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SHORT COMMUNICATION

Generalised Measures of Useful Directed Divergence and Information Improvement with Applications

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ABSTRACT

The present communication describes a new generalised measure of useful directed divergence based on m–1 probability distributions, and a probability distribution closest to these probability distributions has been proposed. The technique has been applied in solving problems related to crops production, export, and industries. Further, a generalised measure of useful information improvement has been developed and its applications in the assessment of balanced military requirements for a country, in ranking and pattern recognition, have been discussed.

NOMENCLATURE

$E_1, E_2,..., E_n$	Events
$p_1, p_2,..., p_n$	Probabilities of events
$u_1, u_2,..., u_n$	Utilities of events
P, Q, P^* $P_1, P_2,..., P_n$	Probability distributions
U, V	Utility distributions
α	Parameter
A	Constant
D	Measure of useful directed divergence
I_n, $\overline{I_n}$	Measures of useful information improvement

1. INTRODUCTION

Consider a probability distribution

$$P = \{(p_1, p_2, \ldots , p_n), \ p_i > 0, \ \sum_{i=1}^{n} p_i = 1\}$$

together with a utility distribution $U = (u_1, u_2,..., u_n)$, where $u_i > 0$ is the utility or importance of the i^{th} event whose probability of occurrence is p_i. Belis and Guiasu[1] proposed a quantitative-qualitative measure of information, called useful information by Longo[2], as

$$H\,(P;\,U) = -\sum_{i=1}^{n} u_i\, p_i \ \log p_i \tag{1}$$

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This measure was also characterised by Sharma, Mitter and Mohan[3], and Aggarwal and Picard[4]. Later on, Bhaker and Hooda[5] characterised two measures of useful information for a generalised probability distribution and utility distribution attached to a random variable.

$$H_1\ (P;\ U) = -\frac{\sum_{i=1}^{n} u_i\ p_i \log p_i}{\sum_{i=1}^{n} u_i\ p_i} \tag{2}$$

and

$$H_\alpha\ (P;\ U) = \frac{1}{1-\alpha} \log\left(\frac{\sum_{i=1}^{n} u_i\ p_i^\alpha}{\sum_{i=1}^{n} u_i\ p_i}\right) \tag{3}$$

If one considers P as a probability distribution of a set of n events on the basis of an experiment whose prior probability distribution is:

$$Q = (q_1,\ q_2, \ldots,\ q_n);\ q_i > 0,\ \sum_{i=1}^{n} q_i = 1$$

then Kullback-Leibler's measure of directed-divergence information that P provides about Q is given by

$$H\ (P/Q) = \sum_{i=1}^{n} p_i \log\left(\frac{p_i}{q_i}\right) \tag{4}$$

In case, one attaches utility distributions U and V to the probability distributions P and Q, respectively, of measure [Eqn (4)], then one has the following two utility information schemes:

$$S = \begin{bmatrix} E_1 & E_2 \ldots, & E_n \\ p_1 & p_2 \ldots, & p_n \\ u_1 & u_2 \ldots, & u_n \end{bmatrix};\ p_i,\ u_i > 0;\ \sum_{i=1}^{n} p_i \leq 1 \tag{5}$$

$$S^* = \begin{bmatrix} E_1 & E_2 \ldots, & E_n \\ q_1 & q_2 \ldots, & q_n \\ v_1 & v_2 \ldots, & v_n \end{bmatrix};\ q_i,\ v_i > 0;\ \sum_{i=1}^{n} q_i \leq 1 \tag{6}$$

Where S and S^* are the posterior and the prior utility information schemes of an experiment.

Hooda[6] characterised a quantitative-qualitative measure of direct-divergence that utility information scheme [Eqn(5)] provides about utility information scheme [Eqn(6)], which was called useful relative information measure and was given by

$$D\ (P/Q;\ U/V) = \sum_{i=1}^{n} \frac{u_i\ p_i}{v_i} \log\left(\frac{p_i}{q_i}\right) \tag{7}$$

The measure [Eqn(7)] reduces to [Eqn(4)] when utilities are identical, ie, $u_i = v_i$ for each i, and vanishes if either $u_i = 0$ or $p_i = 0$ for each i.

