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Dynamic Model for Flow and Droplet Deposition in Direct Ceramic Ink-jet Printing

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ABSTRACT

A rapid fabrication technique, for building microceramic parts, using a drop-on-demand ceramic ink-jet printer is currently under development. This finds application in producing ceramic cores for gas turbines, space applications, and ceramic hybrid electromechanical systems. Another application is in the use of intelligent inks, in which particles assemble themselves so as to interrupt the signals in a particular direction and to produce a suitable output for tactical applications. An attempt has been made to develop a mathematical model for droplet formation from orifice and its deposition on substrate. The distribution of energy supplied to the piezoelectric actuator in the ink-jet print head is modelled and ejected droplet parameters (diameter and velocity) were related to the force imparted by the piezodisc. This model was extended to develop a relation for maximum droplet spread that occurs during impact. Droplet spread determines the lateral resolution of the system and the thickness of each deposited layer.

Keywords: Solid freeform fabrication, microfabrication, droplet deposition, ink-jet printing, drop-on-demand ink-jet printing, tool-less fabrication, intelligent inks, ceramic hybrid electromechanical systems, mathematical model, ceramic hybrid electromechanical systems, CHEMS, SFF, direct ceramic ink-jet printing, splat formation, DCIJP

NOMENCLATURE

D	Diameter of the chamber
L	Length of the chamber
l	Length of the orifice
d_0	Diameter of the orifice
d_{max}	Diameter of the splat droplet
h	Distance between orifice of ink-jet print head and top layer of substrate
v_i	Droplet impact velocity
v_b	Velocity of the droplet at the beginning of its ejection
v_o	Average velocity of the fluid inside orifice
v_c	Average velocity of the fluid inside chamber
δ	Maximum central deflection of the membrane
d	Diameter of the droplet
V	Volume of the droplet
V_o	Volume of the liquid in orifice
E	Young's modulus of the membrane
ν	Poisson's ratio of the membrane (assumed as 0.3)
t	Thickness of the membrane

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I	$\frac{t^3}{12(1-\nu^2)}$
F	Force applied at the centre of membrane by piezodisc
F_1	Central force acting on membrane
ω	Angular velocity of the vibrating piezoelectric crystal
ρ	Density of the solution (apparent density)
σ	Surface tension of the solution
Re	Reynolds number
We	Weber number
μ	Viscosity of the solution (apparent viscosity)
μ_v	Viscosity of the organic vehicle
ϕ_s	Solids loading volume fraction
ϕ_m	Maximum solids loading fraction
K_H	Apparent hydrodynamic shape factor of the particles
θ	Liquid-solid contact angle
ε_1	Energy required for deflecting diaphragm
ε_2	Kinetic energy of the container liquid
ε_3	Frictional energy in orifice
ε_4	Kinetic energy of the droplet at the outlet of the print head
ε_5	Surface tension energy of the droplet at the outlet
ε_6	Energy possessed by the droplet before impact on substrate
ε_7	Surface energy of the droplet after impact
ε_8	Work done in deforming the droplet against viscosity
ε	Total energy supplied to the piezoelectric disc
r	Local radius of the membrane from its centre
h_f	Frictional head
g	Acceleration due to gravity
p	Pressure inside the droplet
a	Characteristic length

1. INTRODUCTION

Microcomponents of fuel cells, gas turbines and many other equipment operating at high temperature are fabricated with ceramic materials due to their ability to withstand and retain strength at high temperatures[1]. Their capability to withstand high temperatures can be attributed to extremely strong covalent and ionic bonds that maintain their high bonding strength and hardness at these temperatures[2]. Another major area of application is the production of mesoscopic ceramic-based devices which would overcome key limitations of microscopic silicon-based devices. Ceramic hybrid electromechanical systems (CHEMSs) have been proposed to overcome some of the disadvantages like sticton, squeeze-film damping, and damage induced by surface tension in liquids during processing while retaining most of the advantages of microelectromechanical systems (MEMSs). In addition, silicon is often not the substrate material of choice for applications in which there are requirements for electrically or thermally insulating substrates, low capacitance, resistance to corrosion, and hermetic sealing.

