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Estimation of Stability and Control Derivatives of Light Canard Research Aircraft from Flight Data

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ABSTRACT

In this paper, stability and control derivatives of light canard research aircraft generated through the flight test using parameter estimation technique has been presented. The maximum likelihood estimation, based on output-error minimisation technique, is used to estimate the derivatives from the aircraft response data. The validity of the estimates has been checked by the cross validation method, wherein the estimated model response is matched with the flight-test data that are not used for estimating the derivatives.

Keywords: Light canard research aircraft, LCRA, flight-test data, parameter estimation technique, maximum likelihood estimation, output-error minimisation, output-error method

NOMENCLATURE

a	Angle of attack
β	Angle of sideslip
8_e	Elevator deflection
δ_a	Aileron deflection
δ_r	Rudder deflection
p	Roll rate
q	Pitch rate
r	Yaw rate
a_n	Normal acceleration
6	Pitch attitude
ϕ	Roll attitude
q	Dynamic pressure
P_{st}	Static pressure
m	Aircraft mass
s	Reference wing area (7.62 m^2)
b	Wing span (7.995 m)
c	Mean aerodynamic chord (0.995 m)
U_0	Trim velocity
g	Gravitational acceleration
X_{α}	Distance of α-sensor from CG along X-axis
X_{β}	Distance of p-sensor from CG along X-axis
X_{a_n}	Distance of a_n sensor from CG along X-axis
I_{xx}, I_{yy}, I_{zz}, I_{xz}	Moment of inertia (computed from mass and CG pos at respective flight cond.)

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Variables with preposition A represents the bias value.

$C_{L_0}, C_{L_\alpha}, C_{L_q}, C_{L_{\delta e}}$	Lift coefficient derivatives
$C_{m_0}, C_{m_\alpha}, C_{m_q}, C_{m_{\delta e}}$	Pitching moment derivatives
$C_{y_0}, C_{y_\beta}, C_{y_p}, C_{y_r}, C_{y_{\delta r}}$	Side force derivatives
$C_{l_0}, C_{l_\beta}, C_{l}, C_{l_r}, C_{l_{\delta a}}, C_{l_{\delta r}}$	Roll moment derivatives
$C_{n_0}, C_{n_\beta}, C_{n_p}, C_{n_r}, C_{n_{\delta a}}, C_{n_{\delta r}}$	Yaw moment derivatives

1. INTRODUCTION

The generation of aerodynamic database through flight test and parameter estimation is well-recognised in the recent years and forms an essential step in any aircraft development programme, viz., flight control system design, simulator model update, etc. The method involves acquiring the necessary flight data by conducting appropriate flight tests, and then applying parameter estimation technique to estimate the desired aerodynamic parameters. In this method, the aircraft system under investigation is modelled by a set of dynamic equations, containing the unknown parameters. The system is excited by a suitable input and the input and the system responses are measured. The values of the unknown parameters are then estimated based on the requirement that the model response (to the same input) matches the actual system response.

The light canard research aircraft (Fig. 1), built at the National Aerospace Laboratories, Banglore, is an all-composite aircraft having canard configuration based on the design plan of Rutan Long-EZ procured from Rutan Aircraft Co, USA. It is a two-seater, high performance aircraft with canard and pusher propeller. It has a tricycle landing gear of which the nose wheel is retractable. It has a wing of moderate sweep with inboard and outboard strakes of appreciable sweep. The canard is an unswept surface of high aspect ratio having full-span trailing edge flaps, which form the elevator for pitch control. The winglets at the wing tips, not only reduce the induced drag but also have rudders in them. Unlike conventional rudder, the two rudders are independent

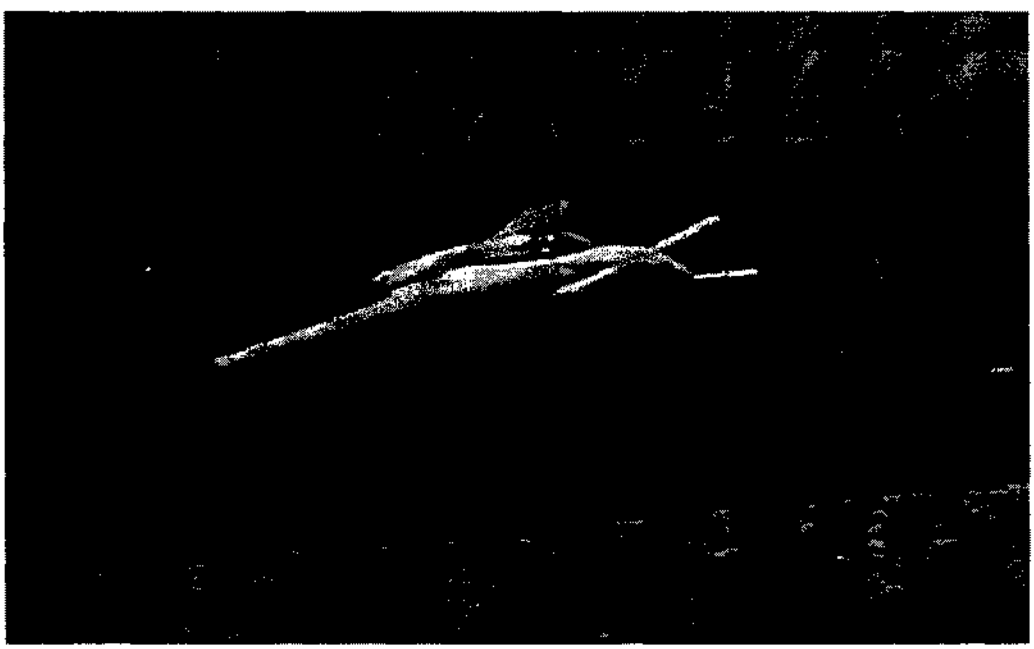

Figure 1. Light canard research aircraft built at the National Aerospace Laboratories

and can operate one at a time and deflect outwards only. Left pedal is connected to the port rudder and the right pedal is connected to the starboard rudder. The ailerons, situated in the wing, provide roll control.

2. MAXIMUM LIKELIHOOD METHOD

Stability and control derivatives, represented as unknown parameters in aircraft dynamical equations, are estimated by the output-error method[1] (OEM) based on maximum likelihood estimation (MLE) technique. In this method, the probability that the aircraft model response-time history attains values near to the measured aircraft response-time history is defined in terms of possible estimate of unknown parameters. Then, the maximum likelihood estimates are defined as those that maximise this probability. Maximum likelihood estimation has many desirable statistical characteristics, for example, it yields asymptotically unbiased, consistent and efficient estimates. The maximum likelihood estimation also provides a measure of reliability of each estimate based on the information obtained from each dynamic manoeuvre, called the Cramer-Rao bound. In the presence of measurement noise, Cramer-Rao bound is analogous to the standard deviation and provides an estimate of the uncertainty interval.

