Defence Science Journal, Vol. 54, No. 1, January 2004, pp. 21-29

MHD Flow with Slip Effects and Temperature-dependent Heat Source in a Viscous Incompressible Fluid Confined between a Long Vertical Wavy Wall and a Parallel Flat Wall

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ABSTRACT

This study examines the problem of an MHD free convection flow in the presence of a temperature-dependent heat source in a viscous incompressible fluid between a long vertical wavy wall and a parallel flat wall with constant heat flux and slip flow boundary condition. A uniform magnetic field is assumed to be applied perpendicular to the walls. It is assumed that the flow consists of two parts; a mean part and a perturbed part. Expressions for the zeroth-order and first-order velocity, temperature, skin friction, and Nusselt number at the walls are obtained. The effects of different parameters entering into the problem, viz., free convection parameter, magnetic parameter, and heat source parameter on the zeroth-order and first-order velocity fields, temperature field, skin friction, and Nusselt number at the walls are shown graphically and discussed numerically.

Keywords: MHD convection flow, porous medium, skin friction, Nusselt number, slip flow, free convection flow, finite differance technique

1. INTRODUCTION

Viscous fluid over a wavy wall has attracted the attention of relatively few researchers although the analysis of such flows finds applications in different areas, such as transpiration cooling of reentry vehicles and rocket boosters, cross-hatching on ablative surfaces, and film vaporisation in combustion chambers.

In view of these applications, Lekoudis[1], *et al.* presented a linear analysis of compressible boundary layer flows over a wavy wall. Shankar and Sinha[2] studied the Rayleigh problem for a wavy wall. Lessen and Gangwani[3] studied the effect of small amplitude wall waviness upon the stability of the laminar boundary layer. In all these problems, the authors have taken the wavy walls to be horizontal.

The problem of free convective heat transfer in a viscous incompressible fluid confined between a long vertical wavy wall and a parallel flat wall was considered by Vajravelu and Sastri[4], and Das and Ahmed[5]. Patidar and Purohit[6] studied free convection flow of a viscous incompressible fluid in porous medium between two long vertical wavy walls. Rao[7], *et al.* have made an interesting analysis of an MHD convection flow in a vertical wavy channel with temperature-dependent heat source.

The authors have studied the MHD free convection flow in the presence of a temperature-dependent heat source in a viscous incompressible fluid confined between a long vertical wavy wall and a parallel flat wall in slip flow regime with constant heat flux at the flat wall. The nonlinear equations governing

Revised 10 September 2002

the flow have been solved numerically using finite difference technique. Expressions for the zeroth-order and first-order velocity fields, temperature field, skin friction, and Nusselt number at the walls have been obtained for different values of the parameters involved in the solution.

2. FORMULATION & SOLUTION OF THE PROBLEM

The two-dimensional steady laminar free convective hydromagnetic flow along a vertical channel has been considered. The x-axis is taken parallel to the flat wall and the y-axis perpendicular to it. The wavy and the flat walls are represented by $y = \varepsilon \cos(\lambda x)$ and $y = d$, respectively. The flow takes place under buoyancy in the presence of temperature-dependent heat source. The equations governing the steady two-dimensional flow and heat transfer are:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+g\beta(T-T_C)-\frac{\nu}{K}u-\frac{\sigma B_0^2}{\rho}u \tag{1}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\frac{\nu}{K}v \tag{2}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{3}$$

$$\rho C_p\left[u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right]=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+Q(T_C-T) \tag{4}$$

where (u, v) is the velocity field, p is the pressure, σ is the electrical conductivity, B_0 is the uniform magnetic field and the other symbols have their usual meanings. The last term in RHS of Eqn (4) denotes the heat generation varying directly with the temperature difference.

The boundary conditions relevant to the problem are taken as

$$u = 0, \quad v = 0, \quad T = T_C \quad \text{at } y = \varepsilon\cos(\lambda x)$$

$$u = L_1\left(\frac{\partial u}{\partial y}\right), v = 0, \quad \frac{\partial T}{\partial y}=\frac{-q}{k} \quad \text{at } y = d$$

where

$$L_1=\left[\frac{2-m_1}{m_1}\right]L \tag{5}$$

L being the mean free path and m_1 the Maxwell's reflexion coefficient.

