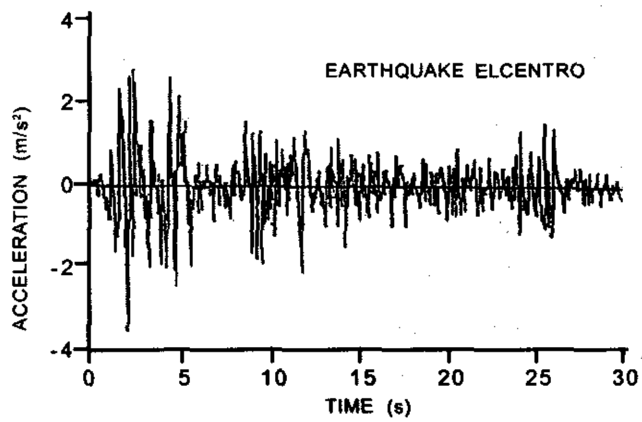


Figure 2. Acceleration time history of N-S component of El-Centro earthquake.

$$\frac{d}{dt}\left[\frac{\partial KE}{\partial \theta}\right]-\frac{\partial (KE)}{\partial \theta}+\frac{\partial (PE)}{\partial \theta}=M_\theta \qquad (1)$$

On mathematical treatment of kinetic and potential energies, the equation of motion is obtained as

$$I\ddot{\theta}-0+\left(\sum_{i=1}^{np} m_{1i}r_i-\sum_{i=1}^{nsp} fb_i r_i+m_d\right)g\ \sin\theta=M_\theta$$

$$\text{or}\quad [I]\ddot{\theta}+\left[\left(\sum_{i=1}^{np} m_{1i}r_i-\sum_{i=1}^{nsp} f\,b_i r_i+m_d\right)g\frac{\sin\theta}{\theta}\right]\theta=M_\theta$$

$$\text{or}\quad [I^*]\ \{\ddot{\theta}\}\ +\ [K]\ \{\theta\}\ =\ \{M_\theta\} \qquad (2)$$

This shows that $[I^*]$ consists of the mass moment of inertias of all the elements including the deck, about the hinge and M_θ is the moment due to non-conservative forces. For combined earthquake and wave loadings, the equation of motion in terms of rotational DOF may be written as

$$[I^*]\ [\ddot{\theta}]+[C]\{\dot{\theta}\}+[K]\{\theta\}=\{M_\theta^h(t)\}+\{M_\theta^{eq}(t)\} \qquad (3)$$

where $\{M_\theta^h(t)\}$ is the moment due to hydrodynamic loading including the effect of nonlinearities and current, and $\{M_\theta^{eq}(t)\}$ is the moment due to earthquake loading. The forcing functions due to wave and current only[6,7] are expressed as

$$F^h(t)=[M_e]\{\dot{u}-\dot{x}\}+[M]\{\ddot{x}\}+0.5\rho C_D[A]$$
$$\{(\dot{u}-\dot{x}+V_c)\,|\dot{u}-\dot{x}+V_c|\} \qquad (4)$$

Under the combined effect of wave loadings and earthquake loadings, the drag and inertia forces will be modified by replacing $(\dot{x})$ by $(\dot{x}_g+\dot{x})$ and $(\ddot{x})$ by $(\ddot{x}_g+\ddot{x})$ that leads to the following expression:

$$F^{eq}(t)=[1M_c]\{\ddot{x}\}+[M]\{\ddot{u}\}+0.5\rho C_D[A]\times$$
$$\{(\dot{u}-(\dot{x}_g+\dot{x}+V_c))\}\big|\dot{u}-(x_g+\dot{x}+V_c)\big| \qquad (5)$$

For the consideration of earthquake alone, the forcing function[8] is modified by replacing $(\ddot{U})=0$ and $(\dot{U})=0$. Thus

$$F^{eq}(t)\ \ alone\ =[M_e]\{\ddot{x}_g\}+0.5\rho C_D[A]$$
$$\{(\dot{u}-(\dot{x}_g+\dot{x}))\}\,\big|-(\dot{x}_g+\dot{x})\big| \qquad (6)$$

3. PROCEDURE

Stepwise time integration scheme of New mark-β has been adopted to counter the nonlinearties involved in iterative manner. Response time histories of the articulated tower, under random wave loading due to earthquake and waves, are thus obtained.

