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Analysis of Axisymmetric Crushing of Frusta

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ABSTRACT

The paper presents a curved-fold model with variable straight length for the axisymmetric crushing of thin frusta. The folding considered in the model is partly inside and partly outside. The variation of circumferential strain during the formation of a fold has been taken into account. The size of the fold and mean, as well as variation of crushing load, has been computed mathematically. The study is purely analytical and does not involve any empirical constant; and hence, can be used in general. The model's predictions have been compared with experimental results and a reasonably good agreement has been observed.

Keywords: Thin-walled structures, energy absorbers, cylindrical tubes, frusta, energy absorbing device, curved-fold model, axisymmetric crushing

NOMENCLATURE

θ	Angle of taper of the frusta
f_y	Yield stress of material
t	Thickness of tube
α, β	Angles subtended by arc *AB* (or *B'C*) and *CC'* (or *D'E*) at their respective centres
ρ, σ	Radii of curvature in lower and upper limbs in a fold, respectively
α_1, β_1	Angles subtended by arcs *AA'* and *D'E* at their respective centres
D_1, D_2	End diameter of tube
m	Folding parameter
L, L'	Fold length of lower and upper limbs, respectively
R, R'	Mean radii of frustum at *A'* and *D'*, respectively
aL, aL'	Length of straight portion in lower and upper limbs, respectively
bL, bL'	Length of curved portion in lower and upper limbs, respectively
cL, cL'	Length of one straight and one curved portion in lower and upper limbs, respectively
ρ_m, ρ_m'	Final radii of curvature in lower and upper limbs in a fold, respectively
α_m	Maximum hinge Angle

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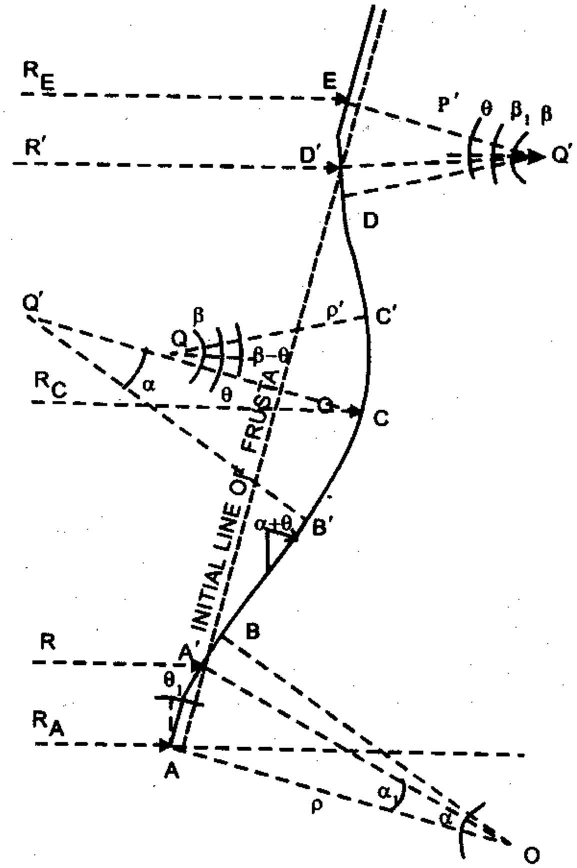

Figure 1. Model for crushing of frusta for case1 (case 1: *m* lies between 0 and *b*).

- *Incremental Energy Absorbed in Bending of Lower Limb*

Radii of tube at A and C are

$$R_A = R + \rho\,\{\cos(\alpha_1 + \theta) - \cos\theta\} \tag{10}$$

$$R_C = R + \rho \begin{Bmatrix} \cos\theta - 2\cos(\alpha + \theta) \\ + \cos(\alpha_1 + \theta) \end{Bmatrix} + aL\sin(\alpha + \theta) \tag{11}$$

Energy to bend the hinges for a small increment in angle by $d\alpha$ is:

$$dW_b = 2\pi M_p (R_A + R_C)\, d\alpha \tag{12}$$

Substituting the values of R_A and R_C from Eqns (10) and (11), one obtains:

$$dW_b = 4\pi M_p \begin{bmatrix} R + \dfrac{bL}{\alpha}\begin{Bmatrix} \cos(\alpha_1 + \theta) \\ - \cos(\alpha + \theta) \end{Bmatrix} \\ + \dfrac{aL}{2}\sin(\alpha + \theta) \end{bmatrix} d\alpha \tag{13}$$

