

SHORT COMMUNICATION

## Calculation of Overall Effective Extinction Cross Section of Several Cold Smoke Infrared Ammunitions

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### ABSTRACT

The effective extinction cross section is an important parameter of the cold smoke infrared ammunition. This paper discusses the calculation of the maximal effective extinction cross section of the smoke produced by several smoke ammunitions and gives the optimal distance between the centres of the two adjacent smoke spheres. This study can provide theoretical basis for proper use of the smoke ammunition and the optimisation of the discharger.

**Keywords:** Cold smoke infrared ammunition, effective extinction cross section, optimal distance, infrared technology, countermeasure devices, smoke ammunition, dischargers, cold smoke agent

### 1. INTRODUCTION

With the development of infrared technology, more and more infrared reconnaissance and guided systems are used in the modern battlefields. As a result, both the fighting forces use countermeasure devices to well protect their own personnel and equipment. As one of these devices, the cold smoke infrared ammunition, is widely used because of its low cost, good effect, easy operation, and free mobility. In practice, the smoke ammunition is projected between the potential target and the threatening infrared systems by the dischargers. When the smoke ammunition explodes, the smoke agent is thrown into the air and forms obscuring smoke. The obscuring smoke can greatly attenuate the infrared radiation emitted by the target, and thus, deteriorates the efficiency of the infrared systems severely. Much has been done in selecting the cold smoke agent<sup>1,3</sup>. Sometimes, one needs to make several smoke ammunitions to explode

simultaneously and close to each other to get a large area of smoke. Usually, the shape of the obscuring smoke made by one ammunition is approximately spherical. Because of the effect of the turbulence, the density of the smoke can be regarded uniform. When the infrared radiation passes through this smoke sphere, the path length is not equal at different sites (for instance, when the infrared radiation passes through the centre of the sphere, its path length is the longest. On the contrary, when the infrared radiation passes at the edge of the sphere, the path length is the shortest). The cross section through which the transmissivity is less than a specified value, is the effective extinction cross section. That is to say, the effective extinction cross section made by a smoke ammunition is not the whole cross section of the smoke. In fact, it equals to the cross section of a certain homocentric sphere with a smaller radius. When one uses several smoke infrared ammunitions to create a large area of obscuring smoke and, to

make full use of the smoke agent, one has the two adjacent smoke spheres overlap partly. Here, the overall effective extinction cross section cannot be calculated by simply adding every effective extinction cross section. The practical effective extinction cross section may augment as a result of overlapping. An important issue is the determination of the optimal distance between the adjacent smoke spheres. A precondition is that the two adjacent smoke ammunitions' effective extinction cross section should be continuous. Besides this, the effective extinction cross section should approach the maximum. Moreover, if there are three or more smoke ammunitions exploding simultaneously, arranging the exploding positions of the smoke ammunitions to create the maximal overall effective extinction cross section is worth investigating. In this paper, the above-mentioned issues have been discussed.

**2. DERIVATION OF OPTIMAL DISTANCE OF THE CENTRES OF TWO ADJACENT SMOKE SPHERES**

Figure 1 shows the model of the obscuring smoke made by a smoke ammunition. Here,  $R$  is the radius of the whole smoke sphere,  $r$  is the radius of the circular effective extinction cross section,  $l$  is the path length of the infrared radiation through the smoke sphere,  $I_0$  is the incident irradiance and  $I$  is the transmitted irradiance.

Based on the Bouguer-Lambert's law<sup>4</sup>, one gets:

$$I = I_0 \exp(-pp/l) \tag{1}$$

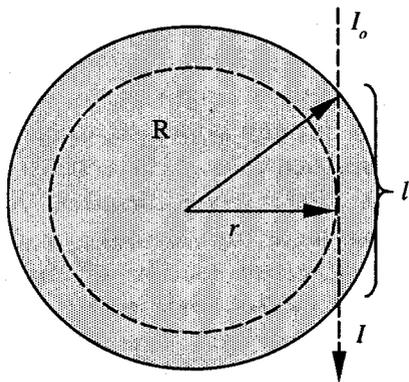


Figure 1. Effective extinction cross section of a smoke sphere

where  $P$  is the mass extinction coefficient of the smoke agent and  $p$  is the density of the smoke. Equation (1) can also be written as

$$I = I_0 \exp\left(-\frac{m}{V}P/l\right) \tag{2}$$

where  $m$  is the mass of the obscurant and  $V$  is the volume of the smoke sphere.