4. NUMERICAL EXAMPLE

The study is carried out on a model[9] of a single-hinged articulated tower (SHAT) described in Table 1. The ground acceleration time history of N-S component of E1–Centro earthquake used in the numerical study is shown in Fig. 2. A

Table 1. Single-hinged articulated tower model

Parameters	Values
Height of tower (l)	400 m
Water depth (d)	350 m
Deck mass (M_D)	2.5E6 kg
Structural mass (SMT) of tower	2.0E4 kg/m
Mass of ballast (M_{BL})	44840 kg/m
Height of Ballast (H_{BL})	120 m
Height of buoyancy chamber (H)	40 m
Position of buoyancy chamber (P_{BC})	325 m
Effective diameter of chamber	
Effective diameter for buoyancy (D_{EB})	19.5 m
Effective diameter for added mass	7.5 m
Effective diameter for drag (D_{ED})	20.0 m
Effective diameter for inertia (D_{EI})	7.5 m
Effective diameter for tower shaft	
Effective diameter for buoyancy (D_{EB})	7.5 m
Effective diameter for added mass	4.5 m
Effective diameter for drag (D_{ED})	17.0 m
Effective diameter for inertia (D_{EI})	4.5 m

regular wave (5 m, 10 s) and a random wave (H_s = 7.97 m, T_z = 9.8 s) have been considered to observe the behaviour due to the combined effect of wave and earthquake loadings. For investigating the combined effect of current and earthquake, a current velocity of 1.0 m/s, uniform throughout the depth, is considered.

However, the sea is random in nature. Especially when the sea bed too is vibrating, the expected behaviour of ocean waves may not be regular but random. Keeping this in view, the articulated tower is further analysed for few of the important cases under long crested random sea environment. Power spectral density functions(PSDFs) are also presented and few of the statistical characteristics are tabulated.

5. RESULTS & DISCUSSION

Tower hinge rotations are the dynamic DOF employed for the determination of response. It is an important design parameter that ascertains the satisfactory performance of the articulated tower in hostile ocean environment. Table 2 shows the statistical characteristics of the response for the combination of regular and random waves with earthquake loading. For comparison, the same results are also obtained for waves and earthquake alone. Starting time T_s = 400.1s and 405.1s refer to the crest and trough locations of the regular wave for a waveheight and wave period of 5 m and 10 s, respectively.

Beginning of an earthquake loading occurs such that its first acceleration peak lies between the trough and crest of the regular wave. Maximum heel angle at T_s = 400.1s (crest) is 7.49×10^{-2} rad while in T_s = 405.1s (trough), the heel angle is 8.90×10^{-2} rad. Mean values, likewise, are 5.36×10^{-5} rad and 2.30×10^{-5} rad, respectively. These values differ appreciably as earthquake at the time corresponding to trough or crest of the regular wave. This effect has been shown in Fig. 3 that also clearly shows the phase lag in the heel angle response. The magnitude in the two cases are differing wrt the striking time. Standard deviation

Table 2. Heel angle (θ) response of SHAT under earthquake loading (values in radian)