Later on, considering u_i independent of p_i and directly proportional to its importance, Taneja and Tuteja[7] characterised axiomatically the quantitative-qualitative measure of relative information as

$$D\ (P/Q;\ U) = \sum_{i=1}^{n} u_i\ p_i \log\left(\frac{p_i}{q_i}\right) \tag{8}$$

The measure [Eqn(8)] was characterised and generalised for complete probability distributions by various authors. Bhaker and Hooda[5] considered P and Q as the posterior and prior generalised probability distributions, respectively, of an experiment having utility distribution U and characterised the following measures of useful relative information:

$$D\ (P/Q;\ U) = \frac{\sum_{i=1}^{n} u_i\ p_i \log\left(\frac{p_i}{q_i}\right)}{\sum_{i=1}^{n} u_i\ p_i} \tag{9}$$

and

$$D\alpha\ (P/Q;\ U) = \frac{1}{1-\alpha} \log \frac{\sum_{i=1}^{n} u_i\ p_i^{\alpha} q_i^{1-\alpha}}{\sum_{i=1}^{n} u_i\ p_i}, \alpha \neq 1 \qquad (10)$$

The measure [Eqn(10)] reduces to [Eqn(9)] when $\alpha \to 1$ and further to Kullback-Leibler's measure in case the utilities are ignored, ie, $u_i = 1$ for each i.

The measure $D(P/Q;\ U)$ is a valid measure only if it satisfies the following conditions:

Condition 1

$D\ (P/Q;\ U) > 0$

Condition 2

$D\ (P/Q;\ U) = 0$ if $p_i = q_i$ for each i.

Condition 3

$D\ (P/Q;\ U)$ is a convex or pseudo-convex function of $p_1, p_2, \ldots, p_n$ as well as of $q_1, q_2, \ldots, q_n$.

Condition 4

$D\ (P/Q;\ U)$ is permutationally symmetric, ie, it does not change when triplets (p_1, q_1, u_1), $(p_2, q_2, u_2), \ldots, (p_n, q_n, u_n)$ are permuted among themselves.

It may be noted that the measures Eqn (8), Eqn (9) and Eqn (10) satisfy all the conditions except *Condition* 1 which holds if

$$\sum_{i=1}^{n} u_i\, p_i \geq \sum_{i=1}^{n} u_i\, q_i$$

Some physical problems appear where $D\ (P/Q;\ U)$ is minimised with constraints $\sum_{i=1}^{n} u_i\, p_i \geq \sum_{i=1}^{n} u_i\, q_i$.

For example a manufacturer makes n different types of articles in proportions $q_1, q_2, \ldots, q_n$ having profits of $u_1, u_2, \ldots, u_n$, respectively. On the basis of labour and raw material, one plans to change the proportion of $p_1, p_2, \ldots, p_n$ however, close to $q_1, q_2, \ldots, q_n$, such that the average profit may not decrease, ie,

$$\sum_{i=1}^{n} u_i\, p_i \geq \sum_{i=1}^{n} u_i\, q_i$$

To obtain p_i's, one minimises:

$$D(P/Q;\ U) = \frac{\sum_{i=1}^{n} u_i\ p_i\ \log\left(\frac{p_i}{q_i}\right)}{\sum_{i=1}^{n} u_i\ p_i}$$

with constraints $\sum_{i=1}^{n} u_i\, p_i \geq \sum_{i=1}^{n} u_i\, q_i$

Kapur[8] has suggested a weighted directed-divergence measure corresponding to Kullback-Leibler's measure as

$$D(P/Q;\ W) = \sum_{i=1}^{n} w_i \left(p_i \log \frac{p_i}{q_i} - p_i + q_i \right) \qquad (11)$$

This measure satisfies all the above-mentioned conditions but its application needs to be studied.

2. GENERALISED MEASURES OF USEFUL DIRECTED DIVERGENCE

Most of the measures mentioned in Section 1 are additive in nature. The non-additive weighted directed-divergence measure or useful relative information measure was first introduced by Hooda[9] and Taneja[10] for generalised probability distributions. They characterised the following measure by different methods:

$$D^{\beta}(P/Q;U) = \frac{\sum_{i=1}^{n} u_i\ p_i \left[p_i^{\beta-1} q_i^{1-\beta} - 1\right]}{\left(2^{\beta-1} - 1\right)\sum_{i=1}^{n} p_i} \qquad (12)$$

Ram[11] has made an extensive study of non-additive generalised measures of useful relative information and J-divergence.