Figure 2 shows the block diagram of the output-error method, which is iterative in nature. The aircraft dynamics is mathematically postulated with initial guess values of unknown parameters (stability and control derivatives). This model response (for the same input) is compared with flight-measured aircraft response and the resulting response residual is used in cost function. The minimisation algorithm is used to estimate the unknown parameters, which minimise the cost function. These new estimates are then used to update the mathematical model, which, in turn, provides a new estimated response, and hence, a new response error. Thus, the mathematical model is continuously updated iteratively until a predetermined convergence criterion is satisfied.

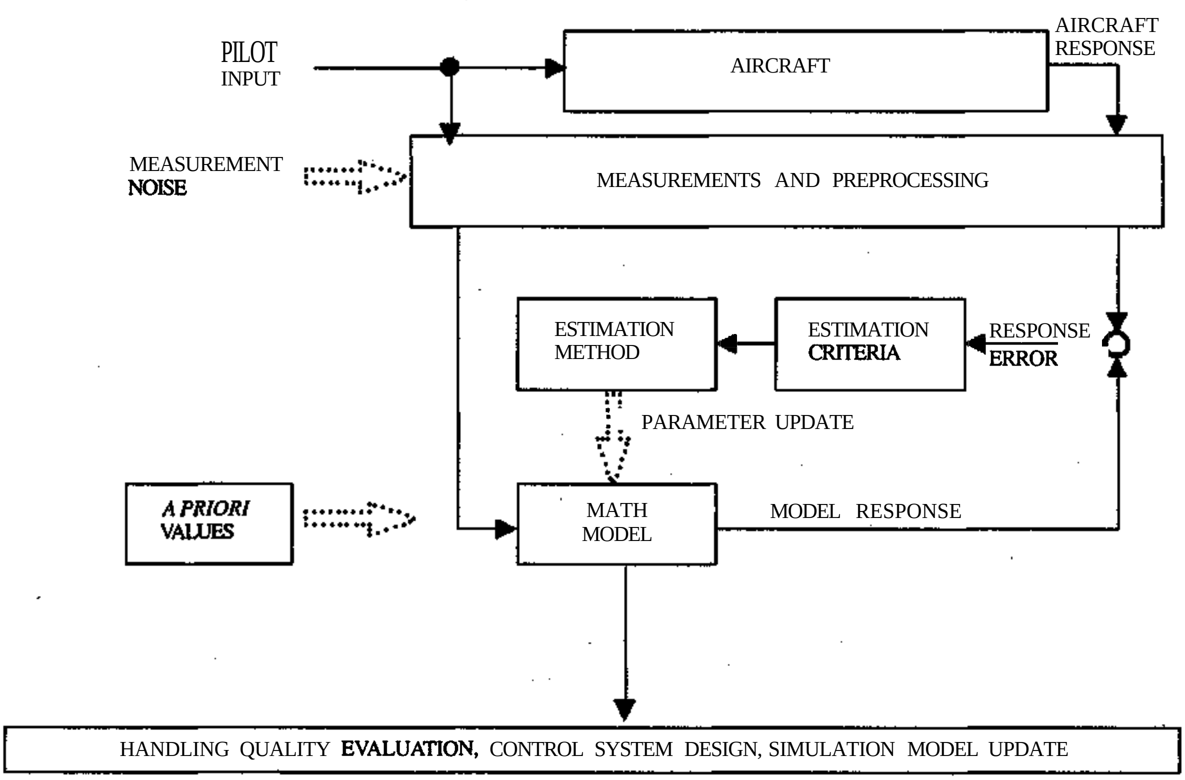

Figure 2. Parameter estimation of aircraft by the output-error method based on maximum likelihood estimation technique

3. FLIGHT TEST

The flight-test data for parameter estimation are generally acquired by carefully planned and conducted flight-test manoeuvres on the vehicles to derive maximum information on the characteristics of the vehicle. For this purpose, a flight-test program was drawn and the specification document was prepared, indicating the necessary manoeuvres to be executed and the quality and quantity of data that need to be acquired for parameter estimation. This flight-test program was carried out in 12 sorties of 45 min each, to cover all the flight conditions/aircraft configurations. The flight-test manoeuvres suitable for parameter estimation, viz., short period and Dutch roll were carried out at two altitudes, viz., 1524 m and 2743 m and at three different speeds, viz., 65 knot, 85 knot and 105 knot at each altitude. The experiments were repeated for forward CG and aft CG at several flight conditions. Also, experiments were carried out for nose wheel up as well as nose wheel down conditions to check the incremental effect of nose wheel on aerodynamic characteristics of the aircraft. Table 1 shows the complete flight-test matrix. Each experiment at each flight condition/configuration was repeated twice for consistency and also for cross validation purpose, wherein the average estimate from the first two experiments was used to simulate the model response and to compare with the flight response from the third experiment.

3.1 Longitudinal Derivatives Estimates from Short-period Flight Data

To estimate the longitudinal derivatives of the light canard research aircraft, during the flight test, the aircraft was excited with short-period manoeuvres by giving doublet input to the elevator. The different response signals of the aircraft as listed in Table 2 are measured during the flight test. The aircraft short-period model explained below is used to estimate the relevant longitudinal parameters from these response signals.

3.1.1 *Short-period Model*

State Equations

$$\dot{\alpha} = -\frac{\bar{q}s}{mU_0}\left[C_{L_0} + C_{L_\alpha}\alpha + C_{L_q}\frac{\bar{c}}{2U_0}q + C_{L_{\delta e}}\delta e\right] + q + \frac{g}{U_0}$$

$$\dot{q} = \frac{\bar{q}s\bar{c}}{I_{yy}}\left[C_{m_0} + C_{m_\alpha}\alpha + C_{m_q}q\frac{\bar{c}}{2U_0} + C_{m_{\delta e}}\delta e\right]$$

Measurement Equations

$$\alpha_m = \left(\alpha - \frac{X_\alpha}{U_0}q\right) + \Delta\alpha$$

$$q_m = q + \Delta q$$

$$a_{n_m} = \frac{1}{8}\left[\frac{\bar{q}s}{m}\left(C_{L_0} + C_{L_\alpha}\alpha + C_{L_q}\frac{\bar{c}}{2U_0}q + C_{L_{\delta e}}8e\right) + X_q q\right] + \Delta a_n$$

The trim velocity is computed from measured static and dynamic pressures as given below:

$$U_0 = \sqrt{\frac{2\bar{q}}{\rho}}$$

where p is the density of air which is given by

$$P = \frac{P_{st} * 68911.8}{RT}$$

R is the gas constant = 287.05 and T is the temperature of air = 273.15 + t °C (K)

Out of the eight derivatives present in the above equations, only the significant four derivatives, viz., $C_{L_\alpha}, C_{m_\alpha}, C_{m_q}$, and $C_{m_{\delta e}}$ are estimated directly. The derivative $C_{L_{\delta e}}$ is computed from $C_{m_{\delta e}}$ by knowing the moment arm length as given below:

$$C_{L_{\delta e}} = C_{m_{\delta e}}\frac{\bar{c}}{l_h}$$

where l_h is the distance between the aerodynamic centre of wing to the canard.