Sea	Wave parameter	Loading combinations	Earthquake starting time	Max.	Min.	RMS	Mean	SD	Earthquake versus strong wave
		Regular wave alone		8.88R-4	-8.51E-4	5.56E-4	1.1153E-6	5.56E-4	-
REGULAR	WH = 5 m, WP = 10 s	Earthquake and regular wave	T_s = 400.1.s	7.49E-2	-9.28E-2	2.1549E-2	5.36E-5	2.1549E-2	6 times
			T_s = 405.1.s	8.99E-2	-7.76E-2	2.3223E-2	-2.306E-5	2.3229E-2	7 times
	(Strong regular wave alone 30 m, 15 s)			1.256E-2	-12.56E-2				
Earthquake alone			T_s = 400 s	9.11E-2	-9.77E-2	2.56E-2	2.361E-4	2.5599E-2	1.5 times
		Random wave alone		3.272E-3	-3.25E-3	1.1748E-2	3.8862E-6	1.174E-3	
RANDOM		Earthquake + Random wave	T_s = 400 s	7.08E-2	-9.29E-2	2.372E-2	-1.237E-3	2.369E-2	13.5 %
	H_s = 7.975 m, T_z = 9.8 s	Earthquake + Random wave + Current (V_c = 1 m/s)	T_s = 400 s	1.05E-1	-5.23E-2	3.079E-2	2.22E-2	2.133E-2	-
		Strong random wave alone (18m, 15 s.) H_s T_z = 17.76 m, 14.64 s		6.127E-2	-1.48E-1	4.11E-2	-2.47E-2	3.29E-2	-

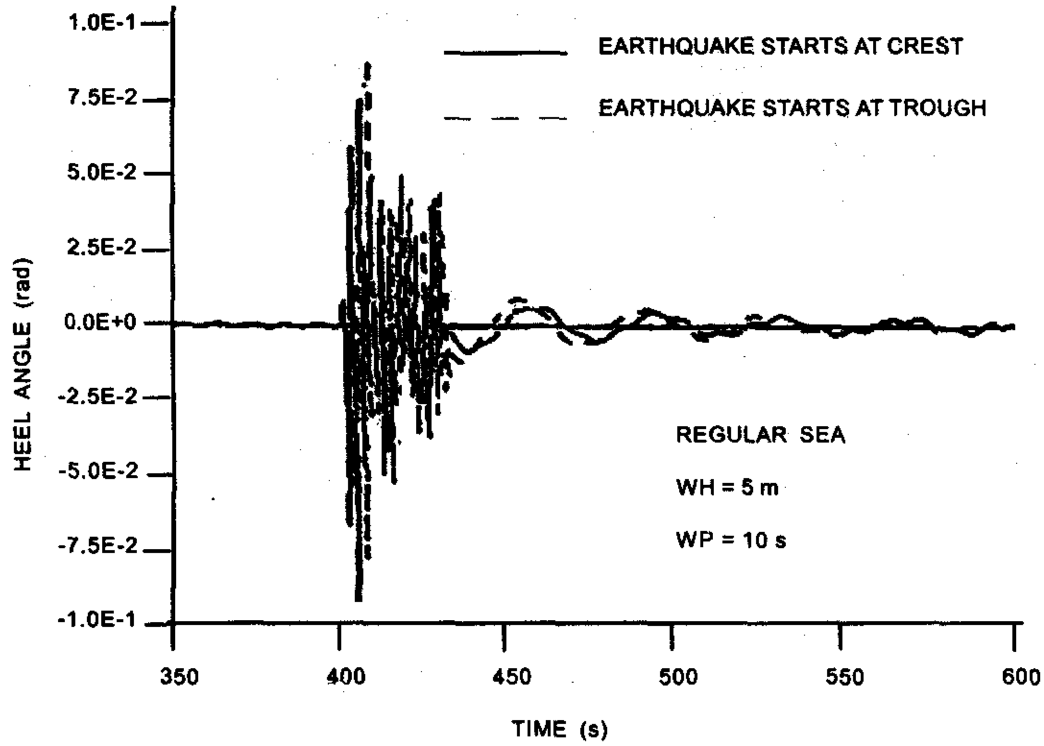


Figure 3. Effect of earthquake initialisation on heel angle of SHAT in regular sea.