Let $P_r = (p_{1r}, p_{2r}, ..., p_{nr})$; $r = 1, 2, ..., m$ be m probability distributions of random variables of an experiment with utility distribution $U = (u_1, u_2, ..., u_n)$ such that $u_i > 0$ is qualitative characteristics of an event E_i and is independent of its probability of occurrence. Then, the sum of useful directed-divergence measures of P_1 from the other $m-1$ probability distributions is given by

$$D[P_1/(P_2, P_3, ..., P_m); U] = \sum_{r=2}^{m} D(P_1/P_r; U)$$

$$= D(P_1/P_2; U) + D(P_1/P_3; U) + ... + D(P_1/P_m; U)$$

$$= \frac{\sum_{i=1}^{n} u_i p_{i1} \log\left(\frac{p_{i1}}{p_{i2}}\right)}{\sum_{i=1}^{n} u_i p_{i1}} + \frac{\sum_{i=1}^{n} u_i p_{i1} \log\left(\frac{p_{i1}}{p_{i3}}\right)}{\sum_{i=1}^{n} u_i p_{i1}} + ... + \frac{\sum_{i=1}^{n} u_i p_{i1} \log\left(\frac{p_{i1}}{p_{im}}\right)}{\sum_{i=1}^{n} u_i p_{i1}}$$

$$= \left[\sum_{i=1}^{n} u_i p_{i1} \log\left(\frac{p_{i1}}{p_{i2}}, \frac{p_{i1}}{p_{i3}}, ..., \frac{p_{i1}}{p_{im}}\right)\right] \Big/ \sum_{i=1}^{n} u_i p_{i1}$$

$$= \left[\sum_{i=1}^{n} u_i p_{i1} \log\left(\frac{p_{i1}}{(p_{i2} \cdot p_{i3} ...., p_{im})^{\frac{1}{m-1}}}\right)^{m-1}\right] \Big/ \sum_{i=1}^{n} u_i p_{i1}$$

$$= (m-1) \frac{\sum_{i=1}^{n} u_i p_{i1} \left(\log \frac{p_{i1}}{p_i^*}\right)}{\sum_{i=1}^{n} u_i p_{i1}} \tag{13}$$

where p_i^* is the geometric mean of i^{th} component of $P_2, P_3, ... P_m$, and

$$\sum_{i=1}^{n} u_i p_{i1} \geq \sum_{r=2}^{m} \sum_{i=1}^{n} u_i p_{ir} \, (m-1)$$

Let
$$\sum_{i=1}^{n} p_{i1}^* = A \tag{14}$$

so that

$$p^* = \left(\frac{p_2^*}{A}, \frac{p_3^*}{A}, ..., \frac{p_m^*}{A}\right) \tag{15}$$

is a probability distribution, then

$$D[P_1/(P_2, P_3, ..., P_m); U]$$

$$= (m-1)\left(\frac{\sum_{i=1}^{n} u_i p_{i1} \log\left(\frac{p_{i1}}{p_i^*/A}\right)}{\sum_{i=1}^{n} u_i p_{i1}} - \log A\right)$$

$$= (m-1)\,[D\,(P_1/P^*; U) - \log A] \tag{16}$$

This is minimum when $P_1 = P^*$, where A and P^* are given by Eqns(14) and (15), respectively. Thus, P^* is the probability distribution which is closest to P_2, ..., P_m in the sense that the sum of useful directed-divergence measure from P^* to $(m-1)$ distributions $P_2, ..., P_m$ is minimum. This technique can be applied to solve various types of problems.

Problem 1

In 1997 and 1998, a farmer has sown wheat, gram, and barley in area proportions (0.5, 0.5, 0.2) and (0.45, 0.25, 0.30), respectively. The yield of these crops per unit area was estimated as 900 kg, 500 kg and 600 kg, respectively. Estimate area proportions for these crops for the year 1999, however, close to the given proportions, such that the expected yield may be more than the average of yields of 1997 and 1998.