Table 1. Flight-test matrix

Flight number	Nose wheel		CG		Weight /inertia			Altitude		Speed		
	Up	Down	Aft	Fwd	Max (kg)	Min (kg)	I_{yy} (kg-m^2)	1524 (m)	2743 (m)	65 (knot)	85 (knot)	105 (knot)
386	X		X		646		760.3	X		X		
386	X		X		646		760.3	X			X	
386	X		X		646		760.3	X				X
385	X		X		640		760.1		X	X		
385	X		X		640		760.1		X		X	
385	X		X		640		760.1		X			X
383	X		X			579	762.5	X		X		
383	X		X			579	762.5	X			X	
383	X		X			579	762.5	X				X
382	X		X			573	762.3		X	X		
382	X		X			573	762.3		X		X	
382	X		X			573	762.3		X			X
374	X			X	666		853.7	X		X		
374	X			X	666		853.7	X			X	
374	X			X	666		853.7	X				X
373	X			X	660		853.5		X	X		
373	X			X	660		853.5		X		X	
373	X			X	660		853.5		X			X
379	X			X		582	822.4	X		X		
379	X			X		582	822.4	X			X	
379	X			X		582	822.4	X				X
377	X			X		576	822.2		X	X		
377	X			X		576	822.2		X		X	
377	X			X		576	822.2		X			X
380		X		X		582	822.4	X		X		
380		X		X		582	822.4	X			X	
380		X		X		582	822.4	X				X
378		X		X		576	822.2		X	X		
378		X		X		576	822.2		X		X	
378		X		X		576	822.2		X			X
375		X		X	666		853.7	X		X		
375		X		X	666		853.7	X			X	
375		X		X	666		853.7	X				X
376		X		X	660		853.5		X	X		
376		X		X	660		853.5		X		X	
376		X		X	660		853.5		X			X

It is difficult to estimate $C_{L_{\dot{\alpha}}}$ and $C_{m_{\dot{\alpha}}}$ independently using this technique because these correlate with other parameters and influence their accurate estimates. Their estimates do not really represent $C_{L_{\dot{\alpha}}}$ and $C_{m_{\dot{\alpha}}}$, instead these represent the bias estimates of C_L and C_m which have no significance from the flight mechanics point of view. The C_{L_q} is a secondary derivative and generally poorly identifiable.

The estimated derivatives at each flight condition /configuration are plotted along with standard deviation against trim angle of attack and compared

Table 2. Aircraft response signals recorded during the flight test of light canard research aircraft

Response signal		Remarks
T	Time (s)	
δ_e	Elevator deflection (deg)	Very reliable. Used as input in short-period model
δ_a	Aileron deflection (deg)	Very reliable. Used as input in Dutch roll model
δ_r	Rudder deflection (deg)	Very reliable. Used as input in Dutch roll model(= Left - Right rudder deflection)
p	Roll rate (deg/s)	Very reliable. Used as observation in Dutch roll model
q	Pitch rate (deg/s)	Very reliable. Used as observation in short-period model
r	Yaw rate (deg/s)	Very reliable. Used as observation in Dutch roll model
a_n	Norm Accn.(g)	Noisy but reliable. Used as observation in short-period model
$a*$	Forwd Accn.(g)	Very noisy and cannot be used
a_y	Lat Accn.(g)	Very critical for lateral derivatives estimate but not available
a_x	Forwd Accn.(g)	3 DM output. Not available most of the time
a_y	Lat Accn.(g)	3 DM output. Not available most of the time
a_n	Norm Accn.(g)	3 DM output. Not available most of the time
θ	Pitch Attitude (deg)	3 DM output. Not available most of the time
a	Angle of attack (deg)	Available but not calibrated. Used as observation in short-period model
p	Sideslip angle (deg)	Available but not calibrated. Used as observation in Dutch roll model
p_{st}	Static pressure	Reliable. Used for computing trim velocity, U_0
$\bar{q}$	Dynamic pressure (PSI)	Reliable. Used in the math model and also to compute trim velocity
	Marker	To indicate start and end of each flight experiment

with corresponding analytically predicted[2] value as shown in the Fig. 3. On comparison, it has been found that the estimated lift coefficient derivative C_{L_α} (average 5.6 per rad) is consistently lower than the corresponding analytically predicted value (6.1 per rad) which is very close to the theoretical limit of 2π. This difference is showing up because there is additional information in the flight data, which is captured by the parameter estimation technique. Similarly, the estimates of static stability derivative C_{m_α}, pitch damping derivative C_{m_q}, and control derivative $C_{m_{\delta e}}$ are lower than the corresponding analytically predicted values.

3.1.2 *Computation of Short-period Roots, Natural Frequency & Damping*

From the above state model, the system matrix

$$A=\begin{bmatrix} -\frac{\bar{q}\,s}{mU_0}C_{L_\alpha} & 1-\frac{\bar{q}\,s\,\bar{c}}{2\,mU_0}C_{L_q} \\ \frac{\bar{q}\,s\,\bar{c}}{I_{yy}}C_{m_\alpha} & \frac{\bar{q}\,s\,\bar{c}^2}{2U_0\,I_{yy}}C_{m_q} \end{bmatrix}$$

With C_{L_q} not being estimated and also due to $\frac{\bar{q}\,s\,c}{2\,mU_0}C_{L_q}$ being small, the above matrix is approximated as

$$A=\begin{bmatrix} -\frac{\bar{q}s}{mU_0}C_{L_\alpha} & 1 \\ \frac{\bar{q}\,s\,\bar{c}}{I_{yy}}C_{m_\alpha} & \frac{qsc^2}{2U_0\,I_{yy}}C_{m_q} \end{bmatrix}$$

In dimensional form, the above matrix is:

$$A=\begin{bmatrix} Z_w & 1 \\ M_\alpha & M_q \end{bmatrix}$$

From this system matrix, the frequency and damping of the short-period mode and also its roots are computed using MATLAB function **damp.m**.

The natural frequency and the damping of the short- period mode at each flight condition/configuration are plotted against the trim angle of attack as shown in the Fig. 4.

