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Fault Detection in Systems–A Fuzzy Approach

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ABSTRACT

The task of fault detection is important when dealing with failures of crucial nature. After detection of faults in a system, it is advisable to suggest maintenance action before occurrence of a failure. Fault detection may be done by observing various symptoms of the system during its operational stage. Sometimes, symptoms cannot be quantified easily but can be expressed in linguistic terms. Since linguistic terms are fuzzy quantifiers, these can be represented by fuzzy numbers. In this paper, two cases have been discussed, where a fault likely to affect a particular system/systems, is detected. In the first case, this is done by means of a compositional rule of inference. The second case is based on modified similarity measure. For both these cases, linguistic terms have been expressed as trapezoidal fuzzy numbers.

Keywords: Fuzzy number, fuzzy relation, occurrence indication relation, conformability indication relation, FMEA, fault detection, failure mode and effect analysis, modified similarity measure, MSM

1. INTRODUCTION

The system-failure engineering is primarily concerned with failures and related problems which may include reliability, safety, security, etc. Failure is an unavoidable phenomenon which can be observed in various circumstances, such as space shuttle explosion, nuclear reaction accident, airplane crash, chemical plant leak, etc. The causes of failure are diverse and can be physical human, logical, and even financial. When an engineer designs a component or a system or develops a process, his main objective in such a design is to prevent unacceptable failures to reach the customer. One of the valuable tools in both safety and quality control of systems is the failure mode and effect analysis (FMEA) which has been extensively used in automotive industries, and most manufacturers stipulate the FMEA as the requisite method for ensuring that the quality is built into the design and manufacturing processes of new products.

When dealing with failures of crucial nature, the task of fault detection becomes very important. By fault, it means a system state which deviates from the desired system state. The task of fault

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detection may include detecting a fault by observing various symptoms of the system during its operation, detecting where the fault has occurred, and assessing the damage. However, the observed symptoms are frequently vague. For example, in identifying the leaking location in the cooling system of a boiling water reactor (BWR), some of the symptoms observed (according to Pouliezos and Stavrakakis[1]) are the pressure decrease in the main streamline; high temperature in the building and an increase in the flow rate of the sump in the building. Fuzzy methodology is a natural tool for incorporating symptoms of this kind. Also, it is difficult to quantify such symptoms. Nevertheless, it may be easy to express these in terms of linguistic phrases, eg, often occurred, seldom occurred, never occurred, etc, which are vague in nature. Because of this vagueness and uncertainty in the system-fault relationship, the usage of fuzzy methodology needs to be explored.

After detection of faults in a system, it is advisable to suggest maintenance action before occurrence of the failure. In the maintenance of systems, repair maintenance and preventive maintenance are the two important constituents. Preventive maintenance is important for systems, wherein failure is of critical nature, eg, defence systems, nuclear power plants, human systems, etc. Repair maintenance is needed to run the system in the most efficient manner and to maximise the expected profit to the possible extent. Without proper and timely maintenance, even highly reliable systems may not remain in dependable state for long periods, as expected.

According to Sorsa and Koivo[2], the problem of fault detection could be solved by any of the three methods: (i) the estimation method, (ii) the rule-based reasoning, and (iii) the pattern-recognition technique. Frank[3] used the estimation method for fuzzy residual generation.

Tsukamoto[4], *et al.*, Asse[5] *et al.*, and Bastani[6], *et al.* have used rule-based reasoning for fault detection problems. Under this, a set of fuzzy relational inequalities is used to describe the intensity of the deterministic relationships existing between the faults (viewed as causes) and the determined symptoms (viewed as effects). If S is the vector of fuzzy symptoms, F is the vector of fuzzy faults, and R is a fuzzy relational matrix describing the intensity of the casual interdependencies existing between the faults and the determined symptoms, then

$$S = F \odot R$$

An alternative idea using a direct symptom-driven fuzzy reasoning strategy was given by Sanchez[7]. He used a heuristic symptoms-faults interdependency R' instead of the casual faults-symptoms relationships, that is

$$F = S_o \odot R'$$

Though rule-based reasoning has been used by many workers, not much has been done in this field when symptoms are expressed in linguistic phrases, which, in turn, can be expressed as fuzzy numbers.

Peltier and Dubuisson[8] showed that pattern-recognition techniques could be used to deal with fault detection problems in cars.