Solution

Let $P_1 = (0.5, 0.3, 0.2)$, $P_2 = (0.45, 0.25, 0.30)$

and U = (900, 500, 600)

then P* = (0.477, 0.276, 0.247)

$$\sum_{i=1}^{3} u_i p_{i1} = 450 \text{ kg} + 150 \text{ kg} + 120 \text{ kg} = 720 \text{ kg}$$

$$\sum_{i=1}^{3} u_i p_{i2} = 405 \text{ kg} + 125 \text{ kg} + 180 \text{ kg} = 710 \text{ kg}$$

$$\sum_{i=1}^{3} u_i p_i^* = 4293 \text{ kg} + 138 \text{ kg} + 148.2 \text{ kg} = 7155 \text{ kg}$$

It is seen that

$$\sum_{i=1}^{3} u_i p_i^* \geq \sum_{r=1}^{2} \sum_{i=1}^{3} \frac{u_i p_{ir}}{2}$$

and

$$D\,[P^*/P_1, P_2);\, U] = 0.1115 > 0$$

Thus (0.474, 0.276, 0.247) are area proportions closest to P_1 and P_2 and satisfying the given condition.

Problem 2

Rice, wheat, cotton, and tea are exported to three countries in proportions (0.2, 0.4, 0.3, 0.1), (0.3, 0.2, 0.3, 0.2) and (0.2, 0.3, 0.1, 0.4), respectively. Their export price rate per quintal are Rs 800, Rs 500, Rs 600 and Rs 400, respectively. These goods are to be exported to 4th country in proportions closest to the proportion of three countries, such that the expected export to 4th country may not be less than the average of exports to these countries. Estimate the proportions.

Solution

Let P_1 = (0.2, 0.4, 0.3, 0.1)
P_2 = (0.3, 0.2, 0.3, 0.2)
P_3 = (0.2, 0.3, 0.1, 0.4)
U = (800, 500, 600, 400)

then $\sum_{i=1}^{4} u_1 p_{i1} = 500$; $\sum_{i=1}^{4} u_1 p_{i2} = 600$

$$\sum_{i=1}^{4} u_i p_{i3} = 530$$

$$P^* = (0.22, 0.28, 0.31, 0.19)$$

and

$$\sum_{i=1}^{4} u_i p_i^* = 578$$

It is seen that

$$\sum_{i=1}^{4} u_i p_i^* > \sum_{r=1}^{3} \sum_{i=1}^{4} \frac{u_i p_{ir}}{3}$$

and

$$D[P^*/(P_1, P_2, P_3);\, U] = 0.146 > 0$$

Thus, P^* = (0.22, 0.28, 0.31, 0.19) are the required proportions.

Problem 3

Three items are manufactured by three industries in proportions (0.5, 0.3, 0.2), (0.4, 0.3, 0.3), (0.3, 0.2, 0.5), respectively. The sale price of these items per thousand units are Rs 1000, Rs 800 and Rs 1200, respectively. Another industry also plans to manufacture these items in the proportions closest to the given proportions, such that the expected sale may be more than the average of sales of these industries estimate the proportions.

Let P_1 = (0.5, 0.3, 0.2)
P_2 = (0.4, 0.3, 0.3)
P_3 = (0.3, 0.2, 0.5)
U = (1000, 800, 1200)

then P* = (0.405, 0.273, 0.322)

$$\sum_{i=1}^{3} u_i\, p_{i1} = 980\ ;\ \sum_{i=1}^{3} u_i\, p_{i2} = 1000$$

$$\sum_{i=1}^{3} u_i\, p_{i3} = 1060 \text{ and } \sum_{i=1}^{3} u_i\, p_{i1} = 1009.4$$

here $\sum_{i=1}^{3} u_i\, p_i^{*} < \sum_{i=1}^{3}\sum_{r=1}^{3} \frac{u_i\, p_{ir}}{3}$ and

$$D[P^{*}/(P_1, P_2, P_3);\ U] = -0.167.$$

In this case, the estimated proportions are not valid, since $D\,[P^{*}/(P_1, P_2, P_3);\ \mathrm{U}] < 0$. From these illustrations, or otherwise, one can infer that

$$\sum_{i=1}^{n} u_i\, p_i^{*} <> \sum_{r=2}^{m}\ \sum_{i=1}^{n} \Leftrightarrow D[P^{*}/(P_2, P_3,\ldots, P_m);\ \mathrm{U}] <> 0$$