- *Incremental Energy Absorbed in Hoop Action of Lower Limb*

The expressions for incremental energy due to hoop strain for various regions have been derived as

Region AA'

$$dW_1 = 2\pi t f_y L^2 \begin{bmatrix} \dfrac{m^2 \sin(\alpha_1 + \theta)}{\alpha} + \\ \dfrac{mb\cos(\alpha_1 + \theta)}{\alpha^2} + \\ \dfrac{b}{\alpha^3}\{\sin\theta - \sin(\alpha_1 + \theta)\} \end{bmatrix} d\alpha \tag{14}$$

Region $A'B$

$$dW_2 = 2\pi t f_y L^2 \begin{bmatrix} \dfrac{(m - b)\sin(\alpha_1 + \theta)}{\alpha} + \\ \dfrac{b}{\alpha^2}\begin{Bmatrix} m\cos(\alpha_1 + \theta) \\ - b\cos(\alpha + \theta) \end{Bmatrix} \\ + \dfrac{b^2}{\alpha^3}\begin{Bmatrix} \sin(\alpha + \theta) \\ - \sin(\alpha_1 + \theta) \end{Bmatrix} \end{bmatrix} d\alpha \tag{15}$$

Region BB'

$$dW_3 = 2\pi t f_y L^2 \begin{bmatrix} \dfrac{a}{\alpha}\begin{Bmatrix} b\sin(\alpha + \theta) \\ - m\sin(\alpha_1 + \theta) \end{Bmatrix} \\ + \dfrac{a^2}{2}\cos(\alpha + \theta) \end{bmatrix} d\alpha \tag{16}$$

Region $B'C$

$$dW_4 = 2\pi t f_y L^2 \left[\begin{matrix} \frac{b}{\alpha}\begin{Bmatrix} b\sin(\alpha+\theta) \\ -m\sin(\alpha_1+\theta) \end{Bmatrix} \\ +b^2\cos(\alpha+\theta) \\ -\frac{b^2}{\alpha^2}\cos(\alpha+\theta) \\ +\frac{b^2}{\alpha^3}\{\sin(\alpha+\theta)-\sin\theta\} \end{matrix} \right] d\alpha \qquad (17)$$

- *Total Incremental Energy Absorbed in Lower Limb*

Total incremental energy for lower limb is obtained by adding Eqns (14) to (17), thus

$$dW_{T1} = \frac{\pi f_y t^2}{\sqrt{3}} \left[\begin{matrix} 2R + \frac{2bL}{\alpha}\begin{Bmatrix} \cos(\alpha_1+\theta) \\ -\cos(\alpha+\theta) \end{Bmatrix} \\ + aL\sin(\alpha+\theta) \end{matrix} \right] d\alpha$$
$$+ 2\pi f_y t L^2 \left[\begin{matrix} \frac{(2m-1)m}{\alpha}\sin(\alpha_1+\theta) \\ +\frac{bc}{\alpha}\sin(\alpha+\theta) + \frac{2}{\alpha^2}\begin{Bmatrix} mb\cos(\alpha_1+\theta) \\ -b^2\cos(\alpha+\theta) \end{Bmatrix} \\ +\frac{2b^2}{\alpha^3}\{\sin(\alpha+\theta)-\sin(\alpha_1+\theta)\} \\ +(0.5a+b)\cos(\alpha+\theta) \end{matrix} \right] d\alpha \qquad (18)$$

Total energy absorbed in full crushing of lower limb, $W_{T1} = \int_0^{\alpha_m} dW_{T1}$ or

$$W_{T1} = \frac{\pi f_y t^2}{\sqrt{3}} \left[\begin{matrix} 2R\alpha_m + aL\begin{Bmatrix} \cos\theta \\ -\cos(\alpha_m+\theta) \end{Bmatrix} \\ +2bLX^* \end{matrix} \right] + 2\pi t f_y L^2 Y^* \qquad (19)$$

where

$$X^* = \int_0^{\alpha_m} \frac{1}{\alpha}\{\cos(\alpha_1+\theta) - \cos(\alpha+\theta)\} d\alpha \qquad (20)$$