From Eqn (2), one obtains:

$$l = -\frac{V}{m\beta} \ln \frac{I}{I_0} \tag{3}$$

If the transmissivity is expressed by  $T = \frac{I}{I_0}$  and  $V = \frac{4\pi R^3}{3}$ , Eqn (3) becomes:

$$l = -\frac{4717R^3}{3np} \ln \tau \tag{4}$$

Equation (4) gives the shortest path length in the smoke sphere, which meets the expected transmissivity ( $T$ ) for the given  $R, m$ , and  $P$ ,

After derivation, one can also get the radius corresponding to the maximal effective extinction cross section ( $r_m$ ):

$$r_m = \left( \frac{\beta m}{>37i|\ln T|} \right)^{\frac{1}{2}} \tag{5}$$

and the radius of the whole smoke sphere is:

$$R_m^2 = \frac{\sqrt{3}\beta m}{2\pi|\ln \tau|} \tag{6}$$

Practically, to obtain a large smoke area, sometimes more than one ammunition is used. In this case, one hopes that the effective extinction areas connect with each other. That is to say on the line that connects the centres of the smoke spheres, the attenuation should exceed the expected level everywhere.

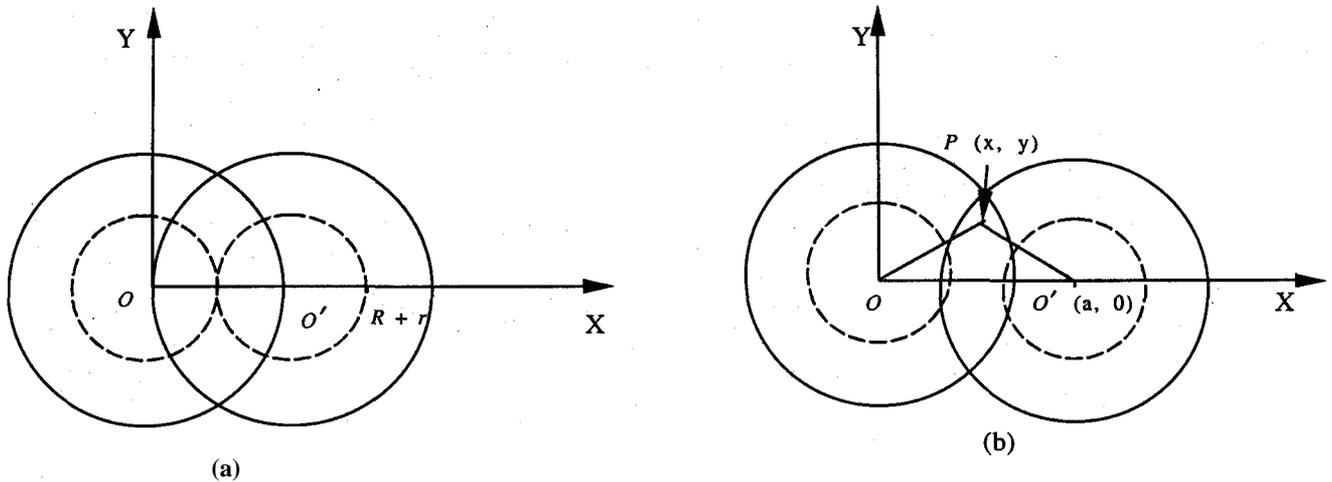


Figure 2. Overlapping of the two spheres: (a) two smaller circles are tangent and (b) two smaller circles are separated

Consider the case of two smoke spheres. If the distance between the centres of two smoke spheres is  $2r$ , as shown in the Fig. 2 (a), the above requirement is satisfied in the overlapping field of the two larger circles, even at some point outside the two smaller circles, since at any point on the line connecting the centres of the two smoke spheres, the equivalent path length exceeds the expected level. Hence, the overall effective extinction cross section of the two smoke spheres cannot be obtained by simply adding the effective extinction cross sections of the two spheres. Actually, it is larger than their sum. From Fig. 2 (a), one can also find that the distance between the centres of two smoke spheres should not exceed  $R + r$ . Otherwise, the path length cannot reach the expected value, where  $x$  is hardly bigger than  $r$ . This will cause the disconnection of the smoke screen. From this, one concludes that the distance between the two smoke spheres should not be shorter than  $2r$ , to avoid the overlapping of the two smaller circles, which will counteract the increase of the overall effective extinction cross section. So, the distance between the centres  $|OO'|$  of the two spheres should not be less than  $2r$  and not more than  $R + r$ . It can be proved that the optimal distance from point  $O$  to point  $O'$  is  $R + r$ .