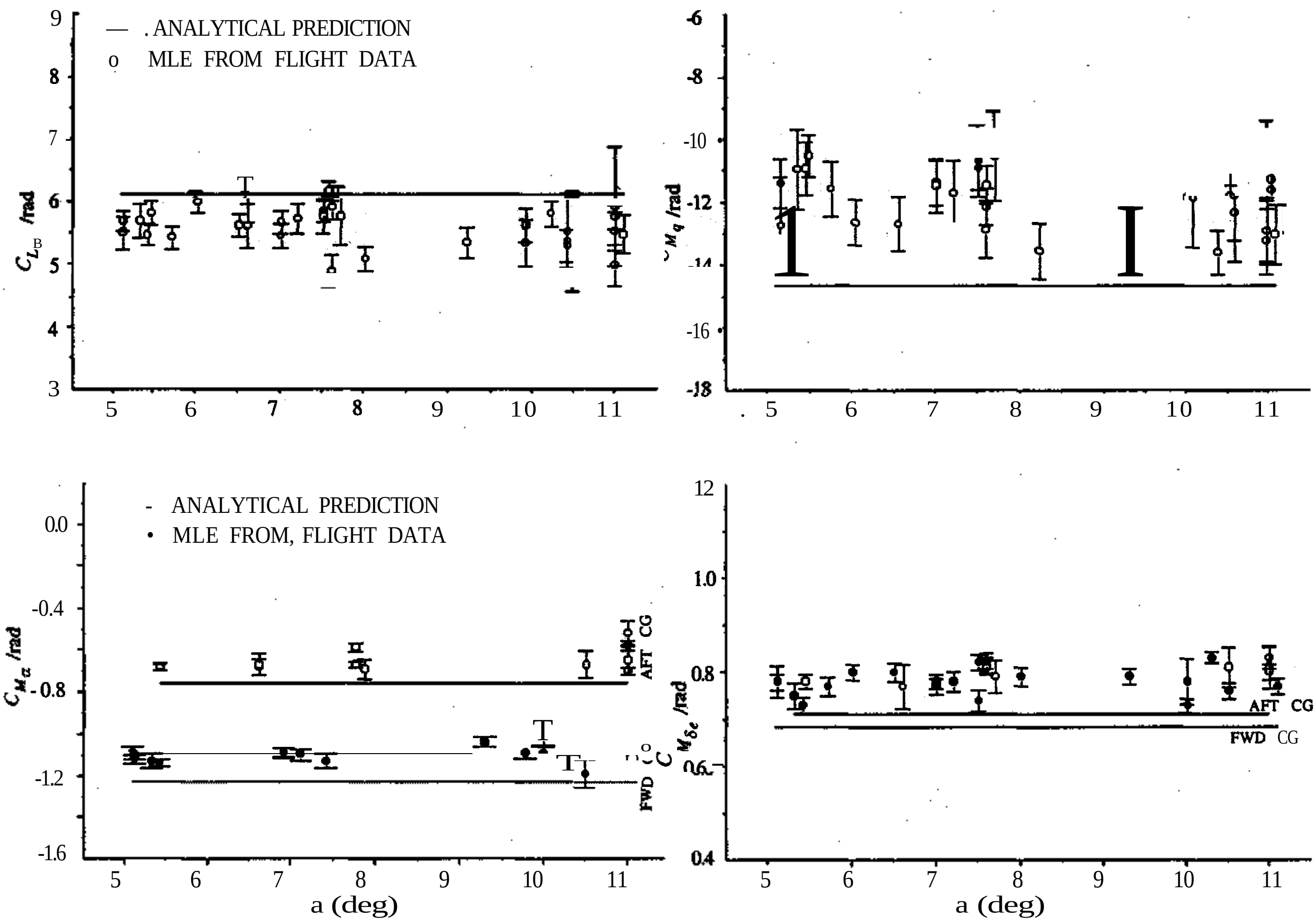

Figure 3. Light canard research aircraft (longitudinal **derivatives)**

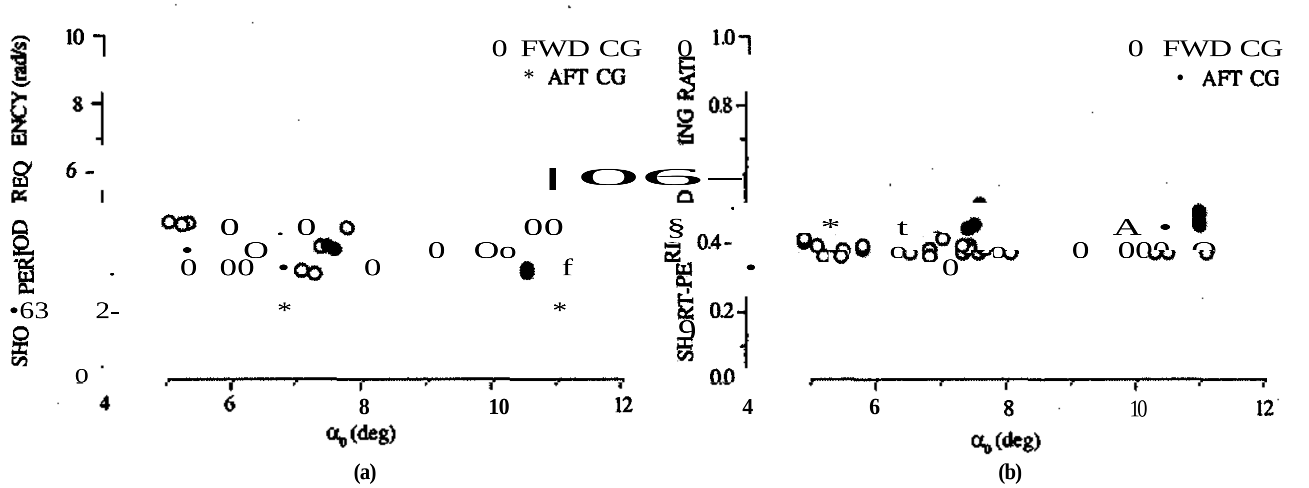

Figure 4. Plots of (a) natural frequency and (b) damping ratio of the short-period mode at different flight conditions/configurations

3.2 Lateral-directional Derivatives Estimates from Dutch Roll Flight Data

The lateral-directional derivatives of the light canard research aircraft are estimated in the following way:

- During the flight test, the aircraft is excited with the Dutch roll manoeuvre by giving doublet input to the aileron, followed by doublet input to the rudders.
- The different response signals of the aircraft as listed in Table 2 are measured during the flight test.
- The aircraft-coupled lateral-directional model given below is used to estimate the relevant lateral-directional parameters from these response signals.

3.2.1 *Lateral-directional Model*

State Equations

$$\dot{\beta}=\frac{\bar{q}s}{mV_0}\left[\left\{\begin{array}{l} C_{y_0}+C_{y_\beta}\,\beta+C_{y_p}\,p\dfrac{b}{2V_0}+ \\ C_{y_r}\,r\dfrac{b}{2V_0}+C_{y_{\delta r}}\,\delta_r \end{array}\right\}\cos\beta\right]$$

$$+\,p\sin\alpha_0-r\cos\alpha_0+\frac{o}{V_0}$$

$$\dot{p}=\frac{\bar{q}\,sb}{I_{xx}I_{zz}-I_{xz}^2}\left[\begin{array}{l} I_{zz}\left\{\begin{array}{l} C_{l_0}+C_{l_\beta}\beta+C_{l_p}\,p\dfrac{b}{2V_0} \\ +C_{l_r}\,r\dfrac{b}{2V_0}+C_{l_{\delta_a}}\delta_a+C_{l_{\delta_r}}\delta_r \end{array}\right\} \\ +I_{xz}\left\{\begin{array}{l} C_{n_0}+C_{n_\beta}\beta+C_{n_p}\,p\dfrac{b}{2V_0} \\ +C_{n_r}\,r\dfrac{b}{2V_0}+C_{n_{\delta_a}}\delta_a+C_{n_{\delta_r}}\delta_r \end{array}\right\} \end{array}\right]$$

$$-\,q_m\,r\,\frac{I_{xz}^2-I_{yy}I_{zz}+I_{zz}^2}{I_{xx}I_{zz}-I_{xz}^2}+p\,q_m\,\frac{I_{xz}\left(I_{xx}-I_{yy}+I_{zz}\right)}{I_{xx}I_{zz}-I_{xz}^2}$$