Intelligent ceramic microstructures, that can be used to report on their environment by changing from one colour to another under mechanical, chemical, or thermal stresses, is currently being developed by the Sandia National Laboratories and the University of New Mexico, USA. These microstructures which are self-assembling and as durable as seashells, may lower costs by reducing the need for expensive manufactured devices like stress detectors, chemical analysers, and thermometers. These structures can be used in aircraft, rockets, and shuttles for monitoring various parameters onboard.

Microfabrication of ceramic parts at the submillimeter scale is a time-consuming and expensive process. Lithographic techniques, used for ceramic microfabrication, fabrication of integrated circuits and microelectromechanical systems (MEMSs), take several weeks to go from CAD drawings to completed chips and require very expensive facilities and extreme processing conditions. An alternate approach in which multiple small volumes of ceramic material is deposited at computer-defined positions, could enable tool-less all-additive fabrication of such devices on a much faster and less expensive basis.

A number of reliable and reproducible ceramic powder forming techniqes are emerging currently. These techniques come under solid freeform fabrication (SFF). Especially notable are stereolithography (SLA)[3,4], selective laser sintering (SLS)[5], laminated object manufacturing (LOM)[6], 3-D printing (3DP)[7], fused deposition of ceramics (FDC)[8], and direct ceramic ink-jet printing techniques (DCIJP)[9-11].

The DCIJP technique for assembling ceramic particles into a complex shape, layer by layer ready for sintering, is ideally suited for digital processing, in the manufacture of advanced materials in the miniaturised form. The present paper discusses the methodology of the DCIJP and models ink droplet formation using distribution of energy at several locations of the ink-jet printer. The droplet deposition over the substrate is also modelled and the equation for finding splat diameter has been formulated.

1.1 Working Principle

The DCIJP technology consists of ink-jet printers with orifice diameters in the range 20-200 μm to print an organic ink which contains a colloidal solution of submicrometer ceramic powder. Droplets of the ink are delivered through the orifice and deposited according to a computer-generated pattern.

Schematic diagram of a typical drop-on-demand printer is shown in the Fig. 1. In the print head, a piezoelectric crystal expands in response to an electrical driving signal, deforming a membrane, causing pressure impulse within the ink chamber, expelling a single droplet from the orifice. The chamber is refilled through the inlet by capillary action at the orifice. Multiple droplets deposited onto the substrate leave the printed pattern. On drying, a green part is formed. This green part is sintered to get the required strength and hardness.

Drop-on-demand ink-jet printing is preferred to continuous ink-jet printing because the latter operates at much higher droplet generation rates with the possibility of ink contamination during recirculation process. Hence, drop-on-demand ink-jet printing is ideally suited for ceramic particles printing application.

2. MATHEMATICAL MODEL

Previous researchers have attempted to develop models based on solution to Navier-Stokes equation for droplet ejection through drop-on-demand printer. Notable among these is the model by Fromm[12], who developed a characteristic dimensionless grouping of physical constants (ratio Re/We) that represents the influence of the viscous, inertial, and surface tension forces on fluid flow:

$$\frac{Re}{We} = \frac{\sqrt{\sigma \rho a}}{\mu} \tag{1}$$

For most of the commercial ink-jet printers, this characteristic dimensionless grouping lies between 1 and 10. Equation 1 determines of fluid composition for experimental study. In a later work, Reis and Derby[13] explored the influence of dimensional grouping in Eqn (1) on ink-jet behaviour through fluid-dynamics simulation. It predicts that if Re/We is too small, viscous force is predominant, and a large pressure pulse is required for droplet ejection. If this ratio is small, there will be satellite droplet formation behind the main drop. The influence of pressure pulse on droplet formation is not attempted here.

In a related work by Bhola and Chandra[14], expression is derived for droplet spread based on simple energy conservation model for impact cooling of molten wax drops. The kinetic energy and the surface energy of the droplet before impact were equated to the surface energy and the work done in deforming the droplet against viscosity after impact, leading to an expression for maximum spread factor.