$$Y^* = \int_0^{\alpha_m} \frac{1}{\alpha}\{m(m-1)\sin(\alpha_1+\theta) - b\sin(\alpha+\theta)\} d\alpha$$
$$+ \frac{b^2}{\alpha_m^2}\{\sin(\alpha_{1m}+\theta) - \sin(\alpha_m+\theta)\}$$
$$+ \frac{b}{\alpha_m}\{b\cos(\alpha_m+\theta) - m\cos(\alpha_{1m}+\theta)\}$$
$$(0.5a^2+b^2)\{\sin(\alpha_m+\theta) - \sin\theta\} \qquad (21)$$

where

$$\alpha_{1m} = \frac{m\alpha_m}{b} \qquad (22)$$

- *Total Energy Absorbed in Upper Limb*

Total potential energy for upper limb can be obtained by multiplying α, α_1, α_m, α_{1m} by e^2 and replacing L by hL and θ by $-\theta$ in the energy equation of lower limb. The final expression will thus be:

$$W_{T2} = \frac{\pi f_y t^2}{\sqrt{3}} \left[\begin{matrix} 2Rh\alpha_m + ahL\begin{Bmatrix} \cos\theta \\ -\cos\begin{pmatrix} e^2\alpha_m \\ -\theta \end{pmatrix} \end{Bmatrix} \\ +2bhLX_1^* \end{matrix} \right] + 2\pi t f_y h^2 L^2 Y_1^* \qquad (23)$$

where

$$X_1^* = \int_0^{\alpha_m} \frac{1}{\alpha}\{\cos(e^2\alpha_1 - \theta) - \cos(e^2\alpha - \theta)\} d\alpha \qquad (24)$$

$$Y_1^* = \int_0^{\alpha_m} \frac{1}{\alpha}\begin{Bmatrix} m(m-1)\sin(e^2\alpha_1 - \theta) \\ -b\sin(e^2\alpha - \theta) \end{Bmatrix} d\alpha + \frac{b^2}{e^4\alpha_m^2}$$
$$\left\{\sin(e^2\alpha_{1m}+\theta) - \sin(\alpha_m + e^2\theta)\right\} + \frac{b}{e^2\alpha_m}$$
$$\left\{b\cos(e^2\alpha_m - \theta) - m\cos(e^2\alpha_{1m} - \theta)\right\}$$
$$+(0.5a^2+b^2)\left\{\sin(e^2\alpha_m + \theta) + \sin\theta\right\} \qquad (25)$$

- *Total Energy Absorbed in Formation of One Complete Fold*

The total energy absorbed in the formation of one complete fold will thus be:

$$W_T = W_{T1} + W_{T2}$$

$$= \frac{\pi f_y t^2}{\sqrt{3}} \left[2R\alpha_m (1+h) + aL \left\{ \begin{array}{l} (1+h)\cos\theta \\ -\cos(\alpha_m + \theta) \\ -h\cos(e^2\alpha - \theta) \end{array} \right\} + 2bL(X^* + h^2 X_1^*) \right] + 2\pi t f_y L^2 (Y^* + h^2 Y_1^*) \quad (26)$$

- *Mean Crushing Load*

Assuming that the energy dissipation in the axisymmetric axial crushing of frusta takes place in the form of flexural and circumferential deformation, therefore, the external work done can be equated to the sum of energy absorbed in bending and circumferential stretching. Total work done by applied load is given by

$$W_P = P_m \delta_T \quad (27)$$

where, δ_T is the effective crushing distance corresponding to one complete fold

$$\delta_T = \left[L(1+h) - t - \frac{bL}{\alpha_m}\left(1 + \frac{h}{e}\right) \right] \cos\theta \quad (28)$$

Equating the work done by the applied load given by Eqn (27) to the total energy absorbed in the formation of fold given by Eqn (26), the mean crushing load, therefore, can be calculated as

$$\frac{P_m}{P_o} = \frac{\frac{t}{2\sqrt{3}} \left\{ \begin{array}{l} 2R\alpha_m (1+h) \\ + aL \left\{ \begin{array}{l} (1+h)\cos\theta - \cos(\alpha_m + \theta) \\ - h\cos(e^2\alpha_m - \theta) \end{array} \right\} \\ + 2bL(X^* + hX_1^*) \end{array} \right\} + 2\pi t f_y L^2 (Y^* + h^2 Y_1^*)}{R\left[L(1+h) - t - \frac{bL}{\alpha_m}\left(1 + \frac{h}{e}\right) \right] \cos\theta} \quad (29)$$

where $P_o = 2\pi R t f_y$ (30)

