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Adaptive Meshing for Deep-drawing Process

Mohd Ahmed and G.S. Sekhon

Indian Institute of Technology Delhi, New Delhi–110 016

ABSTRACT

This paper incorporates the concept of adaptive meshing for finite-element analysis of the deep-drawing process. In adaptive meshing, the mesh is automatically refined both in the areas of insufficient accuracy and sharp stress gradients. The Zienkiewicz-Zhu error estimator based upon the difference between the finite-element solution and the corresponding smoothened solution is used to judge the accuracy both at the element and the global levels. The post-processing for determining more accurate solutions is done by fitting a higher order polynomial expansion to the finite-element solution in nodal patches. An illustrative problem is solved and the adaptive refinement at different stages of deformation is presented.

Keywords: Adaptive meshing, deep-drawing, finite-element analysis, large deformations

NOMENCLATURE

σ'_{ij}	Deviatoric stress
$\bar{\sigma}$	Effective stress
$\dot{\varepsilon}_{ij}$	Strain rate
$\dot{\bar{\varepsilon}}$	Effective strain rate
η_{allow}	Prescribed error percentage
$\|e\|$	Energy norm
e_u, e_σ	Errors in displacement and stress field, respectively.

1. INTRODUCTION

The computational accuracy of a finite-element simulation is classically improved using a uniformly refined mesh. Such refinement tends to increase the problem size and the cost. User-defined selective refinement in areas of sharp gradients is therefore preferred in large-scale simulation where there is some prior knowledge of the solution. However, prior knowledge of the exact solution is seldom available. Therefore, an adaptive technique that automatically determines the areas of insufficient accuracy and refines the mesh accordingly is viewed as a better approach for reducing the of computational cost.

Considerable research has been done for devising error estimators for use in adaptive mesh refinement[1-3]. The Zienkiewicz-Zhu[4] (ZZ) developed an error estimator that has proven to be quite economical and effective device. A general analysis of the convergence of ZZ error estimator has been given by Ainsworth[5], *et al.* Various methods of refinement involving either subdivision of elements[6] (*h*-refinement) or increase of the order of the polynomial defining the shape function (*p*-refinement) have been proposed. However, till date, not much attention has been given to the study of deep-drawing processes through adaptive finite elements.

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2. FINITE-ELEMENT FORMULATION

The finite-element formulation for sheet metal forming processes is classified into solid formulation and flow formulation[7]. The flow formulation used in the present study, idealises the material as rigid-plastic or rigid visco-plastic and the weak formulation of a boundary value problem is obtained using the principle of virtual work along with a penalty term to enforce incompressibility. It can be expressed by the following equation[8]:

$$\int_{\Omega} \bar{\sigma}\, \delta\dot{\bar{\varepsilon}}_i \, d\Omega + K \int_{\Omega} \varepsilon_V \, \delta\dot{\varepsilon}_i \, d\Omega - \int_{S3} \tau \, \delta v_i \, d\Gamma f = 0 \quad (1)$$

where K, a penalty constant, is a large positive constant equal to $(10^6 - 10^8)\ \mu$.

Yield criterion is:

$$f(\sigma_{ij}) = C$$

$$\bar{\sigma} = \left(\frac{3}{2}\sigma'_{ij}\, \sigma'_{ij}\right)^{\frac{1}{2}} = \bar{\sigma}\left(\bar{\varepsilon}, \dot{\bar{\varepsilon}}\right) \quad (2)$$

Constitutive equations are:

$$\varepsilon_{ij} = \frac{\partial f(\sigma'_{ij})}{\partial \sigma'_{ij}} \lambda \quad : \quad \sigma'_{ij} = \frac{2}{3}\frac{\bar{\sigma}}{\dot{\bar{\varepsilon}}} \varepsilon_{ij} \quad (3)$$

with

$$\dot{\bar{\varepsilon}} = \left(\frac{2}{3}\varepsilon_{ij}\, \varepsilon_{ij}\right)^{\frac{1}{2}}$$

Strain displacement relation

$$\dot{\varepsilon}_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \quad (4)$$