In the present study, the following two methods have been presented using trapezoidal fuzzy numbers:

- When a fuzzy set of symptoms is observed in different systems and documentation-relating symptoms with faults is available, the fuzzy set of possible faults for different systems can be inferred by means of compositional rule of inference.
- When a fuzzy set of symptoms is observed in a particular system and the normal range of severity of symptoms that can be expected with different faults, are given. Help of a modified similarity measure is taken to determine the distance between the observed symptoms and the different symptoms associated with the faults.

Before the formal solution/methodologies are presented, some important concepts and definitions from a fuzzy set theory are given.

2. DEFINITIONS

- *Fuzzy Relations*: A fuzzy set defined on the cartesian product of crisp sets X_1, X_2, ...X_n is known as a fuzzy relation. Here, the tuples (x_1, x_2,..., x_n) may have varying degrees of membership within the relation. Any relation between two sets X and Y is known as a binary relation and is usually denoted by R (X, Y).

- *Composition of Two Binary Relations*: Consider two binary relations $P(X,Y)$ and $Q(Y,Z)$. The composition of these two relations is denoted by

$$R\ (X,Z) = P\ (X,\ Y) \circ Q\ (Y,\ Z) \qquad (1)$$

and is defined as a subset R (X, Z) of $X \times Z$ such that $(x, z) \in R$ if and only if there exists at least one $y \in Y$, such that $(x, y) \in P$ and $(y, z) \in Q$.

The composition operation for fuzzy relations can take several forms. One of the forms of this operation on fuzzy relations is the maximum product composition. It is denoted by $P(X,Y) \circledcirc Q(Y,Z)$ and defined by

$$\mu_{P \circledcirc Q}\ (x,\ z) = \max\ [\mu_P(x,y) \cdot \mu_Q(y,z)]$$
$$y \in Y \text{ for all } x \in X \text{ and } z \in Z \qquad (2)$$

An α-cut of a fuzzy set A is a crisp set A_α that contains all the elements of the universal set X having a membership grade in A greater than or equal to the specified value of α. Thus

$$A = \{\ x \in X \mid \mu_A(x) \geq \alpha\},\ 0 \leq \alpha \leq 1 \qquad (3)$$

- *Fuzzy Number*: A convex and normalised fuzzy set defined on R whose membership function is piecewise continuous is called a fuzzy number. A fuzzy set is called normal when at least one of its elements attains the maximum possible membership grade, ie,

$$\text{For all } x \in R,\ V_x\ \mu_A\ (x) = 1$$

where V stands for maximum.

A fuzzy set is convex if and only if each of its α-cut is a convex set. Equivalently, one may say that a fuzzy set A is convex if and only if

$$\mu_A(\lambda r + (1 - \lambda)s) \geq \min\ [\mu_A(r),\ \mu_A(s)] \qquad (4)$$

for all r, $s \in R^n$ and all $\lambda \in [0,1]$.

- *Trapezoidal Fuzzy Numbers*: A fuzzy number A is a trapezoidal fuzzy number denoted by (a_1, a_2, a_3, a_4), ($a_1 \leq a_2 \leq a_3 \leq a_4$) if its membership function A is given by

$$\mu_A(x) = \begin{cases} 0 & , x < a_1 \\ \dfrac{x - a_1}{a_2 - a_1} & , a_1 \leq x \leq a_2 \\ 1 & , a_2 \leq x \leq a_3 \\ \dfrac{a_4 - x}{a_4 - a_3} & , a_3 \leq x \leq a_4 \\ 0 & x > a_4 \end{cases} \qquad (5)$$

When $a_2 = a_3$, then trapezoidal fuzzy number becomes a triangular fuzzy number.

A trapezoidal fuzzy number can also be characterised by the interval of confidence at level α.

Thus, for all $\alpha \in [0,1]$

$$\tilde{A}_\alpha = [(a_2 - a_1)\ \alpha + a_1,\ -\ (a_4 - a_3)\ \alpha + a_4] \qquad (6)$$

where an interval of confidence in R is an ordinary subset of R which represents a type of uncertainty.

- *Associated Ordinary Number*: If $\tilde{A} = (a_1, a_2, a_3, a_4)$ is a trapezoidal fuzzy number, then its associated ordinary number is given by

$$' = \frac{a_1 + a_2 + a_3 + a_4}{4} \qquad (7)$$

Multiplication, maximum and minimum operations on trapezoidal fuzzy numbers do not necessarily give a trapezoidal fuzzy number. However, one can approximate the results of these operations by a trapezoidal fuzzy number[9].