As shown in the Fig. 2 (b), on the assumption that the distance of the two smoke spheres is  $a$ , ie,  $|OO'| = a$ , one can get the total path length of the infrared radiation through a

certain point  $P(x,y)$  in the overlapping area of the two smoke spheres. For the circles with the centres  $O$  and  $O'$ , the path lengths are:

$$l_1 = \sqrt{2^2 R^2 - |OP|^2} = \sqrt{2^2 R^2 - (x^2 + y^2)} \quad (7)$$

and

$$l_2 = 2\sqrt{R^2 - |O'P|^2} = 2\sqrt{R^2 - [(x-a)^2 + y^2]} \quad (8)$$

So the total path length is:

$$l = l_1 + l_2 = 2\sqrt{R^2 - (x^2 + y^2)} + 2\sqrt{R^2 - [(x-a)^2 + y^2]} \quad (9)$$

Apparently, the effective extinction region is formed by the points where the following is satisfied:

$$l \geq 2\sqrt{R^2 - r^2} \quad (10)$$

or

$$\begin{aligned} & \sqrt{R^2 - (x^2 + y^2)} + \sqrt{R^2 - [(x-a)^2 + y^2]} \\ & \geq \sqrt{R^2 - r^2} \end{aligned} \quad (11)$$

Simplifying the inequality [Eqn (11)], one gets:

$$y^2 \leq R^2 - \frac{(R^2 + a^2 - 2ax - r^2)^2}{4(\hat{r}^2 - r^2)} - x^2 \quad (12)$$

The area enclosed by the edges of the two smaller circles and the inequality [Eqn (12)] is:

$$S = 2 \int_{x_1}^{x_2} \sqrt{R^2 - \frac{(R^2 + a^2 - 2ax - r^2)^2}{4(\hat{r}^2 - r^2)} - x^2} dx - 4 \int_{x_1}^r \sqrt{r^2 - x^2} dx \quad (13)$$

where  $x_1$  and  $x_2$  are the abscissa coordinates of the intersecting points of the two smaller circles and the edge of the region defined by inequality [Eqn (12)] and can be expressed by

$$x_1 = \frac{a^2 + r^2 - R^2}{2a} \quad (14)$$

and

$$x_2 = \frac{a^2 - (r^2 - R^2)}{2a} \quad (15)$$

From Eqn (13), one gets:

$$\frac{dS}{da} = R \frac{a^2 + r^2 - R^2}{a^2} + \left\{ \frac{1}{a} \sqrt{[(a+r)^2 - R^2][R^2 - (a-r)^2]} - r \right\} \times \frac{a^2 - r^2 + R^2}{a^2} \quad (16)$$

Let Eqn (16) be zero, one has:

$$a = R + r \quad (17)$$

which means that the effective extinction cross section reaches the maximum value when the distance between the centres of the two spheres is  $R + r$ . So the maximal value of  $S$  is:

$$S_m = 2 \sqrt{\frac{2R}{R-r}} \int_r^R \sqrt{-Rr + (R+r)x - x^2} dx = \frac{(R-r)}{2} \sqrt{\frac{R(R-r)}{2}} \quad (18)$$

Now the edge of the inequality [Eqn (12)] becomes:

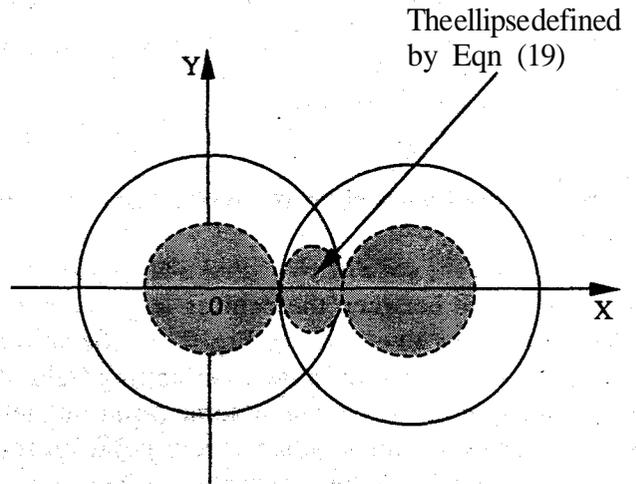


Figure 3. Effective extinction cross section wrt optimal distance between the centres of two spheres.

$$\left( \frac{x - \frac{R+r}{2}}{\left( \frac{R-r}{2} \right)} \right)^2 + \frac{y^2}{\left[ \frac{r^2 - R^2}{2} \right]^2} = 1 \quad (19)$$

which represents an ellipse (Fig 3).