Incompressibility condition

$$\dot{\varepsilon}_V = 0 \quad (5)$$

2.1 Z-Z Error Estimator

The error of an approximate finite-element solution may be expressed in terms of stress or velocity. In particular, e_σ^*, e_u^* are defined as the difference between the exact solution of stress or velocity, σ, u and the corresponding finite-element solution, σ^h, u^h, that is:

$$e_\sigma^* = \sigma - \sigma^h$$

$$e_u^* = u - u^h \quad (6)$$

The error can be evaluated in any appropriate norms. Since the finite-element solution minimises the error in the energy norm, the magnitude of the error in energy norm is a good measure of the overall quality of approximation, and for a metal forming problem, this integral measure of the error in energy norm is defined as

$$\|e\|^2 = \int_{\Omega} (\sigma^{*\prime} - \sigma'^h)^T (2\mu)^{-1} (\sigma^{*\prime} - \sigma'^h)\, d\Omega \quad (7)$$

The relative error in energy norm, η, is defined as

$$\eta = \frac{\|e\|}{\|u\|} \times 100 \text{ per cent} \quad (8)$$

In the above expression, $\|u\|$ is estimated from computed value of σ^h using the following equations.

$$\|\bar{u}\|^2 = \|u^h\|^2 + \|e\|^2 \quad (9)$$

$$\|u^h\|^2 = \left[\sum_{i=1}^{ne} \|u^h\|_i^2\right] \quad (10)$$

3. REFINEMENT STRATEGY

The accuracy of the solution is determined from the global error measure given by Eqn (7). The solution is acceptable if $\eta \leq \eta_{allow}$, where η_{allow} is a prescribed percentage. If not, h-refinement is carried out. In an optimal mesh, the distribution of error is equally divided among the elements. The global error is found from the following equation:

$$\|e\|^2 = \left[\sum_{i=1}^{N} \|e\|_i^2 \right] \tag{11}$$

where N is the total number of elements in the domain.

The permissible global error is given by

$$\|e\|_{allow} = \frac{\eta_{allow} \|e\|}{k} \tag{12}$$

where η_{allow} is the prescribed error percentage, $\|e\|$ is the global strain energy error and k is a factor lying between 1.0 to 1.5 to prevent oscillations[9]. The following relation gives the permissible error in the i^{th} element:

$$\|e\|_{allow(i)} = \frac{\|e\|_{allow}}{\sqrt{N}} \tag{13}$$

where N is the number of elements in the domain. The so-called element refinement parameter, ξ_i, guides the mesh refinement:

$$\xi_i = \frac{\|e\|_i}{\|e\|_{allow(i)}} \tag{14}$$

If $\xi_i > 1$, mesh refinement is needed. The new element size (h_{new}) is found from the following relation:

$$h_{new} = \frac{h_{old}}{\xi_i^{1/p}} \tag{15}$$

where p is the order of the approximating polynomial and h_{old} is the old size of the i^{th} element.

4. ILLUSTRATIVE EXAMPLE

A numerical example has been taken to study the performance of adaptive meshing in the deep-drawing process. The schematic diagram of the deep-drawing operation is shown in Fig.1. Owing to symmetry, only one-half of the blank was modelled. The domain was discretised using six-noded triangular elements. In Fig. 1, the punch radius (R_p) was taken as 50.8 mm and the blank radius (R_b) was taken as 66.0 mm. The thickness of the blank and the velocity of the punch were fixed at 1 mm and 1 mm/s, respectively. The constitutive relation for the blank material was:

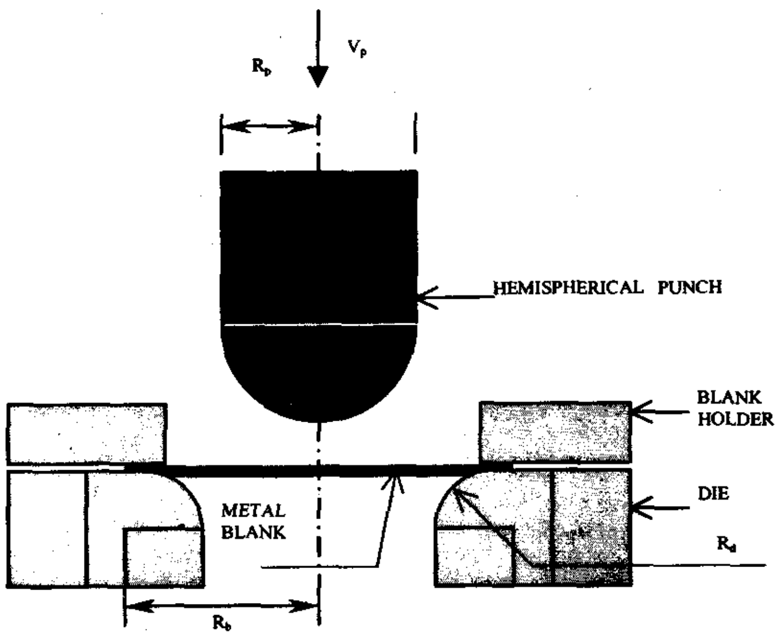


Figure 1. Schematic diagram of deep-drawing operation

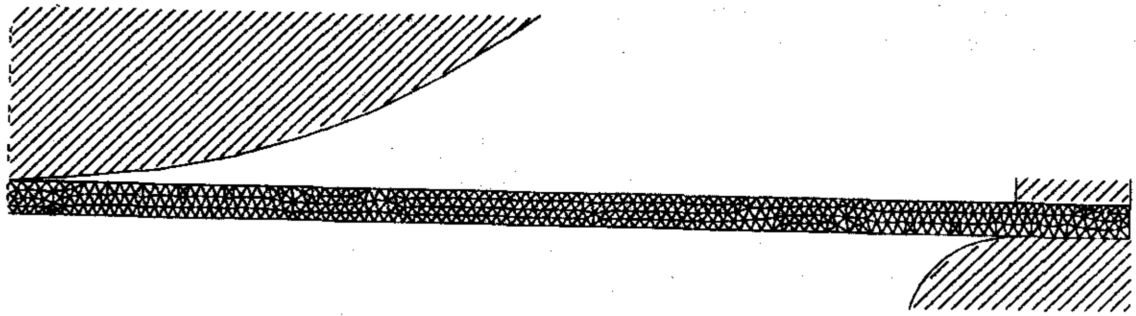


Figure 2. Initial mesh

$$\bar{\sigma} = 589\,[\,0.0001 + \bar{\varepsilon}\,]^{0.216}$$

The total displacement of punch was modelled in a number of incremental steps. The displacement in each increment was such that it caused a maximum strain increment of 1 per cent. The total (21.5 mm) downward displacement was realised in 85 increments and the nominal duration of each increment was 0.25 s.

The initial uniform mesh is shown in Fig. 2. The material of the sheet was modelled as rigid-plastic and the frictional stress at the interface obeys the Coulomb's friction law. A uniform mesh consisting of 2910 degrees-of-freedom (DOFs) was used in the beginning of analysis. Two re-meshings were needed to achieve the target error. A total of 11 h 30 min of CPU time was required for the adaptive analysis. The mesh and the deformed shapes of the blank at punch displacement of 2.5 mm, 15.0 mm and 21.5 mm are shown in Figs 3–5. The DOFs of the refined mesh at punch displacements of 2.5 mm and 15.0 mm were 12534, while at 21.5 mm punch displacement, these actually decreased to 3398. From the mesh plots, it can be observed that after adaptive re-meshing, the density of elements in some portions of the sheet becomes high. For example, in the mesh at punch displacement of 2.5 mm, elements near the punch and blank are finer. The mesh fineness decreased towards the centre. The central portion of the blank had more or less uniform mesh with less density of elements. In the portion of blank in between the blank holder and the die, the finer elements were located near the interface of the punch and blank, and blank holder and blank. In the remaining portions, elements were coarser. The mesh at 15.0 mm punch displacement had similar trend of elements distribution in the different portions. But the distribution of elements at a punch displacement of 21.5 mm was quite different. The portion near the symmetry line had very coarse mesh. The middle portion showed high