The following nondimensional quantities have now been introduced:

$$x^*=\frac{x}{d},\ y^*=\frac{y}{d},\ u^*=\frac{ud}{\nu},\ v^*=\frac{vd}{\nu}$$

$$p^*=\frac{p}{\rho(\nu/d)^2},\ \theta=\frac{T-T_C}{(qd/k)}$$

$G_r=\dfrac{g\beta q d^4}{k\nu^2}$ Grashof number

$M^2=\dfrac{\sigma B_0^2 d^2}{\rho\nu}$ Magnetic parameter

$P_r=\dfrac{\mu C_p}{k}$ Prandtl number

$\alpha=\dfrac{Qd^2}{k}$ Heat source parameter

$\lambda^*=\lambda d$ Nondimensional frequency

$\varepsilon^*=\dfrac{\varepsilon}{d}$ Nondimensional amplitude ratio

$K^*=\dfrac{K}{d^2}$ Permeability parameter

$h_1 = \frac{L_1}{d}$ Rarefaction parameter

The Eqns (1) to (4) can be expressed in the nondimensional form after dropping the asterisks over them as

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+G_r\theta-\frac{1}{K}u-M^2u \tag{6}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial p}{\partial y}+\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}-\frac{1}{K}v \tag{7}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{8}$$

$$P_r\left[u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}\right]=\frac{\partial^2 \theta}{\partial x^2}+\frac{\partial^2 \theta}{\partial y^2}-\alpha\theta \tag{9}$$

with corresponding boundary conditions:

$$u = 0, \quad v = 0, \ \theta = 0 \quad \text{at } y = \varepsilon \cos(\lambda x)$$

$$u = h_1\left(\frac{\partial u}{\partial y}\right), \quad v = 0, \ \frac{\partial \theta}{\partial y} = -1 \quad \text{at } y = 1 \tag{10}$$

It has been assumed that the solution consists of a mean part and a perturbed part so that the velocity field and temperature field are:

$$u(x,y)=u_0(y)+\varepsilon u_1(x,y)$$
$$v(x,y)=\varepsilon v_1(x,y)$$
$$\theta(x,y)=\theta_0(y)+\varepsilon\theta_1(x,y)$$
$$p(x,y)=p_0(x)+\varepsilon p_1(x,y) \tag{11}$$

where the perturbed quantities u_1, v_1, θ_1 and p_1 are small compared with the mean quantities.

In view of the form Eqn (11), the governing Eqns (6) to (9) assume the form:

$$\frac{d^2u_0}{dy^2}-N^2u_0+G_r\theta_0=-C \tag{12}$$

$$\frac{d^2\theta_0}{dy^2}-\alpha\theta_0=0 \tag{13}$$

to the zeroth-order and

$$u_0\frac{\partial u_1}{\partial x}+v_1\frac{\partial u_0}{\partial y}=-\frac{\partial p_1}{\partial x}+\frac{\partial^2 u_1}{\partial x^2}+\frac{\partial^2 u_1}{\partial y^2}+G_r\theta_1-N^2u_1 \tag{14}$$

$$u_0\frac{\partial v_1}{\partial x}=-\frac{\partial p_1}{\partial y}+\frac{\partial^2 v_1}{\partial x^2}+\frac{\partial^2 v_1}{\partial y^2}-\frac{1}{K}v_1 \tag{15}$$

$$\frac{\partial u_1}{\partial x}+\frac{\partial v_1}{\partial y}=0 \tag{16}$$

$$P_r\left[u_0\frac{\partial \theta_1}{\partial x}+v_1\frac{\partial \theta_0}{\partial y}\right]=\frac{\partial^2 \theta_1}{\partial x^2}+\frac{\partial^2 \theta_1}{\partial y^2}-\alpha\theta_1 \tag{17}$$

to the first-order, where

$$C=\frac{-\partial p_0}{\partial x} \quad \text{and} \quad N^2=\left[M^2+\frac{1}{K}\right]$$

The boundary conditions [Eqn (10)] reduce to:

$$u_0 = 0, \quad \theta_0 = 0 \quad \text{at} \quad y = 0$$
$$u_0 = h_1u_0', \quad \theta_0' = -1 \quad \text{at} \quad y = 1 \tag{18}$$

$$u_1 = -Re(e^{i\lambda x}u_0'), \quad v_1 = 0, \quad \theta_1 = -Re(e^{i\lambda x}\theta_0'), \quad \text{at } y = 0$$
$$u_1 = h_1u_1', \quad v_1 = 0, \quad \theta_1' = 0 \quad \text{at } y = 1 \tag{19}$$

where prime denotes differentiation wrt y.