$$\dot{r}=\frac{\bar{q}\,sb}{I_{xx}I_{zz}-I_{xz}^2}\left[\begin{array}{l} I_{xx}\left\{\begin{array}{l} C_{n_0}+C_{n_\beta}\beta+C_{n_p}\,p\dfrac{b}{2V_0} \\ +C_{n_r}\,r\dfrac{b}{2V_0}+C_{n_{\delta_a}}\delta_a+C_{n_{\delta_r}}\delta_r \end{array}\right\} \\ +I_{xz}\left\{\begin{array}{l} C_{l_0}+C_{l_\beta}\beta+C_{l_p}\,p\dfrac{b}{2V_0} \\ +C_{l_r}\,r\dfrac{b}{2V_0}+C_{l_{\delta_a}}\delta_a+C_{l_{\delta_r}}\delta_r \end{array}\right\} \end{array}\right]$$

$$-\,q_m\,r\,\frac{I_{xz}\left(I_{xx}-I_{yy}+I_{zz}\right)}{I_{xx}I_{zz}-I_{xz}^2}+p\,q_m\,\frac{I_{xz}^2-I_{xx}I_{yy}+I_{xx}^2}{I_{xx}I_{zz}-I_{xz}^2}$$

$$\dot{\phi}=p+q_m\sin\phi\tan\theta_0+r\cos\phi\tan\theta_0$$

Measurement Equations

$$\beta_m=\beta-\frac{X_\beta}{V_0}r$$

$$p_m=p,\quad r_m=r$$

From the above model and based on the availability of different flight response signals, the derivatives C_{l_β}, C_{l_p}, $C_{l_{\delta a}}$, C_{n_β}, C_{n_p}, C_{n_r}, $C_{n_{\delta r}}$ and $C_{n_{\delta a}}$ could be estimated accurately. The force derivatives C_{y_0}, C_{y_β}, C_{y_p}, C_{y_r} and $C_{y_{\delta r}}$ have not been estimated because of the non-availability of lateral acceleration a_y. However, during the estimation of other derivatives, the force derivatives are frozen to the following values which are computed using DATCOM method[3]:

$$C_{y_0}=0,\quad C_{y_\beta}=-0.79,\quad C_{y_p}=0$$

$$C_{y_r}=0.30,\quad C_{y_{\delta r}}=0.19$$

The derivatives, viz., C_{n_0}, C_{l_0}, C_{l_r} and $C_{l_{\delta r}}$ are found to be very negligible and their estimates show very large standard deviation, indicating that not much information is available in the flight data to estimate these.

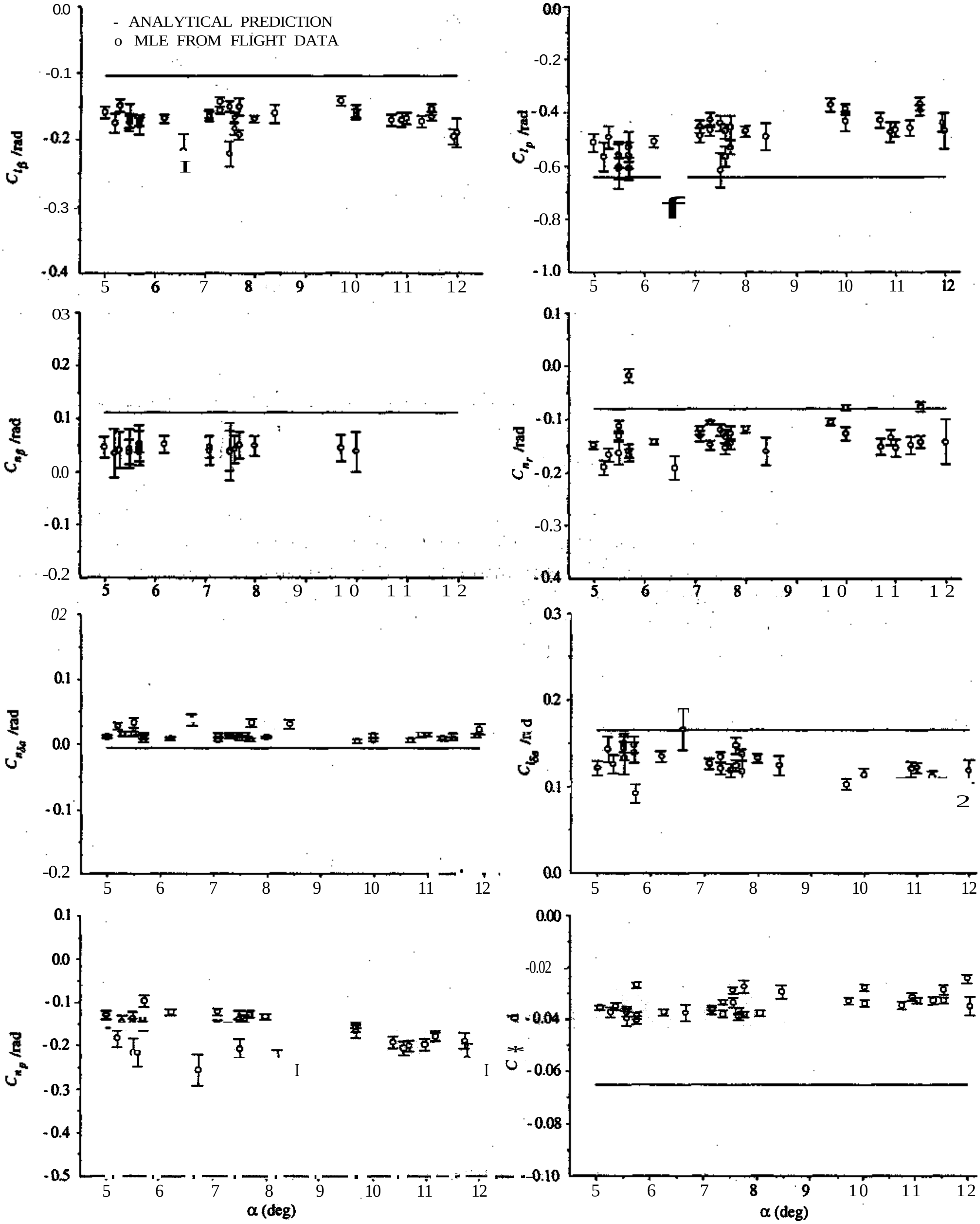

Figure 5. Plots of lateral-directional derivatives estimates along witta standard deviation against trim angle of attack