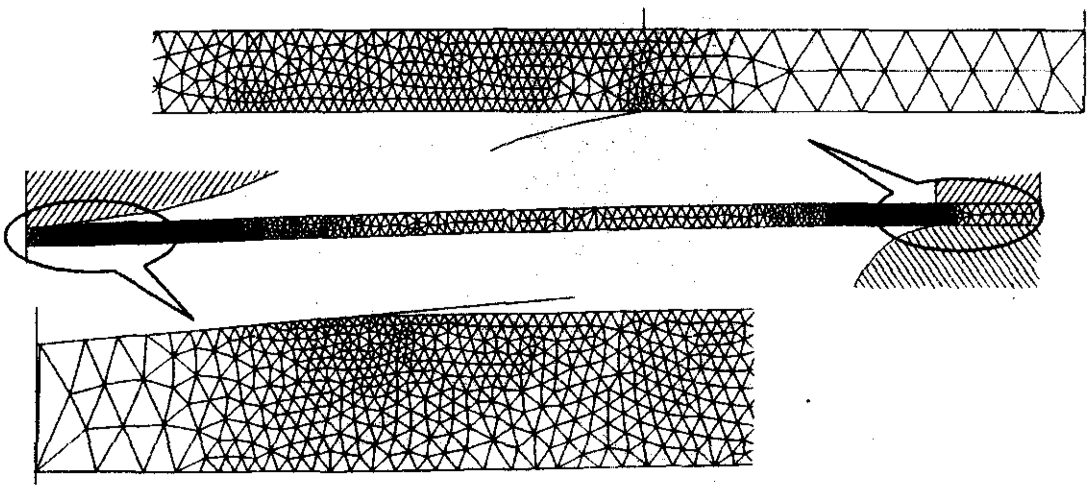


Figure 3. Mesh at punch displacement of 2.5 mm

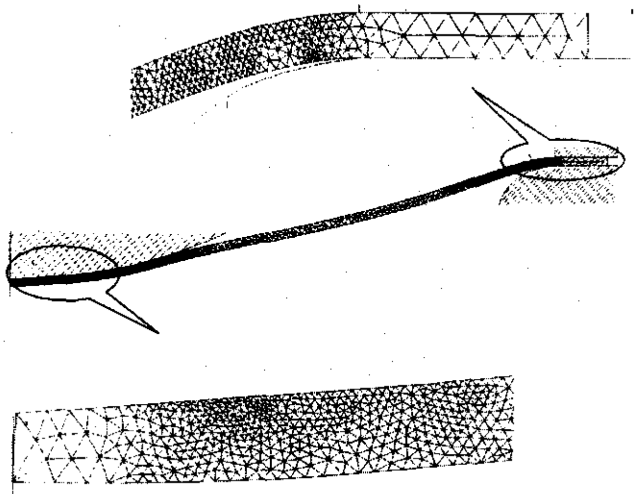


Figure 4. Mesh at punch displacement of 15.0 mm

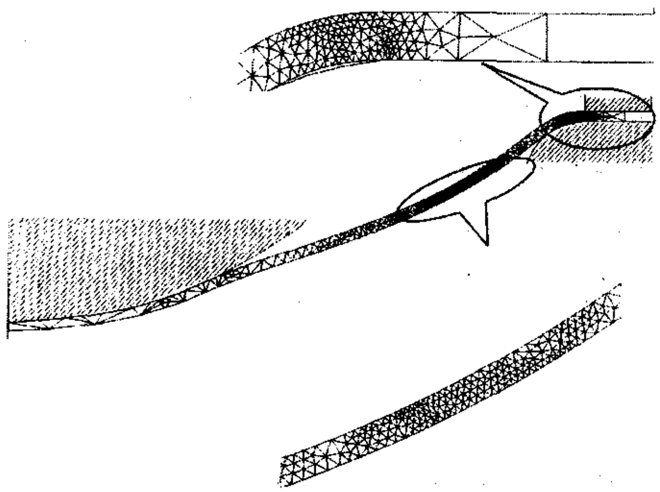


Figure 5. Mesh at punch displacement of 21.5 mm

density of elements. The portion of the blank near the blank holder and the die had finer elements, especially around the die corner. Coarser elements were found along straight portion of die. Figure 6 depicts the evolution of punch load at various stages of deformation. As expected, the punch load increased with the increase of punch displacement.

5. CONCLUSIONS

The finite-element analysis of the deep-drawing process is carried out through adaptive re-meshing. From the mesh plot, it is found that the mesh is automatically regenerated and becomes non-uniform or refined when the solution error exceeds predefined limit at any stage of deformation. The mesh gets finer in places of high stress/velocity gradients and coarser in places of low stress/velocity gradients. The adaptive analysis, therefore, is suitable for predicting seats of large deformations. Also, the finite-element method is applied optimally, i.e., mesh density is varied to capture the deformation behaviour accurately at the smallest expense of the CPU time.