Introducing the stream function ψ defined by

$$u_1 = -\frac{\partial \psi}{\partial y}, v_1 = \frac{\partial \psi}{\partial x}$$

Eliminating p_1 from Eqns (14) and (15)

$$u_0(\psi_{xxx} + \psi_{xyy}) - u_0''\psi_x = 2\psi_{xxyy} + \psi_{xxxx} + \psi_{yyyy}$$
$$- G_r \theta_1, y - N^2 \psi_{yy} - \frac{1}{K}\psi_{xx} \quad (20)$$

$$P_r(u_0 \theta_1, x + \psi_x \theta_0, y) = \theta_1, xx + \theta_1, yy - \alpha\theta_1 \quad (21)$$

In view of Eqn (19), it has been assumed that the general solution for ψ and θ_1 is:

$$\psi(x,y) = \text{Real}\left[\sum_r (\Psi_r \lambda^r)\exp(i\lambda x)\right] \quad (22)$$

$$\theta_1(x,y) = \text{Real}\left[\sum_r (t_r \lambda^r)\exp(i\lambda x)\right]$$
$$(r = 0,1,2..) \quad (23)$$

Substituting these results into Eqns (20) and (21), one obtains the following sets of ordinary differential equations to the order of λ^2:

$$\psi_0^{iv} - N^2\psi_0'' = G_r t_0' \quad (24)$$

$$t_0'' - \alpha t_0 = 0 \quad (25)$$

$$\psi_1^{iv} - N^2\psi_1'' + i(u_0''\psi_0 - u_0\psi_0'') = G_r t_1' \quad (26)$$

$$t_1'' - \alpha t_1 = iP_r(u_0 t_0 + \psi_0 \theta_0') \quad (27)$$

$$\psi_2^{iv} - N^2\psi_2'' + i(u_0''\psi_1 - u_0\psi_1'')$$
$$- 2\psi_0'' + \frac{1}{K}\psi_0 = G_r t_2' \quad (28)$$

$$t_2'' - \alpha t_2 - t_0 = iP_r(u_0 t_1 + \psi_1 \theta_0') \quad (29)$$

The boundary conditions [Eqn (19)] reduce to:

$$\left.\begin{array}{llll} \Psi_0' = u_0', & \Psi_0 = 0, & t_0 = -\theta_0' & \text{at } y = 0 \\ \Psi_0' = h_1\Psi_0'', & \Psi_0 = 0, & t_0' = 0 & \text{at } y = 1 \end{array}\right\} (30)$$

$$\left.\begin{array}{llll} \Psi_i' = 0, & \Psi_i = 0, & t_i = 0, (i \ge 1) & \text{at } y = 0 \\ \Psi_i' = h_1\Psi_i'', & \Psi_i = 0, & t_i' = 0, (i \ge 1) & \text{at } y = 1 \end{array}\right\} (31)$$

The differential Eqns (12) and (13) are solved with the boundary conditions [Eqn (18)] to obtain the mean velocity (u_0) and temperature (θ_0).

$$u_0 = A\cosh(Ny) + B\sinh(Ny) - \frac{G_r\theta_0}{(\alpha - N^2)} + \frac{1}{N^2}$$

$$\theta_0 = \frac{-\sinh(\sqrt{\alpha}y)}{\sqrt{\alpha}\cosh(\sqrt{\alpha})}$$

where

$$d_1 = \tanh(\sqrt{\alpha}), \qquad d_2 = \frac{1}{(\alpha - N^2)}$$

$$d_3 = h_1 N, \qquad d_4 = h_1\sqrt{\alpha}$$

$$d_5 = \left[d_3\cosh(N) - \sinh(N)\right]$$

$$A = -\frac{1}{N^2}$$

$$B = \frac{G_r(d_1 - d_4)d_2}{\sqrt{\alpha}(d_5)} + \frac{1}{N^2 d_5}\left(1 - \cosh(N) - d_3\sinh(N)\right)$$

The differential Eqns (24) to (29) with conditions [Eqns (30) and (31)] are solved numerically by finite difference technique.