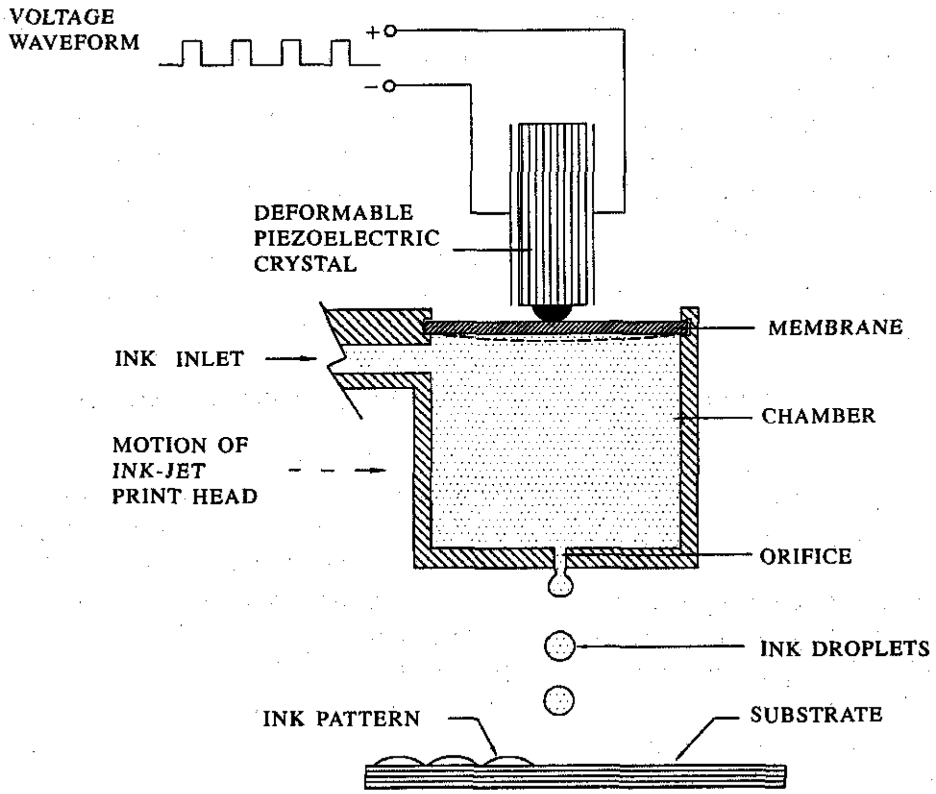

Figure 1. Schematic diagram of piezo drop-on-demand ink-jet printing system

$$\frac{d}{d_{\max}} = \sqrt{\frac{We+12}{3(1-\cos\theta)+4(We/\sqrt{Re})}} \qquad (2)$$

The droplet considered in the above work is free falling under gravity. This model cannot be directly applied to drop-on-demand ink-jet printing applications.

The mathematical model developed gives a relation between the energy imparted by the piezodisc to the ink and droplet ejected out of the orifice. This model is extended to splat formation and the relation between the energy imparted to piezodisc and the splat diameter is obtained. A schematic configuration of a single-chamber piezoelectric crystal print head is shown in the Fig. 2. When a voltage is applied across the piezodisc, the disc expands and a central force F acts on the membrane.

2.1 Droplet Formation

The energy requirement for a single droplet formation is as follows:

- Energy (ε_1) is required for deflecting the membrane
- Kinetic energy (ε_2) of the container liquid due to membrane deflection
- Frictional energy (ε_3) in the orifice
- Kinetic energy (ε_4) of droplet at the outlet of print head, and
- Surface tension energy (ε_5) of the droplet at outlet.

These five energies required are met by the energy (ε) from the central force F.

$$\varepsilon_1 = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \varepsilon_4 + \varepsilon_5 \qquad (3)$$