3. USEFUL INFORMATION IMPROVEMENT MEASURES

Let $\Delta_n = \{P = (p_1, p_2,\ldots, p_m);\ p_i \geq 0,\ i = 1, 2,\ldots, n;\ \sum_{i=1}^{n} p_i = 1\},\ n \geq 1$ be a set of all finite discrete probability distributions. Theil[11] defined a measure of information given by

$$I_n\ (P/Q/R) = \sum_{i=1}^{n} P_i\ \log\left(\frac{r_i}{q_i}\right) \tag{17}$$

where $R = (r_1, r_2,\ldots, r_n) \in \Delta_n$ is the revised prediction probability distribution of a set of n events $E_1, E_2,\ldots, E_n$ whose original predicted probability distribution was $Q = (q_1, q_2,\ldots, q_n) \in \Delta_n$ and the revision is made on the basis of observed probability distribution $\mathrm{P} \in \Delta_n$. The measure Eqn(17) is known as Theil's information improvement, and has been characterised by various authors.

Let $U = (u_1, u_2,\ldots, u_n);\ u_i > 0$, be a utility distribution attached to the set of event, such that u_i is the importance or qualitative characteristic or usefulness of an event E_i. Singh and Bhardwaj[12] defined and characterised the following weighted information measure:

$$I_n\ (p/Q/R;\ U) = \frac{\sum_{i=1}^{n} u_i\, p_i\ \log\left(\frac{r_i}{q_i}\right)}{\sum_{i=1}^{n} u_i\, p_i} \tag{18}$$

However, one can have situations where instead of one revision, several revised utility distributions may be available and an experimenter has to choose one distribution from several revised utility distributions or to have different experimenters (a finite number), who are allowed to choose a suitable revision.

Let $P_j = (p_{1j}, p_{2j},\ldots, p_{nj})$ be m probability distributions, each of which has attached with it an utility distribution $U = (u_1, u_2,\ldots, u_n)$, where $u_i > 0$ is the utility or qualitative characteristic of an event whose probability of occurrence is p_{1j}. However, in general u_i is independent of p_{1j}.

Let P_1 be the true proportion vector which was initially estimated as P_m, but due to new developments it was revised to $p_2, p_3,\ldots, p_{m-1}$, then $m-2$ useful information improvement measures are:

$$I_n(P_1/P_m/P_2;\ U),\ I_n(P_1/P_m/P_3;\ U),\ldots,\ I_n(P_1/P_m/P_{m-1};\ U)$$

The average of these measures is given by

$$\bar{I}_n = \frac{1}{m-2}\left[\frac{\sum_{i=1}^{n} u_i\, p_{i1} \log \prod_{r=2}^{m-1} \frac{p_{ir}}{p_{im}}}{\sum_{i=1}^{n} u_i\, p_{i1}}\right]$$

$$= \frac{\sum_{i=1}^{n} u_i\, p_{i1} \log \frac{p_i^{*}}{p_{im}}}{\sum_{i=1}^{n} u_i\, p_{i1}} \tag{19}$$

where p_i^{*} is the geometric mean of the i^{th} component of $P_2, P_3,\ldots, P_{m-1}$ probability distributions. Thus

$$\bar{I}_n = \frac{\sum_{i=1}^{n} u_i\, p_{i1} \log\left(\frac{p_i^*/A}{p_{im}}\right)}{\sum_{i=1}^{n} u_i\, p_{i1}} + \log A$$

$$= I_n\,(P_1/P_m/P^*;\ U) + \log A \qquad (20)$$

where

$$A = \sum_{i=1}^{n} p_i^*$$

and

$$P^* = \left(\frac{p_1^*}{A}, \frac{p_2^*}{A}, \ldots, \frac{p_n^*}{A},\right) \qquad (21)$$

It has a meaningful interpretation in terms of improvement of useful information in revising the estimate of P_1 from P_m to P^*, where P^* is the probability distribution closest to $P_2, P_3, \ldots, P_{m-1}$.