The perturbed velocity components u_1, v_1, and temperature θ_1 are given by

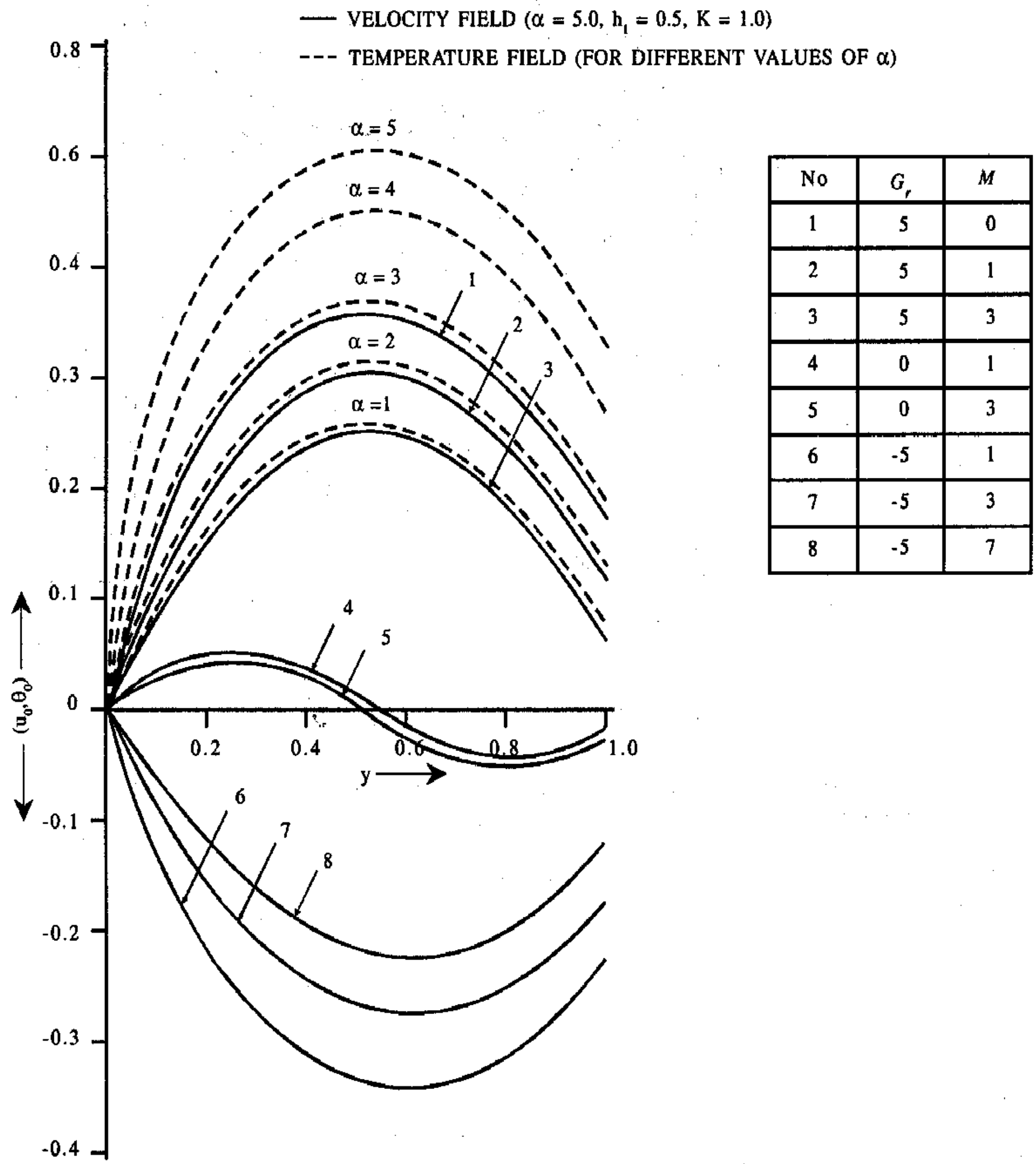
$$\left.\begin{aligned} u_1 &= -\left[\Psi'_r \cos(\lambda x) - \Psi'_i \sin(\lambda x)\right] \\ v_1 &= -\lambda\left[\psi_r \sin(\lambda x) + \psi_i \cos(\lambda x)\right] \\ \theta_1 &= \left[t_r \cos(\lambda x) - t_i \sin(\lambda x)\right] \end{aligned}\right\} \quad (32)$$