- *Size of Fold and Folding Parameter*

Determination of size of fold and folding parameter requires the minimisation of mean crushing load *wrt* fold length and folding parameter, respectively. Thus, the final expression for fold length is given by

$$L = \frac{E_1 - E_2}{E_3} \quad (32)$$

where

$$E_1 = P_m \left[L(1+h) - \frac{2bh}{e\alpha_m} \right] \cos\theta \quad (33)$$

$$E_2 = \frac{\pi f_y t^2}{\sqrt{3}} \left[a \left\{ \begin{array}{l} (1+h)\cos\theta - \cos(\alpha_m + \theta) \\ - h\cos(e^2\alpha_m - \theta) \end{array} \right\} - 2ae^2 h\alpha_m (1 - eh) \sin(e^2\alpha_m - \theta) + 2b(X^* + hX_1^*) + 4bh(1 - eh)\{\cos(e^2\alpha_m - \theta) - \cos(e^2\alpha_{1m} - \theta)\} \right] \quad (34)$$

$$E_3 = 4\pi t f_y (Y^* + h^2 Y_1^*) + 2\pi f_y t h^2 (1 - eh^2) \Big[2m(1-m)\{\sin(e^2\alpha_{1m} - \theta) + \sin\theta\} - 2b\{\sin(e^2\alpha_{1m} - \theta) + \sin\theta\} + \frac{4b}{e^2\alpha_m} \left\{ \begin{array}{l} b\cos(e^2\alpha_m - \theta) \\ - m\cos(e^2\alpha_{1m} - \theta) \end{array} \right\} + 2\{b^2 \sin(e^2\alpha_m - \theta) - m^2 \sin(e^2\alpha_{1m} - \theta)\} + \frac{4b^2}{e^4\alpha_m}\{\sin(e^2\alpha_{1m} - \theta) - \sin(e^2\alpha_m - \theta)\} - (a^2 + 2b^2) e^2\alpha_m \cos(e^2\alpha_m - \theta) \Big] \quad (35)$$

The expression for folding parameter can thus be obtained from:

$$\frac{P_m bL(1-h)\cos\theta}{2\pi f_y t\alpha_m R} - \frac{tae^3(1-h)L\alpha_m \sin(e^2\alpha_m - \theta)}{\sqrt{3}R} - \frac{bL}{\sqrt{3}}\left(\frac{\partial X^*}{\partial m} + h\frac{\partial X_1^*}{\partial m}\right) - L\left(\frac{\partial Y^*}{\partial m} + h^2\frac{\partial Y_1^*}{\partial m}\right) = 0 \quad (36)$$

where

$$\frac{\partial X^*}{\partial m} = \frac{1}{m}\{\cos(\alpha_{1m} + \theta) - \cos\theta\} \quad (37)$$

$$\frac{\partial X_1^*}{\partial m} = \frac{2eL(1-h)}{hR}\{\cos(e^2\alpha_{1m} - \theta) - \cos(e^2\alpha_m - \theta)\} + \frac{1}{m}\{\cos(e^2\alpha_{1m} - \theta) - \cos\theta\} \quad (38)$$

$$\frac{\partial Y^*}{\partial m} = (2m-1)\int_0^{\alpha_m}\frac{1}{\alpha}\sin(\alpha_1 + \theta)d\alpha + (2m-1)\sin(\alpha_{1m} + \theta) - (m-1)\sin\theta \quad (39)$$