(SD) shows the same magnitude of dispersion but the mean values are different. Earthquake-induced responses are further compared with the responses due to independent strong sea state to obtain the relative severity. Heel angle of SHAT due to regular wave (30 m, 15 s) alone is 1.256×10^{-2} rad, while it is increased to 7.49×10^{-2} rad and 8.99×10^{-2} rad, when the tower is hit by an earthquake.

Table 2 also shows another interesting result on comparing response due to earthquake alone. The maximum heel angle for earthquake alone is 9.11×10^{-2} rad, while the same for wave and earthquake is 7.49×10^{-2} rad. This 21.4 per cent increase for earthquake alone is mainly due to the absence of stabilising moment caused by hydrodynamic damping. The same is true if the statistical response is compared with that due to random wave and earthquake loadings. In case of random wave in the presence of earthquake, maximum heel angle is 7.08×10^{-2} rad, which is less than 9.11×10^{-2} rad for earthquake alone. Maximum heel angle due to random wave alone is very small in comparison to the same on inclusion of earthquake. Inclusion of the current with wave and earthquake loading cause the maximum heel angle response of 1.05×10^{-1} rad, which is maximum among all cases.

A successful design of SHAT depends on the successful design of the hinge. The hinge should be able to sustain the fluctuating shear force and the resulting stresses. For the fatigue-resistant design of the hinge, a shear force time history is enviable. For seismic load-induced shear force analysis, an elaborate and systematic study has been carried out. Table 3 shows the shear force characteristics at tower base. The base shear in case of earthquake loading and wave loading are 126 and 141 times more than that in the case of regular wave alone.

In Table 3 respective factors unexpectedly are 58.6 and 126 for the cases of (random wave + earthquake) and (regular wave + earthquake), respectively. This shows a greater wave attenuation in case of random sea state in comparison to regular waves. Random sea state has infinite harmonics of different frequencies. Hence, it is more capable of controlling the random seismic disturbances.

Table 3. Base shear response of single-hinged articulated tower under earthquake loading (all values in Newtons)

Sea	Wave parameter	Loading combinations	Earthquake starting time	Max.	Min.	RMS	Mean	SD	Earthquake versus Strong wave
		Regular wave alone		3.177E-5	-3.11E-5	2.188E-5	1.399E-2	2.188E-5	-
REGULAR	WH = 5m WP = 10 s	Earthquake + Regular wave	T_s = 400.1 s	4.01E-7	-4.7E-7	6.26E-6	-1.23E-4	6.26E-6	2.5 times
			T_s = 405.1 s	3.92E-7	-4.38E-7	6.533E-6	-5.961E-6	6.53E-6	
	(strong regular wave alone 30 m, 15 s)			1.6E-7	-1.6E-7				denominator
Earthquake alone			T_s = 400.s	3.54E-7	-4.9E-7	5.7175E-6	-8.773E-4	5.7168E-6	-
		Random wave alone		7.3E-5	-7.58E-5	2.369E-5	-1.112E-3	2.368E-5	-
RANDOM		Earthquake + Random wave	T_s = 400.s	4.28E-7	-4.3E-7	6.711E-6	5.25E-5	6.688E-6	6 times
	H_s = 7.975 m T_s = 9.8 s	Earthquake + random wave + current (V_c = 1 m/s)	T_s = 400s	2.576E-7	-5.95E-7	1.0516E-7	-7.653E-6	6.215E-6	-
		Strong random wave alone 18 m 15 s) H_s T_z = 17.76 m, 14.64 s		6.89E-6	-2.27E-7	1.736E-6	7.75E-6	1.56E-6	denominator