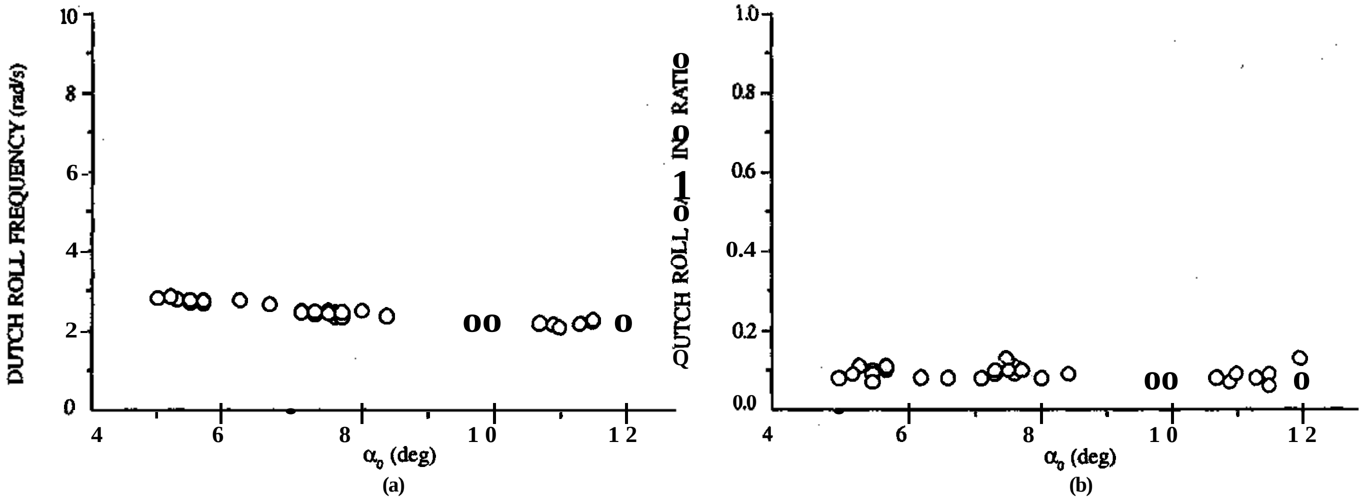

Figure 6. Plots of (a) Dutch roll frequency and (b) Dutch roll damping ratio at each of the flight condition/configuration against trim angle of attack.

The important lateral-directional derivative estimates are plotted along with standard deviation against trim angle of attack and compared with corresponding analytically predicted value[2] as shown in the Fig. 5. On comparisons, it has been found that the aircraft weathercock stability (C_{n_β}) determined from the flight-test data is lower than what is analytically predicted. Similarly, the estimated control derivative ($C_{n_{\delta r}}$) is nearly 50 per cent lower than the analytically predicted value. The standard deviation of these estimates are small, indicating that one can have more confidence in these estimates compared to their corresponding analytically predicted values. From flight mechanics point of view, these observations are valuable, and thus, prove the usefulness of parameter estimation method in ascertaining the values of these derivative.

3.2.2 *Computation of Roots, Natural Frequency & Damping of Lateral-directional Modes*

From the above state model, the system matrix is appended below:

$$A = \begin{bmatrix} \frac{\bar{q}s}{mV_0} C_{y_\beta} & \frac{\bar{q}s}{mV_0}\frac{b}{2V_0} C_{y_p} + \sin\alpha_0 & \frac{\bar{q}s}{mV_0}\frac{b}{2V_0} C_{y_r} - \cos\alpha_0 & \frac{g}{V_0}\cos\theta \\ \frac{qsb}{I_{xx}I_{zz} - I_{xz}^2}\left(I_{zz}C_{l_\beta} + I_{xz}C_{n_\beta}\right) & \frac{qsb}{I_{xx}I_{zz} - I_{xz}^2}\left(I_{zz}C_{l_\beta} + I_{xz}C_{n_\beta}\right) & \frac{\bar{q}sb}{I_{xx}I_{zz} - I_{xz}^2}\left(I_{zz}C_{l_\beta} + I_{xz}C_{n_\beta}\right) & 0 \\ \frac{qsb}{I_{xx}I_{zz} - I_{xz}^2}\left(I_{zz}C_{l_\beta} + I_{xz}C_{n_\beta}\right) & \frac{qsb}{I_{xx}I_{zz} - I_{xz}^2}\left(I_{zz}C_{l_\beta} + I_{xz}C_{n_\beta}\right) & \frac{qsb}{I_{xx}I_{zz} - I_{xz}^2}\left(I_{zz}C_{l_\beta} + I_{xz}C_{n_\beta}\right) & 0 \\ 0 & 1 & \tan\theta & 0 \end{bmatrix}$$

From this system matrix, the frequency and damping of all the lateral-directional modes and also its roots are computed using MATLAB function **damp.m**.

The natural frequency and damping of the Dutch roll mode at each of the flight condition/configuration are plotted against trim angle of attack as shown in the Fig. 6.