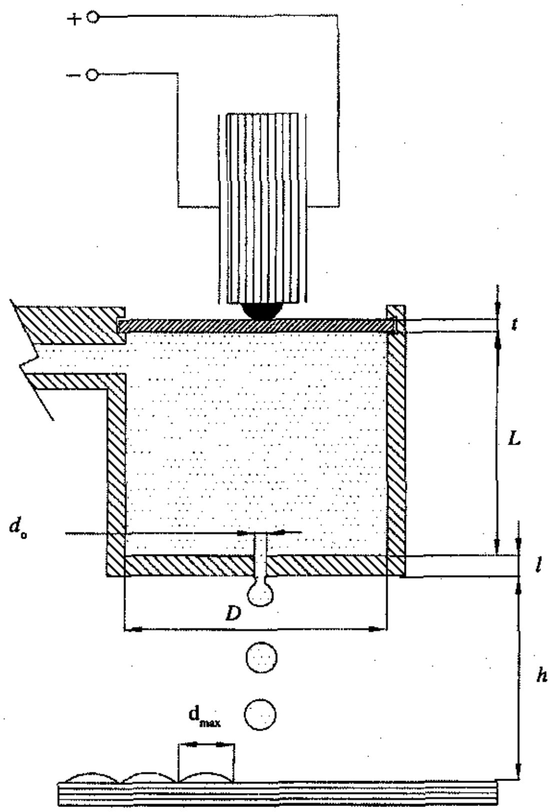

Figure 2. Piezocrystal ink-jet print head

Kinetic energy of liquid in orifice is assumed to be negligible compared to kinetic energy of liquid in a container.

To relate splat diameter (d_{max}) to F, energy of the droplet before impact (ε_6) is equated to surface energy of droplet after impact (ε_7) and the work done (ε_8) in deforming the droplet against viscosity.

$$\varepsilon_6 = \varepsilon_7 + \varepsilon_8 \tag{4}$$

Volume of droplet V is equal to the maximum volume displaced by the membrane. The driving voltage on the piezoelectric device was assumed to be converted into force acting on the centre of the disc. It will cause the deflection of the disk as follows[15]:

$$x = \frac{F_1 r^2}{8\,\pi EI} \ln \frac{2r}{D} + \frac{F_1 (D^2 - 4r^2)}{64\,\pi EI} \tag{5}$$

where

$$I = \frac{t^3}{12\,(1-v^2)} \tag{6}$$

Substituting n = 0.3 and r = 0 in Eqn (5), one gets maximum central deflection:

$$\delta = \frac{0.217 F_1 D^2}{4\varepsilon\ t^3} \tag{7}$$

The total volume of the liquid displaced V can be found by considering the volume displaced in a small layer and integrating it over entire diameter as shown in the Fig. 3.

Volume of the liquid displaced is:

$$dv = 2\pi r dr . x \tag{8}$$

Total volume of the liquid (V) displaced is:

$$V = \int_0^{D/2} 2\pi r \left[\frac{F_1 r^2}{8\,\pi EI} \ln \frac{2r}{D} + \frac{F_1 (D^2 - 4r^2)}{64\,\pi EI} \right] dr \tag{9}$$

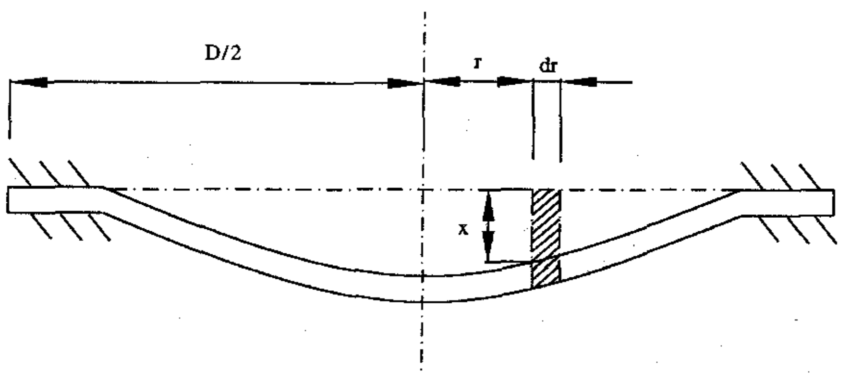

Figure 3. Deflected membrane

$$= \frac{1.7 F_1 D^4}{1024 EI} = \frac{D^2}{3}\delta \tag{10}$$

2.1.1 *Diameter of Droplet Formed*

The volume of the droplet is equal to V

$$\frac{D^2\delta}{3} = \frac{4\pi d^3}{24} \tag{11}$$

$$d = \sqrt[3]{\frac{2D^2\delta}{\pi}} \tag{12}$$

2.1.2 *Energy Required for Deflecting Diaphragm*

$$\varepsilon_1 = \frac{1}{2} F_1 \delta = \left[\frac{2Et^3\delta}{0.217 D^2}\right]\delta \tag{13}$$

2.1.3 *Kinetic Energy of Container Liquid*

Average velocity of oscillating column v_c is considered to find kinetic energy of the container liquid.