$$\frac{\partial Y_1^*}{\partial m} = (2m-1)\int_0^{\alpha_m}\frac{1}{\alpha}\sin(e^2\alpha_1 - \theta)d\alpha + \left(m + 1 - \frac{2me}{h}\right)\{\sin(e^2\alpha_m - \theta) + \sin\theta\} + \frac{L(1-h)}{hR}\left[2be\left\{\sin\begin{pmatrix}e^2\alpha_m\\ -\theta\end{pmatrix} + \sin\theta\right\} - \frac{4b}{e\alpha_m}\left\{b\cos\begin{pmatrix}e^2\alpha_m\\ -\theta\end{pmatrix} - m\cos\begin{pmatrix}e^2{}_{1m}\\ -\theta\end{pmatrix}\right\} + 2e\left\{m^2\sin\begin{pmatrix}e^2\alpha_{1m}\\ -\theta\end{pmatrix} - b^2\sin\begin{pmatrix}e^2{}_{1m}\\ -\theta\end{pmatrix}\right\} - \frac{4b^2}{e^3\alpha_m^2}\left\{\sin\begin{pmatrix}e^2\alpha_{1m}\\ -\theta\end{pmatrix} - \sin\begin{pmatrix}e^2\alpha_m\\ -\theta\end{pmatrix}\right\} + (a^2 + 2b^2)e^3\alpha_m\cos\begin{pmatrix}e^2\alpha_{1m}\\ -\theta\end{pmatrix}\right] + m\sin(e^2\alpha_{1m} - \theta) \quad (40)$$

- *Maximum Hinge Angle*

The expression for deformation is given by

$$\delta = (1+h)L\cos\theta + \frac{2bL}{\alpha}\left\{\left(1 - \frac{h}{e}\right)\sin\theta - \sin(\alpha + \theta) - \frac{h}{e}\sin(e\alpha - \theta)\right\} - aL\{\cos(\alpha + \theta) + h\cos(e\alpha - \theta)\} \quad (41)$$

Now, when $\alpha = \alpha_m$ then $\delta = \delta_T$, therefore Eqn (41) reduces to the following equation from where α_m can be determined as

$$t\alpha_m\cos\theta + bL\left(1 + \frac{h}{e}\right)\cos\theta + 2bL\left\{\left(1 - \frac{h}{e}\right)\sin\theta - \sin\begin{pmatrix}\alpha_m\\ +\theta\end{pmatrix} - \frac{h}{e}\sin\begin{pmatrix}e\alpha_m\\ -\theta\end{pmatrix}\right\} - aL\alpha_m\{\cos(\alpha_m + \theta) + h\cos(e\alpha_m - \theta)\} = 0 \quad (42)$$

2.2.2 *Case 2 – Folding Parameter Lies between b and c*

The model for axisymmertic crushing of the frustum for this case is shown in Fig. 2. The initial line of frusta cuts the straight portion of fold at A'.

- *Incremental Energy Absorbed in Bending of Lower Limb*

The mean radii at A and C are given by

$$R_A = R + \rho\{\cos(\alpha + \theta) - \cos\theta\} + (m - b)L\sin(\alpha + \theta) \quad (43)$$

$$R_C = R + \rho\{\cos\theta - \cos(\alpha + \theta)\} + (c - m)L\sin(\alpha + \theta) \quad (44)$$

Substituting the values of R_A and R_C in Eqn (12), one obtains the energy absorbed in bending of lower limb for a small increment of angle $d\alpha$ as

$$dW_b = 2\pi M_p\left[2R + L(1-2m)\sin\begin{pmatrix}\alpha\\ +\theta\end{pmatrix}\right]d\alpha \quad (45)$$