- *Maximum of Trapezoidal Fuzzy Number*: If $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$, then an approximate trapezoidal fuzzy number will be

$$AVB \sim (a_1 \vee b_1, a_2 \vee b_2, a_3 \vee b_3, a_4 \vee b_4) \quad (8)$$

- *Multiplication of Trapezoidal Fuzzy Number with an Ordinary Number*: If $\tilde{A} = (a_1, a_2, a_3, a_4)$, then the interval of confidence is:

$$\tilde{A}_\alpha = [\ a_1 + (a_2 - a_1)\alpha,\quad a_4 - (a_4 - a_3)\alpha\ ]$$

Multiplying by an ordinary number, one gets:

$$b\tilde{A}_\alpha = b\ [a_1 + (a_2 - a_1)\alpha,\ a_4 - (a_4 - a_3)\alpha]$$

$$[ba_1 + b(a_2 - a_1)\alpha\ ,\ ba_4 - b(a_4 - a_3)\alpha]$$

when $\alpha = 0$ and $\alpha = 1$, then an approximate value is obtained as

$$[ab_1,\ ab_2,\ ab_3,\ ab_4] \quad (9)$$

Normalised distance $\delta\ (\tilde{A}_i,\ \tilde{A}_j)$ between $\tilde{A}_i$ and $\tilde{A}_j$ is given by[9]

$$\delta(\hat{}_i, \hat{}_j) = \frac{[LD(\hat{}_i, \hat{}_j) + RD(\hat{}_i, \hat{}_j)]}{2(\beta_2 - \beta_1)}, 0 \le \delta \le 1 \quad (10)$$

If the interval of confidence of two trapezoidal fuzzy numbers $\tilde{A}_i$ and $\tilde{A}_j$ be respectively

$$\tilde{A}_{i_\alpha} = [(a_2 - a_1)\alpha + a_1,\ -(a_4 - a_3)\ \alpha + a_4]$$

$$\tilde{A}_{j_\alpha} = [(b_2 - b_1)\alpha + b_1,\ -(b_4 - b_3)\alpha + b_4]$$

then

$$LD\ (\tilde{A}_i, \tilde{A}_j) = [(a_2 - a_1)\ \alpha + a_1 - (b_2 - b_1)\alpha - b_1]$$

$$RD\ (\tilde{A}_i, \tilde{A}_j) = [-(a_4 - a_3)\ \alpha + a_4 + (b_4 - b_3)\alpha - b_4],$$

$$\alpha \in [0,\ 1] \quad (11)$$

where β_2 and β_1 are the arbitrary values at the right and at the left chosen in such a way that

$$\beta_2 - \beta_1 \ge \frac{LD(\hat{}_i, \hat{}_j) + RD(\hat{}_i, \hat{}_j)}{2}$$

3. METHODOLOGY

3.1 Case Study I

Determination of the Fuzzy Set of Possible Faults for Different Systems using Compositional Rule of Inference

In this case, faults have been detected by rule-based reasoning, ie, by means of compositional rule of inference, when documentation relating different symptoms to different faults is available. It is assumed that the symptoms and the faults are specified in linguistic terms. Linguistic terms being fuzzy quantifiers can be represented in fuzzy logic by fuzzy numbers. These are then manipulated in terms of operations of fuzzy arithmetic.

Let S be a crisp universal set of all symptoms, F be a crisp universal set of all faults, and P be the universal set of all components/systems.

A fuzzy relation M_s specifying the degree of presence of symptoms for different systems is given. Based on this information, the fuzzy set of possible faults has to be determined for different systems by the compositional rule of inference. This will be done by constructing two types of relations, an occurrence relation and a conformability relation. An occurrence relation gives the frequency of appearance of a symptom with a particular fault, whereas a conformability relation describes the discriminating power of the symptom to confirm a particular fault. Conclusions can be drawn from these two indication relations. The basic steps for this method are:

Step 1. Since documentation concerning relations of the symptoms and the faults involves statements with linguistic terms, express these terms as trapezoidal fuzzy numbers.

Step 2. Based on this documentation, construct matrices of relations[10]

- M_o on the set S x F, where μ_{M_o} (s, f) $(s \in S,\ f \in F)$ indicates the frequency of occurrence of symptoms with fault f.
- M_c on the set S x F where μ_{M_c} (s, f) corresponds to the degree to which symptom s, confirms the presence of fault f.