$$\varepsilon_2 = \frac{(m_c v_c - 0)}{t}\delta \tag{14}$$

$$m_c = \rho\left(\frac{\pi}{4} D^2 L - \frac{D^2\delta}{3}\right) \tag{15}$$

Using the continuity equation for the flow in chamber and orifice

$$v_c = \frac{d_o^2 v_o}{D^2} \tag{16}$$

Maximum velocity v_{max} attained by the piezodisc at its mean position is equal to the velocity of liquid in the chamber. Since the liquid is incompressible and has to maintain contact with diaphragm

$$v_o = \frac{v_{max} D^2}{d_o^2} \tag{17}$$

$$v_{max} = v_c = \frac{\omega\delta}{\pi} \tag{18}$$

where π/ω is the time taken t during which the piezoelectric crystal deformation leads to droplet formation.

$$v_o = \frac{\omega D^2 \delta}{\pi d_o^2} \tag{19}$$

From the Eqns (14) to (16)

$$\varepsilon_2 = \left[\frac{D^2\omega^2\rho\delta}{\pi^2}\left(\frac{\pi}{4}L - \frac{8}{3}\right)\right]\delta \tag{20}$$

2.1.4 *Frictional Energy of Orifice*

According to the Hagen-Poissoulle equation for laminar flow (Re < 2000)

$$h_f = \frac{32\, \mu v\, l_o}{\rho g d_o^2} \tag{21}$$

From Dougherty-Krieger equation,

$$\mu = \mu_v \left(1 - \frac{\phi_s}{\phi_m}\right)^{-K_H \phi_m} \tag{22}$$

$$\varepsilon_3 = \rho V_o g h_f \tag{23}$$

From Eqns (21) to (23) one gets:

$$\varepsilon_3 = \left[\frac{8D^2\omega l^2 \mu_v \left(1 - \frac{\phi_s}{\phi_m}\right)^{-K_H\phi_m}}{d_o^2}\right]\delta \tag{24}$$

In the case of turbulent flow, value of h_f should be taken from the experimental data.

2.1.5 *Kinetic Energy of Droplet at the Outlet of Print Head*

Mass of the droplet is ρV

$$\varepsilon_4 = \frac{1}{2}\rho V v_b^2 = \left[\frac{D^2 \rho v_b^2}{6}\right]\delta \tag{25}$$

2.1.6 *Surface Tension Energy of the Droplet at the Outlet*

$$p = \frac{4\sigma}{d} \tag{26}$$

$$\varepsilon_5 = pV = \left[\frac{4\sigma D^2}{3d}\right]\delta \tag{27}$$

From the Eqn (3), one gets the following relation:

$$\varepsilon = \left[\frac{2Et^3\delta}{0.217D^2}\right]\delta + \left[\frac{D^2\omega^2\rho\delta}{\pi^2}\left(\frac{\pi}{4}L - \frac{\delta}{3}\right)\right]\delta + \left[\frac{8D^2\omega l^2 \mu_v (1 - \frac{\phi_s}{\phi_m})^{-K_H\phi_m}}{d_o^2}\right]\delta + \left[\frac{D^2\rho v_b^2}{6}\right]\delta + \left[\frac{4\sigma D^2}{3d}\right]\delta \tag{28}$$

Since $\varepsilon = F\delta$

$$F = \left[\frac{2Et^3\delta}{0.217D^2}\right] + \left[\frac{D^2\omega^2\rho\delta}{\pi^2}\left(\frac{\pi}{4}L - \frac{\delta}{3}\right)\right] + \left[\frac{8D^2\omega l^2 \mu_v (1 - \frac{\phi_s}{\phi_m})^{-K_H\phi_m}}{d_o^2}\right] + \left[\frac{D^2\rho v_b^2}{6}\right] + \left[\frac{4\sigma D^2}{3d}\right] \tag{29}$$

The above expression is the output of mathematical model for flow through a print head relating the droplet velocity to the central force imparted by the piezodisc.