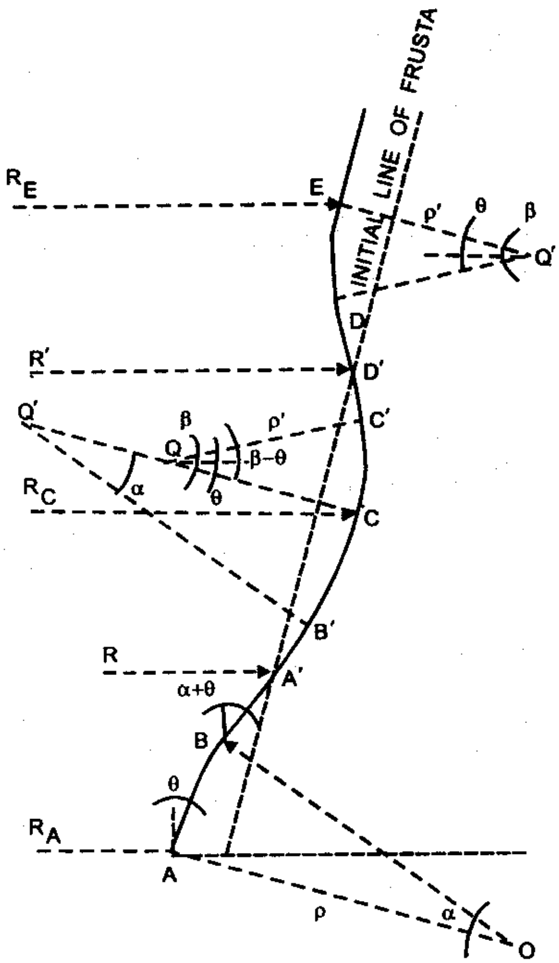

Figure 2. Model for crushing of frusta for case 2 (case 2: *m* lies between *b* and *c*).

- *Incremental Energy Absorbed in Hoop Action of Lower Limb*

The expressions for incremental energy due to hoop strain for various regions are as follow.

Region *AB*

$$dW_1 = 2\pi t f_y L^2 \Big[b(m-b)\cos(\alpha+\theta) + \frac{b^2\sin(\alpha+\theta)}{\alpha} + \frac{b^2\cos(\alpha+\theta)}{\alpha^2} + \frac{b^2}{\alpha^3}\{\sin(\alpha_1+\theta)-\sin\theta\} \Big] d\alpha \qquad (46)$$

Region *BA'*

$$dW_2 = 2\pi t f_y L^2\left[0.5(m-b)^2\cos(\alpha+\theta)\right]d\alpha \qquad (47)$$

Region *A'B'*

$$dW_3 = 2\pi t f_y L^2\left[0.5(c-m)^2\cos(\alpha+\theta)\right]d\alpha \qquad (48)$$

Region *B'C*

$$dW_4 = 2\pi t f_y L^2 \left[\begin{array}{l} b(c-m)\cos(\alpha+\theta) \\ -\dfrac{b^2\cos(\alpha+\theta)}{\alpha^2} \\ +\dfrac{b^2}{\alpha^3}\{\sin(\alpha_1+\theta)-\sin\theta\} \end{array} \right] d\alpha \qquad (49)$$

- *Total Incremental Energy Absorbed in Lower Limb*

Total incremental energy for lower limb is obtained by adding Eqns (46) to (49), thus

$$dW_{T1} = \frac{\pi f_y t^2}{\sqrt{3}}\left[2R + L(1-2m)\sin(\alpha+\theta)\right]d\alpha + 2\pi f_y t L^2\left[\frac{b^2}{\alpha}\sin(\alpha+\theta) + J\cos(\alpha+\theta)\right]d\alpha \qquad (50)$$

where

$$J = ab + 0.5\{(m-b)^2 + (c-m)^2\} \qquad (51)$$

Total energy absorbed in full crushing of the lower limb, $W_{T1} = \int_0^{\alpha_m} dW_{T1}$

$$= \frac{\pi f_y t^2}{\sqrt{3}}\left[2R\alpha_m + L(1-2m)\left\{\cos\theta - \cos\begin{pmatrix}\alpha_m \\ +\theta\end{pmatrix}\right\}\right] + 2\pi t f_y L^2 Y^* \qquad (52)$$

where

$$Y^* = b^2\int_0^{\alpha_m}\frac{1}{\alpha}\sin(\alpha+\theta)\,d\alpha + J\left\{\begin{array}{l} \sin(\alpha_m-\theta) \\ -\sin\theta \end{array}\right\} \qquad (53)$$

- *Total Energy Absorbed in Upper Limb*

Total potential energy for upper limb can be obtained by multiplying $\alpha, \alpha_1, \alpha_m, \alpha_{1m}$ by e^2 and replacing L by hL and θ by $-\theta$ in the energy equation of lower limb. The final expression becomes

$$W_{T2} = \frac{\pi f_y t^2}{\sqrt{3}} \begin{bmatrix} 2hR\alpha_m + hL(1-2m) \\ \{cos\theta - cos(e^2\alpha_m - \theta)\} \end{bmatrix} + 2\pi t f_y h^2 L^2 Y_1^* \qquad (54)$$

where

$$Y_1^* = b^2 \int_0^{\alpha_m} \frac{1}{\alpha} \sin\begin{pmatrix} e^2\alpha \\ -\theta \end{pmatrix} d\alpha + J \begin{Bmatrix} \sin(e^2\alpha_m - \theta) \\ + \sin\theta \end{Bmatrix} \qquad (55)$$