The nondimensional skin friction τ_{xy} is given by

$$\tau_{xy} = \frac{d^2\tau_{xy}}{\rho v^2} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \quad (33)$$

At the wavy wall, $y = \varepsilon \cos(\lambda x)$ and the flat wall, $y = 1$, τ_{xy} becomes:

$$\tau_w = \tau_0^0 + \varepsilon\left[-\cos(\lambda x) u_0''(0) + \sin(\lambda x)\Psi_i''(0)\right] \quad (34)$$



No	G_r	M
1	5	0
2	5	1
3	5	3
4	0	1
5	0	3
6	-5	1
7	-5	3
8	-5	7

Figure 1. The zeroth-order velocity field (u_0) for different values of G_r and M and zeroth-order temperature field (θ_0) for different values of α plotted against y.

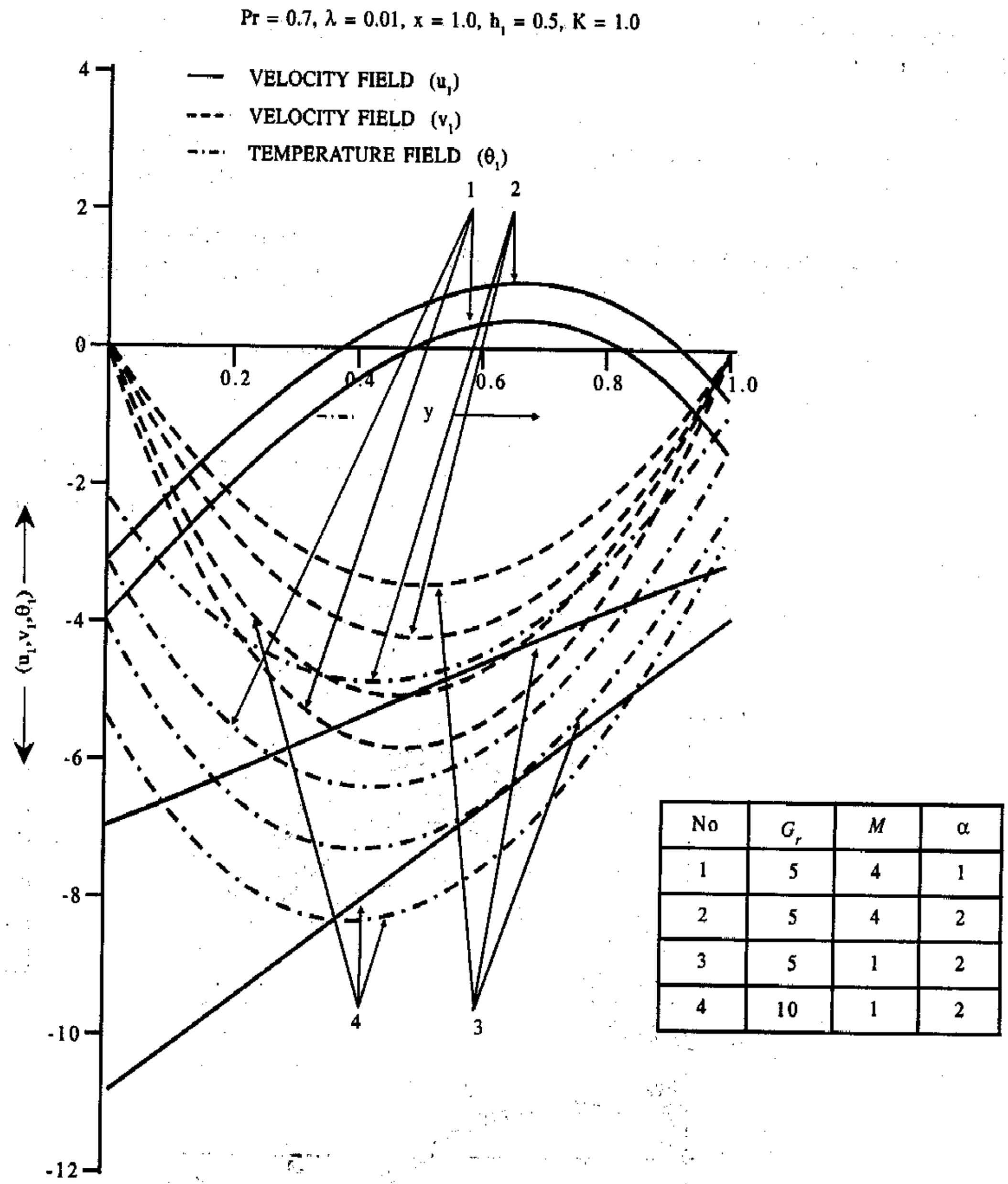


Figure 2. The first-order velocity and the temperature fields (u_1, v_1, θ_1) plotted against y for different values of G_r, M, α.

$$\tau_1 = \tau_1^0 + \varepsilon\left[-\cos(\lambda x)\psi_r''(1) + \sin(\lambda x)\psi_i''(1)\right] \quad (35)$$

where

$$\tau_0^0 = u_0'(0), \tau_1^0 = u_0'(1)$$

are the zeroth-order skin frictions at the walls.

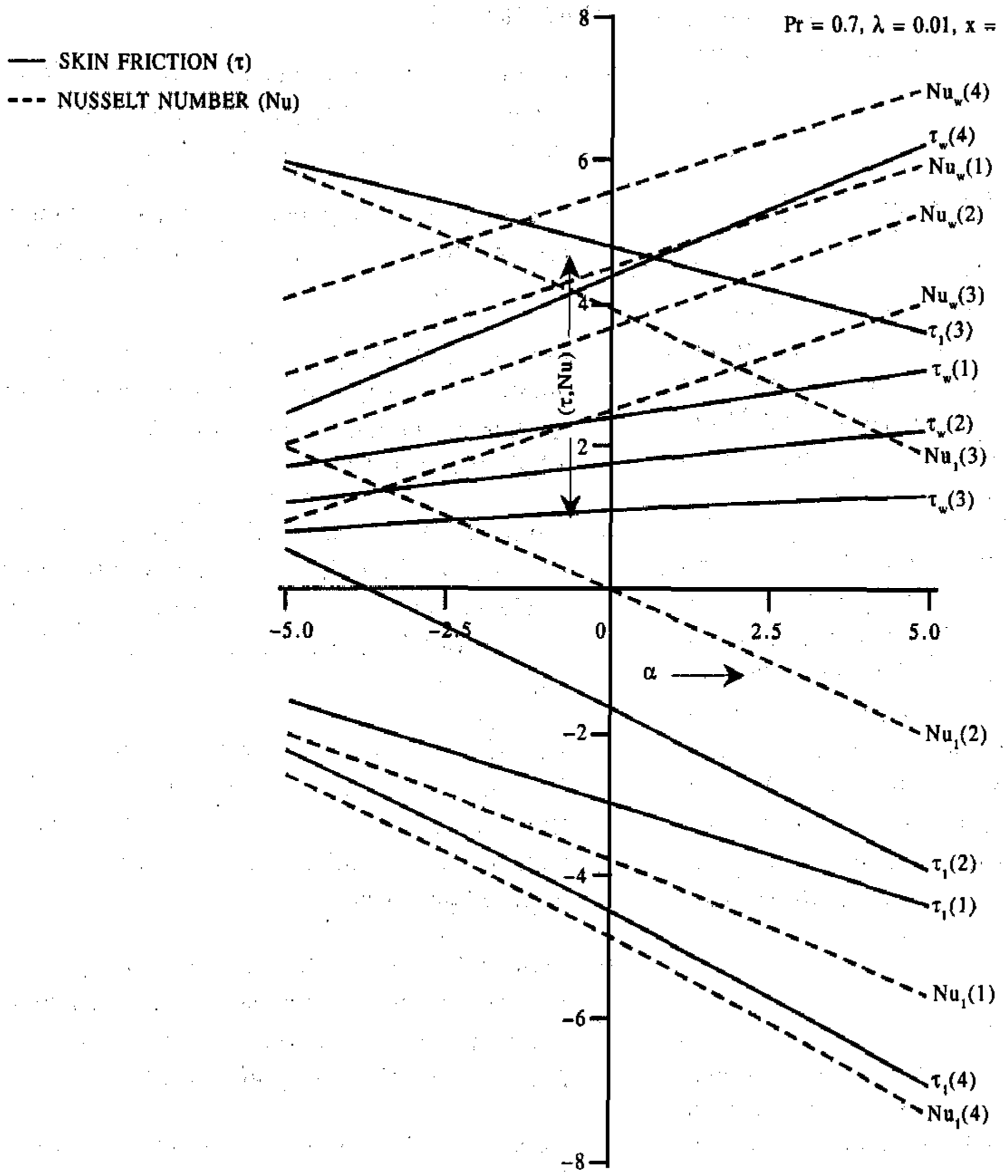
The nondimensional Nusselt number (Nu) is given by