2.1.7 *Spreading (Splat Formation)*

Energy of droplet before impact (ε_6) is the combination of kinetic energy of droplet at orifice outlet (ε_4), surface tension energy (ε_5) of the droplet at the outlet, and the potential energy of the droplet acquired during its fall from height h

$$\varepsilon_6 = \varepsilon_4 + \varepsilon_5 + \rho g V h = \left[\frac{D^2\rho v_b^2}{6}\right]\delta + \left[\frac{4\,\sigma D^2}{3d}\right]\delta + \left[\frac{\rho g h D^2}{3}\right]\delta \tag{30}$$

After impact, when the droplet is at its maximum extension, the kinetic energy is zero and the surface energy (ε_5) is (refer Fig. 4):

$$\varepsilon_7 = \frac{\pi}{4} d_{\max}^2 \sigma (1 - \cos\theta) \tag{31}$$

The work done (ε_8) in deforming the droplet against viscosity[16] is:

$$\varepsilon_8 = \frac{\pi}{3}\rho v_i^2 d d_{\max}^2 \frac{1}{\sqrt{Re}} \tag{32}$$

where

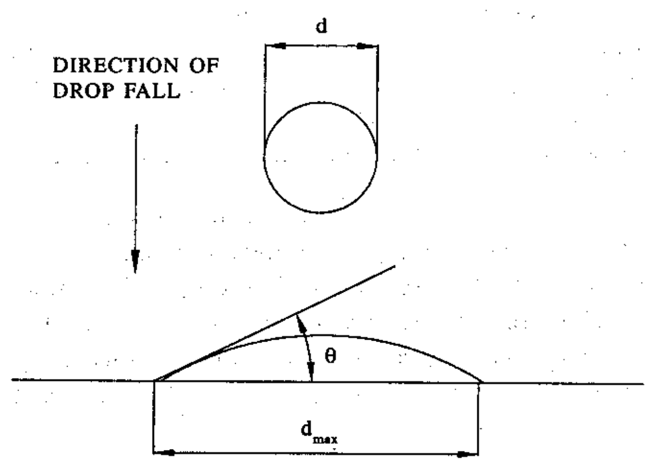

Figure 4. SPLAT formation on substrate

$$Re = \frac{\rho v_i d_o}{\mu} \quad (33)$$

$$v_i = v_b + \sqrt{2gh} \quad (34)$$

From the Eqn (4), one gets the relation:

$$\left[\frac{D^2 \rho v_b^2}{6} + \frac{4\sigma D^2}{3d} + \frac{\rho g h D^2}{3}\right]\delta = \frac{\pi}{4} d_{max}^2 \sigma(1-\cos\theta) + \frac{\pi}{3}\rho\left(v_b + \sqrt{2gh}\right) d d_{max}^2 \frac{1}{\sqrt{Re}} \quad (35)$$

The expression in Eqn (35) is the model for splat formation in relation to the velocity of droplet at the beginning of fall and deflection of the membrane, which are, in turn, related to force induced by the piezodisc.

3. DISCUSSION

Equation (29) gives the mathematical model where the force imparted by the piezodisc on the membrane is related to the velocity of the ejected droplet and membrane deflection. The diameter of the droplet formed is related to the deflection of the membrane. In Eqn (35), the maximum diameter of spread is related to the velocity of ejected droplet and the deflection of the membrane, which is, in turn, related to F. Hence, for a required spread diameter d_{max} to be obtained, v_b and δ have to be controlled. v_b and δ can be controlled by controlling the piezoelectric force using relation in the Eqn (29).

Unlike the previous models by Fromm[12] and Reis[13], who have attempted to find a relation between the properties of the solution and the droplet formation, model in this paper gives a total picture of the droplet formation, right from force imparted, considering the properties of the solution as in the previous models.

4. CONCLUSION

In the present study, a theoretical mathematical model for flow through the print head is developed which helps in relating the velocity and diameter of the ejected droplet with force imparted by piezodisc. This model is further extended to find the splat diameter. The developed model shows that the required spread of the droplet can be obtained by varying the energy given by piezodisc. A computer sends the signal for voltage to be supplied across the piezodisc for producing deformation of the disc based on the required force that can be obtained from the spread to be produced on substrate. The model involves apparent viscosity, surface tension, and density of the ceramic colloidal solution. These parameters depend on the ceramic material used, its carrier, dispersant used, ceramic loading in the liquid, and dispersion methodology applied. Ceramic ink can be prepared and optimised to achieve the required values for the above-mentioned parameters.

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