- *Total Energy Absorbed in Formation of One Complete Fold*

The total energy absorbed in the formation of one complete fold is thus given by

$$W_T = W_{T1} + W_{T2}$$

$$= \frac{\pi f_y t^2}{\sqrt{3}} \begin{bmatrix} 2R\alpha_m(1+h) + L(1+h)(1-2m) \\ \cos\theta - (1-2m)L \begin{Bmatrix} \cos(\alpha_m + \theta) \\ + h\cos(e^2\alpha - \theta) \end{Bmatrix} \end{bmatrix} + 2\pi t f_y L^2 (Y^* + h^2 Y_1^*) \qquad (56)$$

- *Mean Collapse Load*

The final expression to find the mean collapse load corresponding to one complete fold is obtained by equating the total internal energy to the total work done by the applied load as

$$\frac{P_m}{P_o} = \frac{\frac{t}{2\sqrt{3}} \begin{Bmatrix} 2R\alpha_m(1+h) + L(1-2m) \\ (1+h)\cos\theta - L(1-2m) \\ \{\cos(\alpha_m + \theta) + h\cos(e^2\alpha_m - \theta)\} \end{Bmatrix} + L^2(Y^* + h^2 Y_1^*)}{R\left[L(1+h) - t - \frac{bL}{\alpha_m}\left(1 + \frac{h}{e}\right)\right]\cos\theta} \qquad (57)$$

- *Size of Fold and Folding Parameter*

The size of fold obtained by minimising P_m wrt L is given by

$$L = \frac{E_1 - E_2}{E_3} \qquad (58)$$

where

$$E_1 = \frac{P_m}{\pi f_y t}\left[(1+h) - \frac{2bh}{e\alpha_m}\right]\cos\theta \qquad (59)$$

$$E_2 = \frac{t(1-2m)}{\sqrt{3}} \begin{bmatrix} (1+h)\cos\theta - \cos(\alpha_m + \theta) \\ - h\cos(e^2\alpha_m - \theta) \\ - 2e^2 h\alpha_m(1-eh)\sin(e^2\alpha_m - \theta) \end{bmatrix} \qquad (60)$$

$$E_3 = 4(Y^* + h^2 Y_1^*) - 4h^2(1-eh) \left[b^2 \left\{ \sin\begin{pmatrix} e^2\alpha_{1m} \\ -\theta \end{pmatrix} + \sin\theta \right\} + J\alpha_m e^2 \cos\begin{pmatrix} e^2\alpha_m \\ -\theta \end{pmatrix} \right] \qquad (61)$$

The expression for folding parameter can thus be obtained from

$$\frac{P_m bL(1-h)\cos\theta}{2\pi f_y t\alpha_m R} + \frac{t}{\sqrt{3}} \begin{Bmatrix} (1+h)\cos\theta - \cos\begin{pmatrix} \alpha_m \\ +\theta \end{pmatrix} \\ - h\cos(e^2\alpha_m - \theta) \end{Bmatrix} - L\left(\frac{\partial Y^*}{\partial m} + h^2 \frac{\partial Y_1^*}{\partial m}\right) + \frac{te^3(2m-1)(1-h)L\alpha_m \sin(e^2\alpha_m - \theta)}{\sqrt{3}R} = 0$$

where

$$\frac{\partial Y^*}{\partial m} = \sin\theta - \sin(\alpha_m + \theta) \qquad (62)$$

$$\frac{\partial Y_1^*}{\partial m} = \frac{2eL(1-h)}{hR} \begin{bmatrix} b^2\{\sin\theta + \sin(e^2\alpha_m - \theta)\} \\ + Je^2\alpha_m \cos(e^2\alpha_m - \theta) \end{bmatrix} - \{\sin(e^2\alpha_m - \theta) + \sin\theta\} \qquad (63)$$

Contributors

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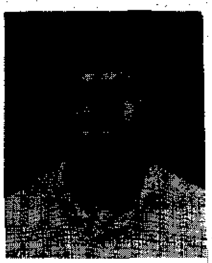

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