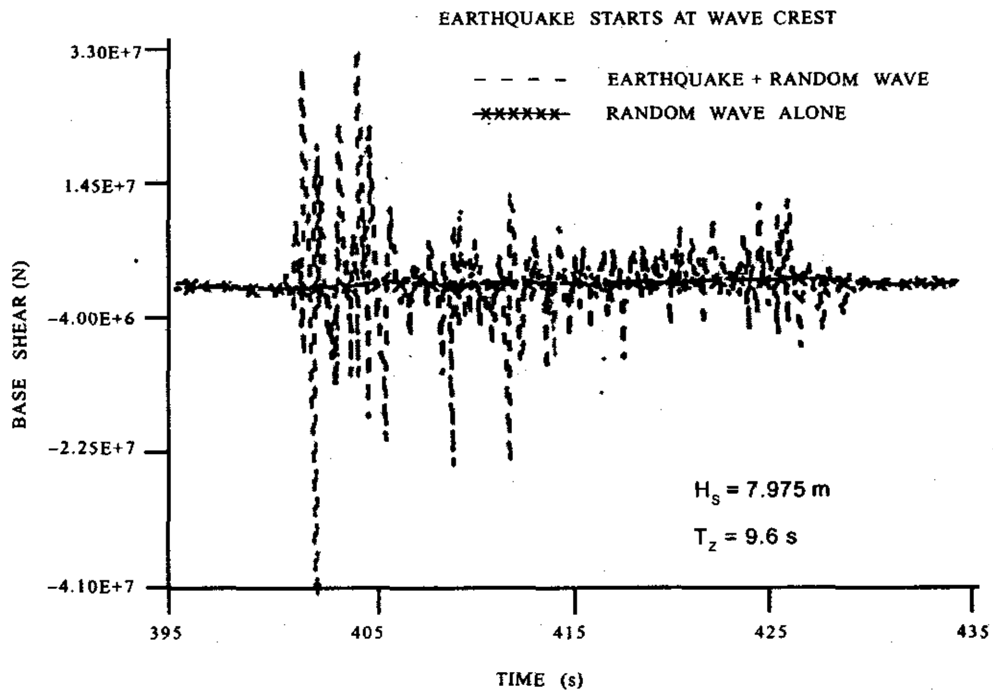


Figure 4. Seismic base shear response of SHAT in long crested random sea state

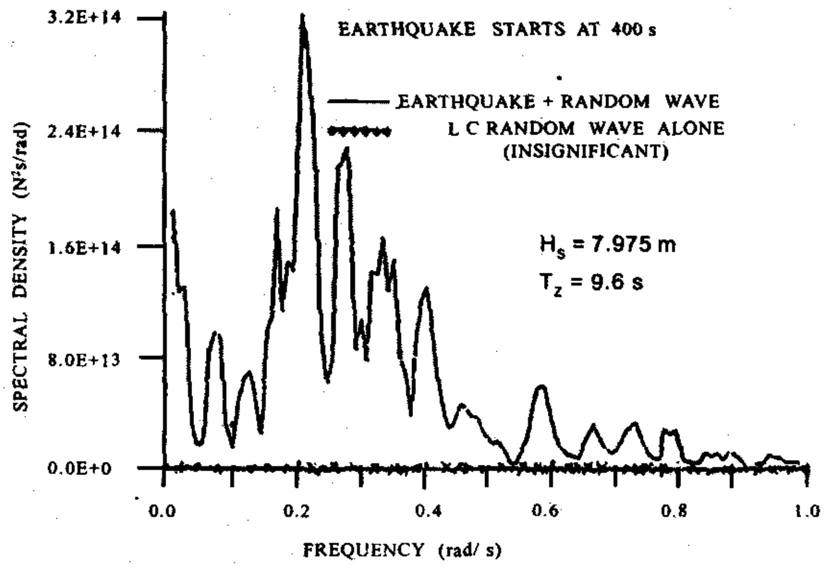


Figure 5. Effect of earthquake on PSDF of base shear of SHAT

Figure 4 shows the base shear time history under seismic load in the presence of random sea state and Fig. 5 shows its corresponding power spectra. Power spectrum that shows the frequency energy content of the earthquake is not attracted due to the compliant nature of the articulated tower. The maximum energy concentration is in the vicinity of the natural frequency of the tower as the structure oscillates at its natural frequency.