The matrix of relation, M_s on the set $P \times S$, where membership grade $\mu_{M_s}(p,s)$ ($p \in P$, $s \in S$) indicate the degree to which the symptom s is present in a system p, is given. (It is supposed that this is obtained on observation of the systems).

Step 3. Using relations M_o, M_c and M_s, calculate the indication relations defined on the set $P \times F$ of systems and faults, the relations being occurrence indication relation (MP_{R_1}) and conformability indication relation (MP_{R_2}), where

$$MP_{R_1} = M_s \odot M_o$$

and

$$MP_{R_2} = M_s \odot M_c.$$

Here $\odot$ stands for the maximum product composition of two binary relations.

Step 4. Draw different types of conclusions regarding the presence of faults in the systems, from the derived relations. For instance, one may make a confirmed diagnosis of a fault for a symptom or strongly confirmed diagnosis or excluded diagnosis, and so on.

3.2 Case II

Determination of the Most Likely Fault Affecting the System by a Similarity Measure Called Modified Similarity Measure.

In this case, faults have been detected by fuzzy clustering. The method uses some form of distance measure to determine the similarity between observed attributes (symptoms) and those present in the existing diagnostic clusters. The method described is a modified version of the method employed by Esogbue and Elder[11,12]. A measure called modified similarity measure (MSM) has been developed for determining the most likely fault.

Let there be a single system which displays certain symptoms of irregularity while functioning. The observer makes a note of the symptoms in terms of linguistic phrases like a particular symptom is very strongly present, not present, etc. Also, each of the fault-symptom relation is described by trapezoidal fuzzy numbers. The importance of the symptoms for detecting faults are given by a matrix depicting weights of relevance. Then, for finding out which fault is most likely to affect the system, a measure called the MSM has been deviced. The steps in this method are briefly outlined as follows:

Step 1. Convert the observed symptoms in the system in terms of trapezoidal fuzzy number.

Step 2. Write down the interval of confidence for all the trapezoidal fuzzy numbers.

Step 3. Find the normalised distance[13] between the system's symptoms with the respective symptoms of the faults.

Step 4. Find MSM. If $\mu_w(s_i, f_i)$ denotes the weight of the symptom s_i for fault f_i, then the MSM is given by

$$D_x f_j = \left[\sum_{i=1}^{n} (\mu_w(s_i, f_i) \,.\, \delta x_{s_i} f_{s_{ij}})^2 \right]^{1/2} \qquad (12)$$

where $\delta x_{s_i} f_{s_{ij}}$ is the normalised distance between the system x's symptoms (s_1, s_2, ..., s_n) and the symptoms of fault, f_j ($j = 1, ..., p$).

Step 5. The most likely fault for the system is the one for which the similarity measure has the minimum value.

4. ILLUSTRATIONS

4.1 Case I

Let there be four symptoms s_1, s_2, s_3, and s_4 and two types of faults, f_1 and f_2. Assume that the analyst has knowledge about the relation between s_1, s_2, s_3, and s_4 with f_1 and f_2 and are given as follows:

s_1 never occurs with fault f_1 and never confirms f_1. It often occurs with fault f_2.

s_2 occurs seldom with f_1 and very seldom confirms f_1. It always occurs with f_2 and always confirms f_2.

s_3 occurs very often with f_1 but seldom confirms f_1. It never occurs with f_2 and never confirms f_2.

s_4 occurs very seldom with f_2 but often confirms f_2.

All missing relational pairs of symptoms and faults are assumed to be unspecified. Since the terms always, often, seldom, etc. are fuzzy quantifiers, one can represent these by trapezoidal fuzzy numbers.

The linguistic terms and the corresponding quadruple representations of fuzzy numbers are shown in Table 1. However, these values are only suggestive[6,11].