Hence, the maximal overall effective extinction cross section of the two overlapping smoke spheres is:

$$S_2 = 2\pi r^2 + S_m = 2\pi r^2 + \frac{\pi(R-r)\sqrt{R(R-r)}}{2\sqrt{2}} \quad (20)$$

Similarly, the maximal overall effective extinction cross section made by  $N$  smoke spheres arranged in a line is:

$$S_N = N\pi r^2 + \frac{\pi(N-1)(R-r)\sqrt{R(R-r)}}{2\sqrt{2}} \quad (2.1)$$

When one uses three or more smoke ammunicions to make the obscuring smoke and to get the maximal overall effective extinction cross section, one should have each smoke sphere border upon the other smoke sphere as much as possible. This means that one should not arrange the smoke ammunicions in a line. For example, if four smoke ammunicions are put in two lines as shown in the Fig. 4, it is known that the maximal overall effective extinction cross section will be:

$$S_4 = 4\pi r^2 + \frac{5\pi(R-r)\sqrt{R(R-r)}}{2\sqrt{2}} + 2S_0 \quad (22)$$

where  $S_0$  is the area of the effective extinction cross section made by the overlapping of the three smoke spheres (Fig 4).

One can find that the maximal overall effective extinction cross section of the four smoke spheres in a line is  $\frac{2\pi(R-r)\sqrt{R(R-r)}}{\sqrt{2}} + 2S_0$  smaller than that in two lines.

### 3. DISCUSSION

In the above section, the expressions of the maximal overall effective extinction cross section generated by several smoke ammunicions for various

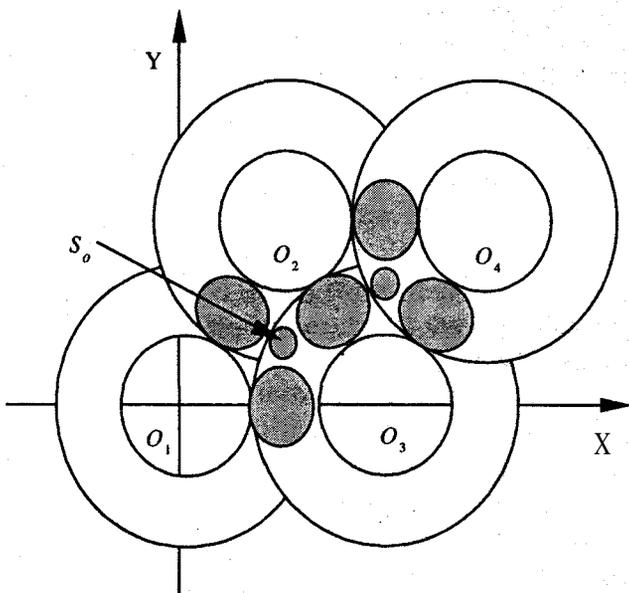


Figure 4. Arrangement of four smoke spheres

arrangements were derived. Following example illustrates the effect on the maximal overall extinction cross section of the arrangement.

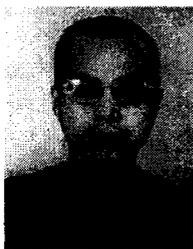
Consider 10 cold smoke ammunicions. The mass extinction coefficient of the smoke agent in the bomb is  $P = 0.6 \text{ m}^2/\text{g}$  and the mass of the obscurants in each projectile is 4 kg. It is hoped that the transmissivity of the effective extinction cross section area is not more than 10 per cent ( $T < 0.1$ ). Using Eqn (5), one gets that one smoke sphere's maximal effective radius is 13.8 m and the smoke's effective extinction cross section is  $598 \text{ m}^2$  when the smoke sphere's radius is 16.9 m. When 10 smoke ammunicions are in a line, the maximal overall effective extinction cross section is  $6204.2 \text{ m}^2$  according to Eqn (21). If these are in two lines, as shown in the Fig. 4, the maximal overall effective extinction cross section is  $6378.4 \text{ m}^2$ . It is also known that the maximal effective extinction cross section calculated by simply adding every effective extinction cross section of the 10 smoke spheres is  $5980 \text{ m}^2$ . It is obvious that different arrangements considerably affect the maximal overall effective extinction cross section.

This paper gives the optimal distance of two adjacent smoke ammunicions spheres, and discusses the optimal arrangement of several smoke ammunicions. This will help one to make full use of the obscurants and to optimally design the discharger.

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