$$Nu = \frac{1}{\theta} = \left[\theta_0(y) + \varepsilon\theta_1(x, y)\right]^{-1} \quad (36)$$

At the wavy wall, $y = \varepsilon\cos(\lambda x)$ and the flat wall $y = 1$, Nu takes the form:

$$Nu_w = \left[Nu_0^0 - \varepsilon\{\cos(\lambda x)\,\theta_0'(0) + \sin(\lambda x)t_i(0)\}\right]^{-1} \quad (37)$$

$$Nu_1 = \left[Nu_1^0 + \varepsilon\{\cos(\lambda x)t_r(1) - \sin(\lambda x)t_i(1)\}\right]^{-1} \quad (38)$$



No	G_r	M
1	5	4
2	5	9
3	5	12
4	10	4

Figure 3. Skin friction (τ) and Nusselt number (Nu) plotted against α for different values of G_r and M

where

$$Nu_0^0 = \theta_0(0), Nu_1^0 = \theta_0(1)$$

are the zeroth-order Nusselt numbers at the walls.

3. DISCUSSION & CONCLUSION

To understand the physical solution, the numerical values have been calculated for the zeroth-order velocity and temperature fields (Fig. 1), the first-order velocity field and temperature field (Fig. 2), skin friction, and Nusselt number (Fig. 3) for different values of G_r (free convection parameter), M (magnetic parameter), and α (heat source parameter).

In the Fig. 1, the zeroth-order velocity field (u_0) is plotted against y for fixed values of $\alpha = 5.0$, $h_1 = 0.5$, $K = 1.0$ and different values of G_r and M. It has been observed that for $G_r \geq 0$, the velocity u_0 decreases throughout the channel when M increases. For $G_r = 0$, velocity u_0 becomes negative near the flat wall while it remains positive near the wavy wall. Further, it is seen from the graph numbers 6, 7 and 8 (in Fig. 1) that for $G_r < 0$, the velocity u_0 becomes negative throughout the channel. In this case when M is increased, velocity u_0 is increased. In this figure the zeroth-order temperature field (θ_0) is also plotted against y for different values of α. It is being observed that when α is increased, temperature θ_0 is increased.

In the Fig. 2, the first-order velocity and temperature fields (u_1, v_1, θ_1) are plotted against y for fixed values of P_r = 0.7, λ = 0.01, x = 1.0, h_1 = 0.5, K = 1.0 and different values of G_r, M and α. It is being observed that when M and α are increased, velocity u_1 and temperature θ_1 are increased but the phenomena reverses for the case of G_r. Further, it is seen that when G_r and M are increased, velocity v_1 is decreased but the phenomena reverses for the case of α.

In the Fig. 3 the skin friction (τ) and Nusselt number (Nu) are plotted against α for fixed values of P_r = 0.7, λ = 0.01, x = 1.0, h_1 = 0.5, K = 1.0 and different values of G_r and M. It is being observed that when M is increased, skin friction (τ_w) and Nusselt number (Nu_w) at the wavy wall are decreased but the phenomena reverses for the case G_r. Further, it is seen that when M is increased, skin friction (τ_1) and Nusselt number (Nu_1) at the flat wall are increased but the phenomena reverses for the case of G_r.

ACKNOWLEDGEMENTS

The authors are thankful to the learned referee for their valuable suggestions for the improvement of the paper. Dr N. C. Jain is thankful to University Grants Commission and the University of Rajasthan for providing financial assistance.

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