Table 1. Linguistic terms and their corresponding fuzzy numbers

Linguistic terms	Fuzzy numbers
Always	(0.9,1,1,1)
Very often	(0.6, 0.8, 0.8, 1)
Often	(0.5, 0.7, 0.7, 1)
Unspecific	(0,0,1,1)
Seldom	(0,0.3,0.3,0.5)
Very seldom	(0, 0.1, 0.1, 0.3)
Never	(0, 0, 0, 0.3)

Matrices of relations M_o and M_c are given by

$$M_o = \begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \end{array} \begin{bmatrix} f_1 & f_2 \\ (0, 0, 0, 0.3) & (0.5, 0.7, 0.7, 1) \\ (0, 0.3, 0.3, 0.5) & (0.9, 1, 1, 1) \\ (0.6, 0.8, 0.8, 1) & (0, 0, 0, 0.3) \\ (0, 0, 1, 1) & (0, 0.1, 0.1, 0.3) \end{bmatrix}$$

and

$$M_c = \begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \end{array} \begin{bmatrix} f_1 & f_2 \\ (0, 0, 0, 0.3) & (0, 0, 1, 1) \\ (0, 0.1, 0.1, 0.3) & (0.9, 1, 1, 1) \\ (0, 0.3, 0.3, 0.5) & (0, 0, 0, 0.3) \\ (0, 0, 1, 1) & (0.5, 0.7, 0.7, 1) \end{bmatrix}$$

Assume that a fuzzy relation M_s specifying the degree of presence of symptoms s_1, s_2, s_3, and s_4 for four systems p_1, p_2, p_3 and p_4 are given:

$$M_s = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \\ p_4 \end{array} \begin{array}{c} \begin{array}{cccc} s_1 & s_2 & s_3 & s_4 \end{array} \\ \begin{bmatrix} 0.3 & 0.7 & 0.6 & 1.0 \\ 0.2 & 0.9 & 0.3 & 0.7 \\ 0.5 & 0.4 & 0.8 & 0.1 \\ 1.0 & 0 & 0.6 & 0.6 \end{bmatrix} \end{array}$$

Since one has the binary relations M_o defined on $S \times F$, M_c defined on $S \times F$ and M_s defined on $P \times S$, with a common set S, one has to find the composition of M_o and M_s for the occurrence indication relation MP_{R_1} and the composition of M_c and M_s for the conformability indication relation MP_{R_2}.

Now

$$\begin{bmatrix} 0.3 & 0.7 & 0.6 & 1 \\ 0.2 & 0.9 & 0.3 & 0.7 \\ 0.5 & 0.4 & 0.8 & 0.1 \\ 1 & 0 & 0.6 & 0.6 \end{bmatrix} \odot \begin{bmatrix} (0,0,0,0.3) & (0.5,0.7,0.7,1) \\ (0,0.3,0.3,0.5) & (0.9,1,1,1) \\ (0.6,0.8,0.8,1) & (0,0,0,0.3) \\ (0,0,1,1) & (0,0.1,0.1,0.3) \end{bmatrix}$$

The first value in the matrix MP_{R_1} is calculated as follows:

$$\text{Max } [\, 0.3(0, 0, 0, 0.3),\ 0.7(0, 0.3, 0.3, 0.5),\ 0.6(0.6, 0.8, 0.8, 1),\ 1(0, 0, 1, 1)]$$

Using Eqns (8) and (7), one gets the approximate values for multiplication and maximum.

Then

$$MP_{R_1} = \begin{bmatrix} (0,0,0,0.09) & (0.15,0.21,0.21,0.3) \\ V(0,0.21,0.21,0.35) & V(0.63,0.7,0.7,0.7) \\ V(0.36,0.48,0.48,0.6) & V(0,0,0,0.18) \\ V(0,0,1,1) & V(0,0.1,0.1,0.3) \\ & \\ (0,0,0,0.06) & (0.1,0.14,0.14,0.2) \\ V(0,0.27,0.27,0.45) & V(0.81,0.9,0.9,0.9) \\ V(0.18,0.24,0.24,0.3) & V(0,0,0,0.09) \\ V(0,0,0.7,0.7) & V(0,0.07,0.07,0.21) \\ & \\ (0,0,0,0.15) & (0.25,0.35,0.35,0.5) \\ V(0,0.12,0.12,0.20) & V(0.36,0.4,0.4,0.4) \\ V(0.48,0.64,0.64,0.8) & V(0,0,0,0.24) \\ V(0,0,0.1,0.1) & V(0,0.01,0.01,0.03) \\ & \\ (0,0,0,0.3) & (0.5,0.7,0.7,1) \\ V(0,0,0,0) & V(0,0,0,0) \\ V(0.36,0.48,0.48,0.6) & V(0,0,0,0.18) \\ V(0,0,0.6,0.6) & V(0,0.06,0.06,0.18) \end{bmatrix}$$

$$MP_{R_1} = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \\ p_4 \end{array} \begin{array}{c} \begin{array}{cc} \quad f_1 \quad & \quad f_2 \quad \end{array} \\ \begin{bmatrix} (0.36,0.48,1,1) & (0.63,0.7,0.7,0.7) \\ (0.18,0.27,0.7,0.7) & (0.81,0.9,0.9,0.9) \\ (0.48,0.64,0.64,0.8) & (0.36,0.4,0.4,0.5) \\ (0.36,0.48,0.6,0.6) & (0.5,0.7,0.7,1) \end{bmatrix} \end{array}$$