6. CONCLUSIONS

On the basis of the study, the following conclusions have been drawn:

(a) The initial condition described by the instant of time of the steady state of the articulated tower motion, at which the earthquake strikes, has significant effect on the tower response. The peak values may differ up to 18 per cent due to the change in initial conditions. However, the RMS responses, and the standard deviations are not significantly influenced by the initial conditions.

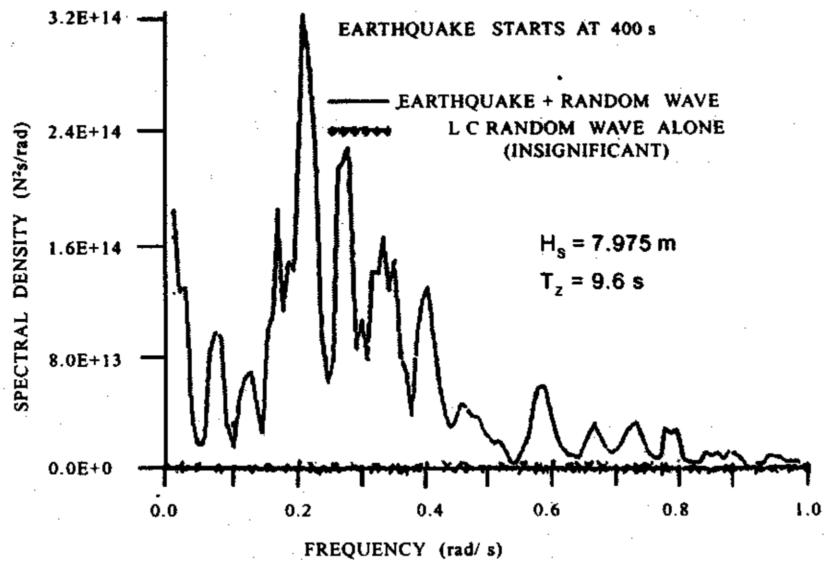
(b) The maximum response for earthquake alone has been observed to be 21 per cent more than that due to the combined loading of the wave and the earthquake. It is because the wave attenuates the seismic response. During the earthquake, the tower tends to vibrate at its own natural frequency while the steady state response again takes place in wave frequency when the earthquake is over.

(c) The time required to achieve the steady state response after the period of earthquake depends upon the sea state at that time. High sea state dampens the seismic response quickly.

(d) The high frequency content of the earthquake does not appreciably contribute to the response of SHAT. The main energy concentration in the PSDF occurs close to the low natural frequencies of the tower. It attributes to the compliant characteristics of the articulated tower.

(e) In the presence of steady current together with random sea and earthquake, the articulated tower tends to oscillate at its natural frequency. Once the period of earthquake is over, the response dies out more rapidly as compared to the case without the current. The presence of the current reduces the seismically-induced standard deviation of responses.

(f) Dynamic responses of structure due to random wave (H_s = 8 m, T_z = 10 s) alone considered in the present study is significantly higher than that due to regular wave (5 m, 10 s) alone.



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(f) Dynamic responses of structure due to random wave (H_s = 8 m, T_z = 10 s) alone considered in the present study is significantly higher than that due to regular wave (5 m, 10 s) alone.

However, inclusion of the earthquake load drastically reduces the overall response due to random sea state in comparison to the response due to the regular wave and the earthquake. Hence, wave attenuation effect enhances in random sea environment.

(g) Short-lived severe responses of SHAT due to earthquake alone dies out quickly when the earthquake is over. In the absence of earthquake and other environmental loads, the articulated tower's oscillations are checked by quadratic hydrodynamic damping due to articulated tower's oscillations and inherent articulated tower's buoyancy. It shows that the inherent characteristics of SHAT help these to sustain the seismic loads.

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