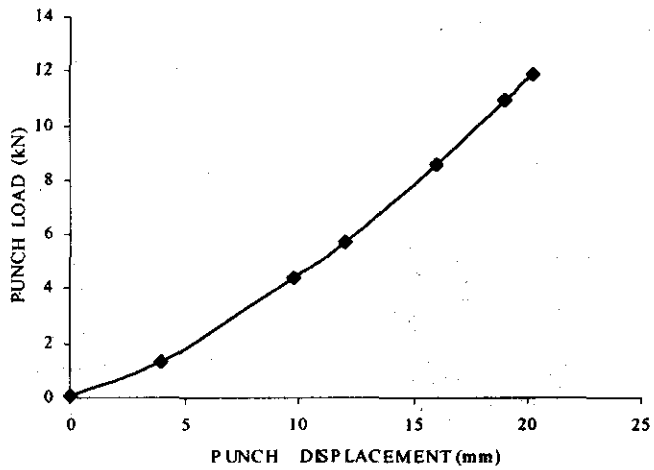


Figure 6. Curve of punch load versus punch displacement.

REFERENCES

1. Gallimard, L.; Ladeveze, P. & Pelle, J.P. Error estimation and adaptivity in elastoplasticity. *Int. J. Num. Meth. Engg.*, 1996, **39**, 189–217.

2. Fourment, L. & Chenot, J.L. Error estimators for visco-plastic materials : Applications to forming processes. *Engg. Comp*, 1995, **12**, 469–90.

3. Lee, N.S. & Bathe, K.J. Error indicators and adaptive re-meshing in large deformation finite-element analysis. *Finite Elem. Anal. Des.*, 1994, **16**, 75-90.

4. Zienkiewicz, O.C. & Zhu, J.Z. A simple error estimation and adaptive procedure for practical engineering analysis. *Int. J. Num. Meth. Engg.*, 1987, **24**, 337–57.

5. Ainsworth, M.; Zhu, J.Z.; Craig, A.W. & Zienkiewicz, O.C. Analysis of the Zienkiewicz-

Zhu: *A-posteriori* error estimator in the finite-element method. *Int. J. Num. Meth. Engg.*, 1989, **28**, 2161–174.

6. Zienkiewicz, O.C.; Liu, Y.C. & Huang, G.C. Error estimation and adaptivity in flow formulation for forming problems. *Int. J. Num. Meth. Engg.*, 1988, **25**, 23–42.

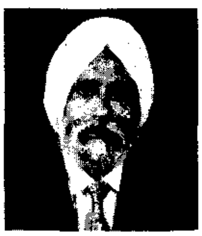
7. Honer, M.E. & Wood, R.D. Finite-element analysis of axisymmetric deep-drawing using a simple two-noded mindlin shell element. *In* Numerical methods for nonlinear problems. Pinridge Press, 1987. pp. 440–49.

8. Oh, S.I. & Kobayashi, S. Finite-element analysis of plane-strain sheet bending. *Int. J. Mech. Sci.*, 1980, **22**, 583–94.

9. Li, X.D. & Wiberg, N.E., *A posteriori* error estimate by element patch post-processing, adaptive analysis in energy and L_2 norms. *Composite Structures*, 1994, **53**, (4), 907–19.

Contributors

Mohd. Ahmed obtained his MSc(Building Engg) in 1993 from Aligarh Muslim University, Aligarh. He is prsently working at the CPWD and also pursuing research at the Indian Institute of Technology Delhi. His areas of research include: Finite-element method and computational mechanics.



Dr GS Shekhon is a Senior Professor in the Dept of Applied Mechanics at the IIT Delhi. He has served as chairman of the Departmental Research Committee; member of the Board of Postgraduate Studies (IIT Delhi) and other universities. His areas of specialisation include: Manufacturing analysis, computational plasticity, and design engineering. He is a fellow of the Institute of Engineers (India); member of the Indian Society for Technical Education; Indian Society of Biomechanics; and International Society of Biorheology.