For drawing conclusions, one finds the associated ordinary numbers of the trapezoidal fuzzy numbers (also called defuzzified values).

$$\text{defuzz}\,.\,MP_{R_1} = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \\ p_4 \end{array} \begin{array}{c} \begin{array}{cc} f_1 & f_2 \end{array} \\ \begin{bmatrix} 0.71 & 0.6825 \\ 0.4625 & 0.8775 \\ 0.64 & 0.415 \\ 0.51 & 0.725 \end{bmatrix} \end{array}$$

Similarly, calculating for MP_{R_2}, one has

$$MP_{R_2} = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \\ p_4 \end{array} \begin{array}{c} \begin{array}{cc} \quad f_1 \quad & \quad f_2 \quad \end{array} \\ \begin{bmatrix} (0,0.18,1,1) & (0.63,0.7,0.7,1) \\ (0,0.09,0.7,0.7) & (0.81,0.9,0.9,0.9) \\ (0,0.24,0.24,0.4) & (0.36,0.4,0.5,0.5) \\ (0,0.18,0.6,0.6) & (0.3,0.42,0.42,0.6) \end{bmatrix} \end{array}$$

and

$$\text{defuzz}\,.\,MP_{R_2} = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \\ p_4 \end{array} \begin{array}{c} \begin{array}{cc} f_1 & f_2 \end{array} \\ \begin{bmatrix} 0.5450 & 0.7575 \\ 0.3725 & 0.8775 \\ 0.2200 & 0.4400 \\ 0.3450 & 0.4350 \end{bmatrix} \end{array}$$

From the defuzzified values of MP_{R_1}, it is seen that fault f_1 is occurring quite strongly in the system p_1, and to some extent, in the system p_3. Also fault f_2 is occurring strongly in systems p_1 and p_3 and very strongly in the system p_2. A fault f can be said to be totally confirmed for the system p if $MP_{R_2}(p,f) = 1$. While looking at MP_{R_2}, one observes that this cannot be said for any of the systems, one can only say that fault f_2 is strongly confirmed for the system p_2 and more or less confirmed for the system p_1.

4.2 Case II

Let there be a system x which indicates symptoms s_1, s_2, s_3, and s_4. The levels of severity of these symptoms are shown in Table 2.

Table 2. Symptoms and the levels of severity of the symptoms

Symptoms	Level of severity
s_1	Almost absent severity
s_2	Very high severity
s_3	Moderate Severity
s_4	High Severity

On the basis of the symptoms, one has to determine a diagnosis for this system from among the three possible faults f_1, f_2, and f_3.

The normal range of severity of each of the four symptoms, that can be expected in a system with the fault, is given in terms of trapezoidal fuzzy numbers(Table 3).

Table 3. Normal range of severity of different symptoms

Faults	Symptoms			
	s_1	s_2	s_3	s_4
f_1	(0,0,0,0.2)	(0.6,7,0.7,1)	(0.5,0.6,0.6,0.7)	(0,0,0,0)
f_2	(0,0,0,0)	(0.9,.95,0.95,1)	(0.3,0.7,0.7,1)	(0.2,0.3,0.3,0.4)
f_3	(0,0,0,0.3)	(0,0,0,0)	(0.7,0.8,0.8,0.9)	(0,0,0,0)

The importance given to different weights of the symptoms in the detection of fault f is given by the following matrix W:

$$W = \begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \end{array} \begin{array}{c} \begin{array}{ccc} f_1 & f_2 & f_3 \end{array} \\ \begin{bmatrix} 0.4 & 0.8 & 1.0 \\ 0.5 & 0.6 & 0.3 \\ 0.7 & 0.1 & 0.9 \\ 0.9 & 0.6 & 0.3 \end{bmatrix} \end{array}$$

Writing down the linguistic phrases, almost absent, very high severity, moderate severity, and high severity in terms of trapezoidal fuzzy numbers, one has the symptoms of x as