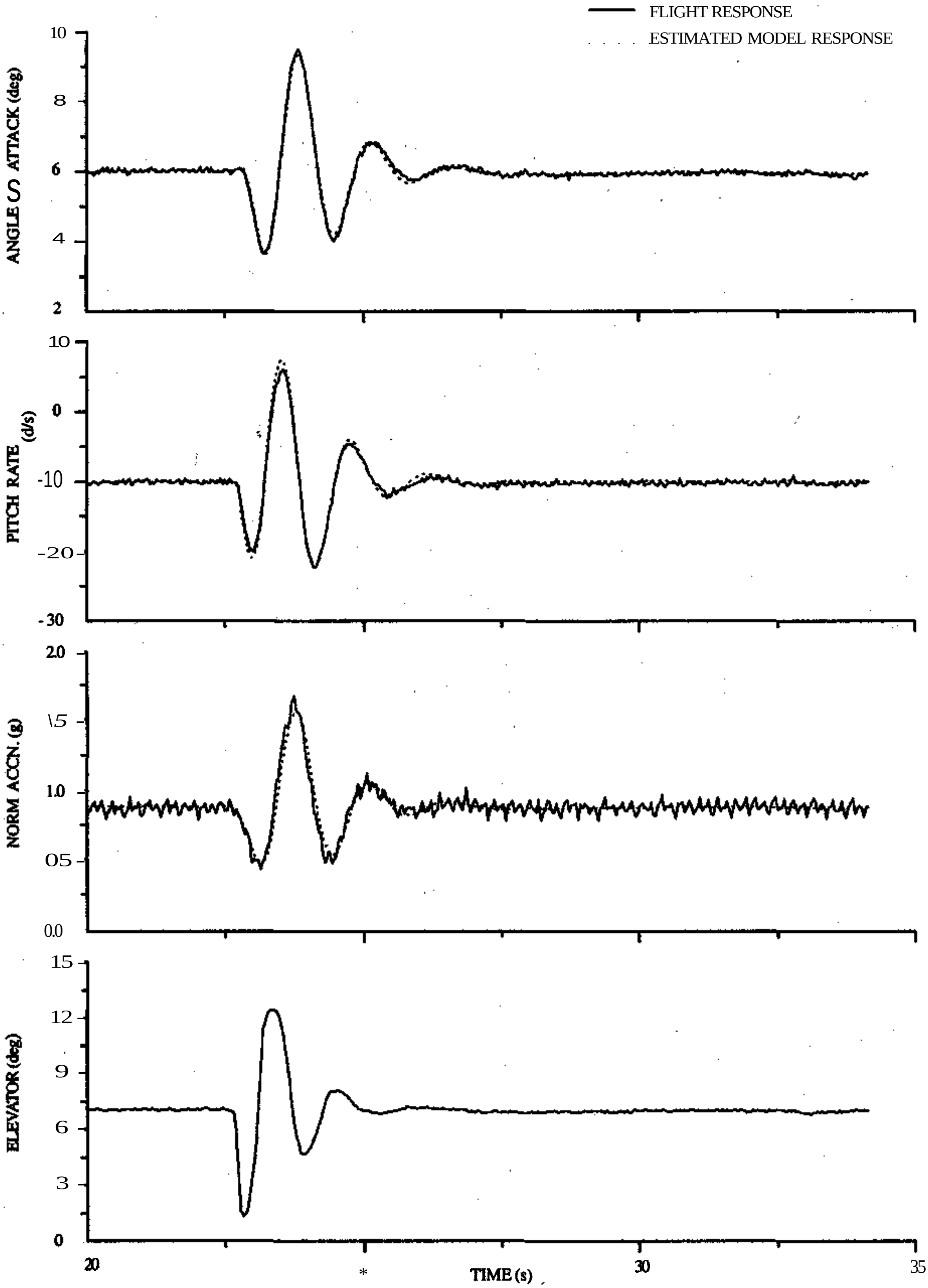

Figure 7. Flight and estimated model responses match of short-period data

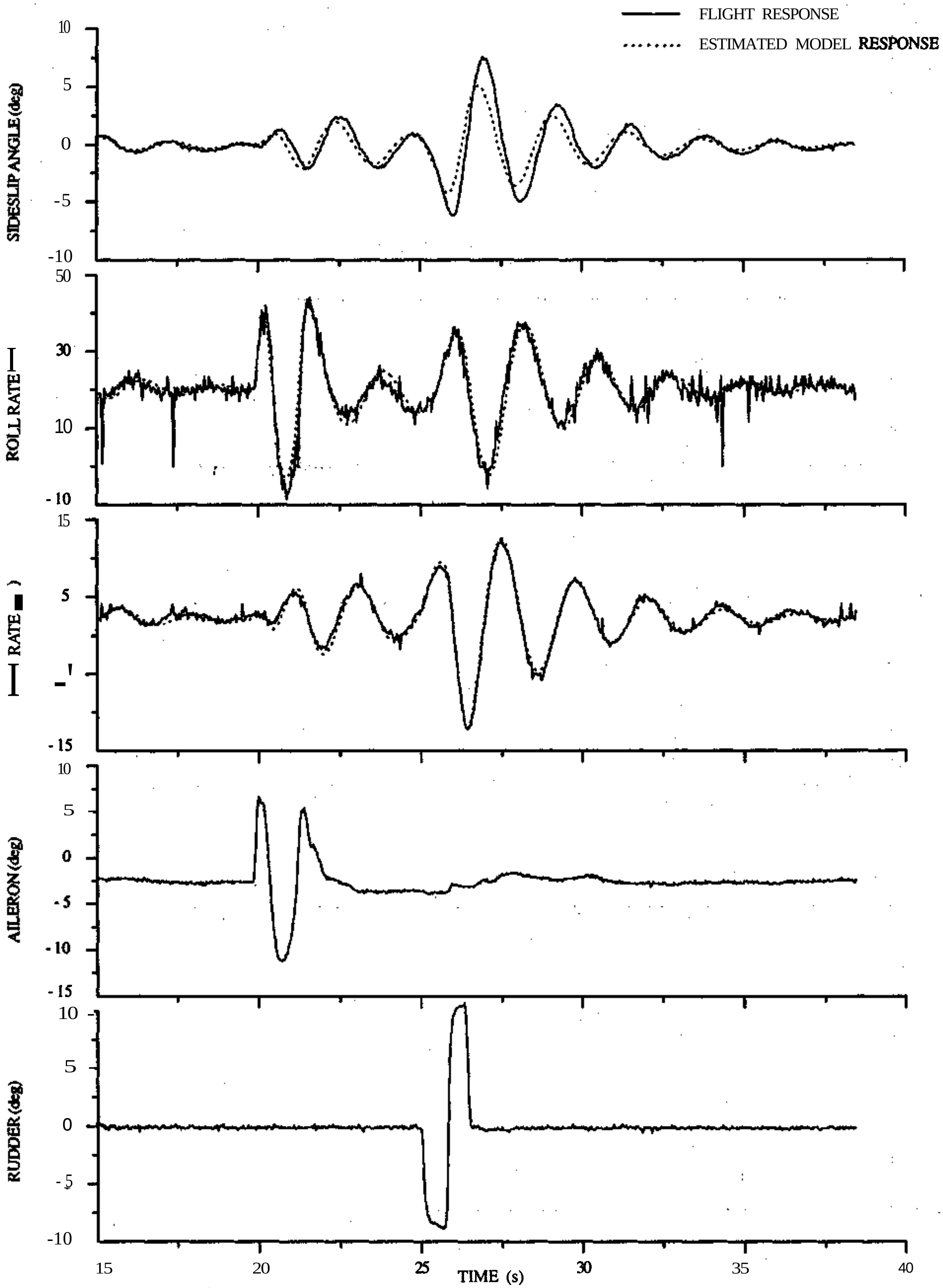

Figure 8. Flight and estimated model responses match of Dutch roll data

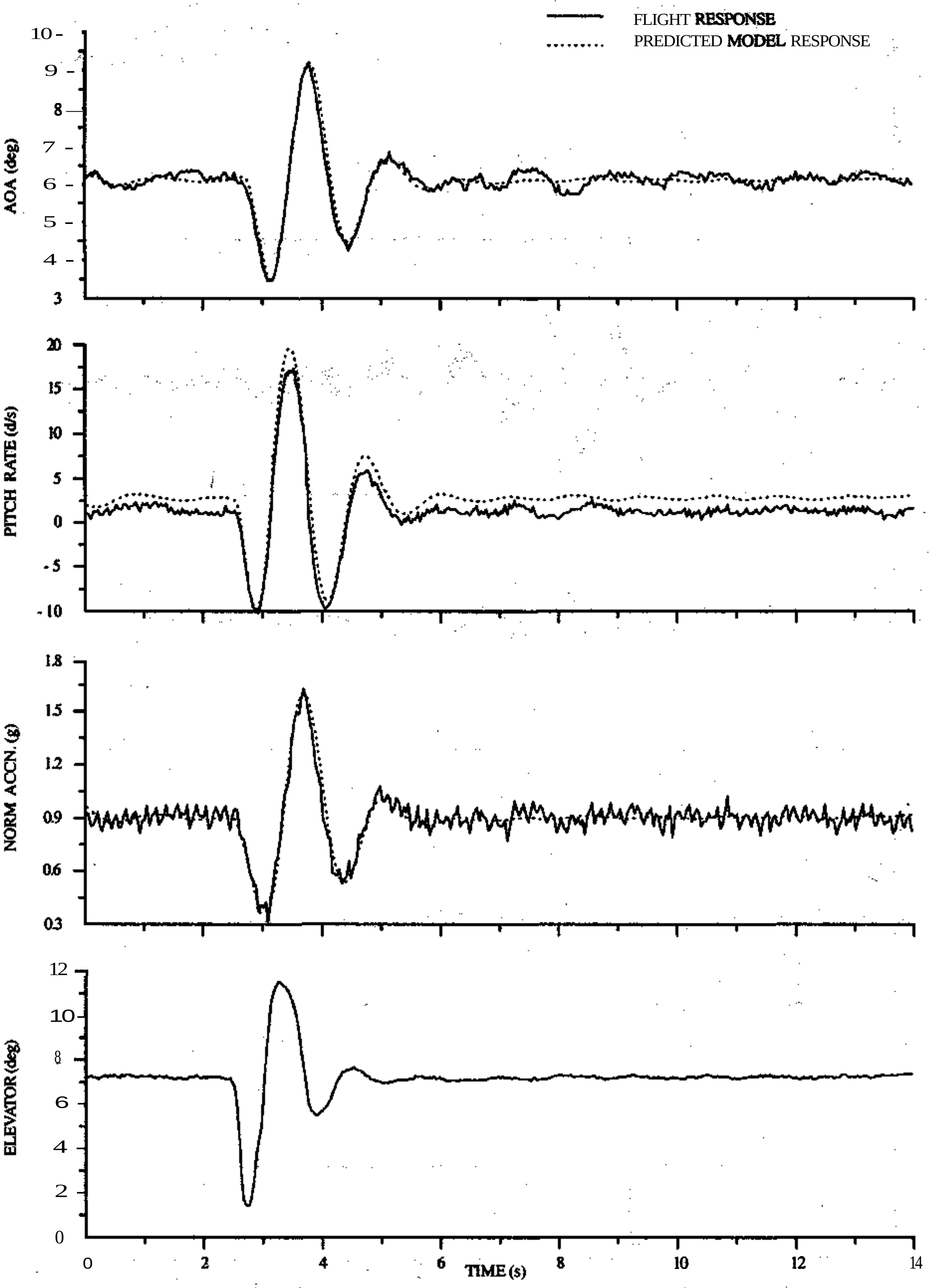

Figure 9. Flight and predicted model responses match of short-period data

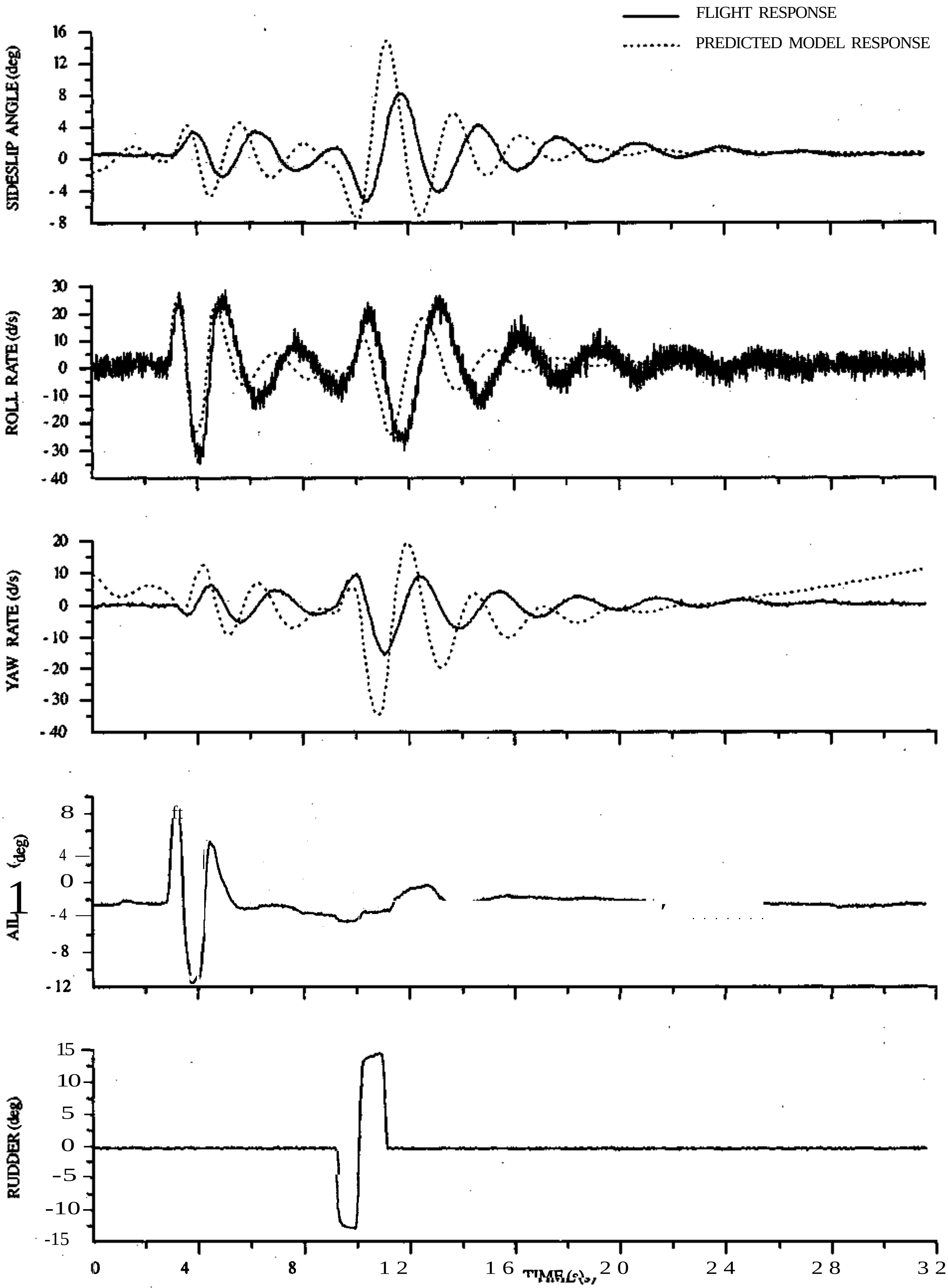

Figure 10. Flight and predicted model responses match of Dutch roll data

4. VALIDATION OF RESULTS OBTAINED

The experiment at each flight condition/configuration was repeated twice for consistency and also for cross validation purpose, wherein the average estimates from the first two experiments were used to simulate the model response and to compare with the flight response from the third experiment. Two cases of such validation are shown in the Figs 7 and 8, thus proving the validity of such estimates. This further enhances the confidence in the estimated derivatives/models. Also, model response (using analytically predicted derivatives) was compared with the flight responses in the Figs 9 and 10, which show that the analytically predicted model does not correctly represent the actual flight response. Hence, one can see that the flight-determined derivatives are estimated by extracting sufficient information from the flight data.

5. CONCLUSIONS

This paper presents the results of investigation related to the estimation of stability and control derivatives of the light canard research aircraft from the flight data. The maximum likelihood estimation, based on output-error minimisation technique is used to estimate the derivatives from the aircraft response data.

Also, the natural frequency and the damping of the aircraft modes computed from the estimated derivatives appear to be consistent with the flight trajectories. The values of some of the derivatives estimated from the flight data are different from their corresponding predicted values, but these are very accurate and derived directly from flight data, which has sufficient information to estimate these. The model validation exercise has enhanced the confidence in these estimates.

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