4. APPLICATIONS

The measures proposed earlier may find various applications. Some of these are:

(i) Let p_{ij} ($i = 1, 2, \ldots, n$; $j = 1, 2, \ldots, m$) represents the proportions of defence budget allocated to i^{th} category of weapons system (say tanks, missiles, fighter planes, etc.) by the j^{th} country and u_i be the operational capability of a particular weapon system. In general, u_i is independent of the country. So, P^* in Eqn (21) can be interpreted as the appropriate choice, ie, the first country should have the proportion of different categories of weapon system closest to other countries. If the quantity given by Eqn(20) is positive, one can say that operational capability of the country has improved by revising the estimate of P_1 from P_m to P^* and revision is advantageous.

(ii) One knows that the reliability plays an important role with design and development of hardware system [Kapur and Lamberson[13]]. This is a probability of a device to perform its purpose adequately for an intended time (mission) under given environmental condition. If reliability factor is attached to mathematical model Eqn(20), then one has proportions integrated with reliability and utility:

Let r_{ij} ($0 \le r_{ij} \le 1$); $i = 1, 2, \ldots, n$; $j = 1, 2, \ldots, m$; is the reliability factor assigned to the i^{th} category of weapon system by the j^{th} country. P_{ij} and u_i have the same meaning as mentioned above in (i).

$$\text{Define } p'_{ij} = \frac{r_{ij}\, p_{ij}}{\sum_{i=1}^{n} u_{ij}\, p_{ij}} = ;\ j = 1, 2, \ldots, m \qquad (22)$$

and

$$p'_j = \left(p'_{1j}, p'_{2j}, \ldots, p'_{nj}\right)$$

then

$$\bar{I}' = \frac{1}{m-2}\left[\frac{\sum_{i=1}^{n} u_i\, p'_{i1} \log \frac{p_i^{**}}{p'_{im}}}{\sum_{i=1}^{n} u_i\, p'_{i1}}\right]$$

where p_i^{**} is the geometric mean of the i^{th} component of $P'_2, P'_3, \ldots, P'_{m-1}$ distributions. Consequently

$$\bar{I} = I_n\left(P'_1/P'_m/P^{**};\ U\right) + \log A$$

where

$$A = \sum_{i=1}^{n} p_i^{**}, \text{ so that}$$

$$P^{**} = \left(\frac{p_1^{**}}{A}, \frac{p_2^{**}}{A}, \ldots, \frac{p_n^{**}}{A}\right)$$

Thus P^{**} is a probability distribution of different categories of weapon system coupled with reliability and utility factors which is the closest to the given proportions of categories of weapon system of other countries. In this way, one can obtain proportions of different categories of weapon system along with reliability and utilities which at least a country should have to meet the challenge of future battle.

(iii) Suppose there are different categories of communication, say, friendly, conciliatory, indifferent, hostile and threatening. Let p_{ij} be the proportion of the i^{th} category of messages, communicated by the j^{th} country. To measure an external threat perception [Banerjee[14]] one needs a measure of directed-divergence measure of one country from another or from a group of countries and the measure given by Eqn (16) can be an appropriate measure.

(iv) One can use the methodology developed in sections 2 and 3 in measuring difference of opinion in a group of persons, in ranking and selection, in pattern recognition, etc. with a proper interpretation of utility distribution according to situation being encountered.

(v) The measure can also be applied in preparation of a country budget where many revisions are effected due to changing situations like drought, floods, inflation, recession, etc.

5. GENERAL DISCUSSION & CONCLUSIONS

(a) The measure [Eqn (20)] developed in the present paper is obtained by taking the average of (m–2) improvements of useful information. One could also consider groups of probability distributions and obtain a new measure by reducing it into three groups of probability distributions. Thus one can develop a new measure by taking other central tendency measure instead of taking arithmetic mean.

(b) There are a number of parametric generalised measure of information available in the literature. One can consider these and apply the same methodology to get more generalised directed-divergence measures coupled with reliability and utility factor, which may have Kullback-Leibler measure with utility as limiting case. The role of involved parameter in the measure gives accountability of the goodness of fit.

(c) The data required to apply these measures to assess military requirements, one needs to get the proportions of defence budget, proportions of different categories of weapon system, applicability and operational capacity with their reliability from various countries. Such data can be collected from our country, but difficult to get the exact figure of other countries.

(d) The estimated proportions are not valid in case of *Problem* 3. Now it is an open problem, "Can one find the required proportions by taking any mean other than geometric or by any new technique?"

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