System	Symptoms			
	s_1	s_2	s_3	s_4
x	(0,0.1,0.1,0.2)	(0.5,0.7,0.7,1)	(0.2,0.4,0.4,0.6)	(0.4,0.6,0.6,0.8)

The intervals of confidence for the trapezoidal fuzzy numbers are given as follows:

For x

$$S_{1x_\alpha} = (0.1\alpha,\ -0.1\alpha + 0.2)$$

$$S_{2x_\alpha} = (0.2\alpha + 0.5,\ -0.3\alpha + 1)$$

$$S_{3x_\alpha} = (0.2\alpha + 0.2,\ -0.2\alpha + 0.6)$$

$$S_{4x_\alpha} = (0.2\alpha + 0.4,\ -0.2\alpha + 0.8)$$

For f_1

$$S_{1_{f1\alpha}} = (0,\ -0.2\alpha + 0.2)$$

$$S_{2_{f1\alpha}} = (0.1\alpha + 0.6,\ -0.3\alpha + 1)$$

$$S_{3_{f1\alpha}} = (0.1\alpha + 0.5,\ -0.1\alpha + 0.7)$$

$$S_{4_{f1\alpha}} = (0,0)$$

For f_2

$$S_{1_{f2\alpha}} = (0,0)$$

$$S_{2_{f2\alpha}} = (0.05\alpha + 0.9,\ -0.5\alpha + 1)$$

$$S_{3_{f2\alpha}} = (0.4\alpha + 0.3,\ -0.3\alpha + 1)$$

$$S_{4_{f2\alpha}} = (0.1\alpha + 0.2,\ -0.1\alpha + 0.4)$$

For f_3

$$S_{1_{f3\alpha}} = (0,\ -0.3\alpha + 0.3)$$

$$S_{2_{f3\alpha}} = (0,\ 0)$$

$$S_{3_{f3\alpha}} = (0.1\alpha + 0.7,\ -0.1\alpha + 0.9)$$

$$S_{4_{f3\alpha}} = (0,\ 0)$$

Taking β_1 and β_2 as the two extreme values of the trapezoidal fuzzy numbers[9], ie, $\beta_1 = 0$ and $\beta_2 = 1$, one finds the normalised distance between s_1 of x and s_1 of f_1, s_2 of x and s_2 of f_1 and so on, for all the three types of faults.

$$\delta_{x_{s_1}} f_{s_{11}} = \frac{0.2\alpha}{2}$$

$$\delta_{x_{s_2}} f_{s_{21}} = \frac{0.1\alpha - 0.1}{2}$$

$$\delta_{x_{s_3}} f_{s_{31}} = 0.2$$

$$\delta_{x_{s_4}} f_{s_{41}} = 0.6$$

For $\alpha = 0$, using formula 3.1, one gets the modified distance as

$$D_{x_{f_1}} = [0 + (0.5 \times 0.05)^2 + (0.2 \times 0.7)^2 + (0.6 \times 0.9)^2]^{1/2}$$

$$= 0.558$$

$$\sim 0.56$$

In the same manner, the distance between symptoms of system x and symptoms of faults f_2 and f_3 can be worked out as

$$D_{x_{f_2}} = 0.25$$

$$D_{x_{f_3}} = 0.465$$

Since $D_{x_{f_2}}$ is minimum, one can conclude that the system's symptoms are most similar to those of fault f_2. one will get same conclusion if one takes any other value of α (say $\alpha = 0.5$ or $\alpha = 1$).

5. CONCLUSION

Methods presented earlier for determining the most likely fault in system/systems use conventional quantitative analysis. However in practice, documentation relating different symptoms to different systems are available in linguistic terms. Also, when an observer makes note of the symptoms in a particular system, one may find it easier to express one's views in linguistic terms. In such cases, the usual conventional methods cannot be applied. In this paper, such problems have been dealt with by expressing the linguistic terms as trapezoidal fuzzy numbers. Since fuzzy numbers are easy to deal with, the proposed methods can provide useful ways of detecting the faults.

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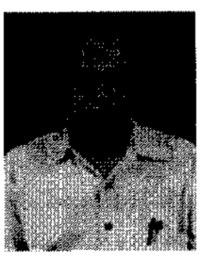

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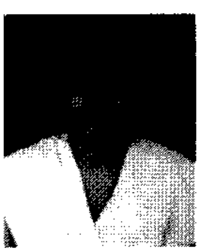

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