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REVIEW PAPER

Finite Element Simulation of Sheet Metal Forming Processes

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ABSTRACT

In the present study, the survey of research work on finite element analysis of metal forming processes has been carried out. A classification of formulations dealing with geometry and material nonlinearity in the context of finite element simulation of forming operations has been recapitulated. The procedures based upon shell and continuum approaches and methods of dealing with frictional contact, are described. Topics of current interest on finite element analysis such as error estimation, projection of error, and adaptive mesh refinement have been reviewed.

Keywords: Sheet metal forming, geometric nonlinearity, material nonlinearity, shell elements, continuum elements, adaptive mesh refinement, recovery procedures, finite element analysis, simulation

1. INTRODUCTION

The finite element method has secured acceptance as a tool of choice for simulating forming processes and assessing the effect of process parameters[1]. The simulation of forming operations on a thin sheet cause complex deformation in the blank[2]. The nature of deformation in different portions of the blank is generally different. It could range from pure stretching to pure bending, to combined stretching and bending. Shape of the blank undergoes continuous change during the sheet forming operation. In mathematical terms, sheet forming operations involve deformation that is nonlinear from both the geometric and the material points of view. Three classes of elements, namely membrane, shell, and continuum (or solid) are of interest in finite element analysis of sheet forming operations[3]. The elements based on the membrane theory are simple, cheap, and consume less computational time. But these do not incorporate bending effects. The shell elements, though approximate, are capable of considering both the stretching and the bending effects, whereas the solid or continuum elements, though requiring extensive computation, are more accurate because through-thickness discretisation can adequately account for transverse shear effects.

An area of concern during the finite element simulation of sheet metal forming process is the possible error in the numerical solution. The accuracy of results of simulation depends upon the approximation of mathematical modelling, manner of domain discretisation, choice of scheme for solving system

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equations, and the method employed for computing derivatives of the state variable[4]. Different error estimation and adaptive mesh generation techniques have been developed to enhance the reliability of a finite element simulation[5]. The error estimators can be classified into three main types, namely the residual type introduced by Babuska and Rheinboldt[6], the interpolation type propounded by Erikssan and Johnson' and the post-processing type proposed by Zienkiewicz-Zhu[8]. A brief survey of the literature on geometric nonlinearity, material nonlinearity, and frictional contact and other concern in the context of finite element simulation of forming operations have been presented.

2. GEOMETRIC NONLINEARITY & MATERIAL NONLINEARITY

Four kinds of formulations have been proposed in the literature for dealing with the motion and deformation of solid bodies. Truesdell[9] named these formulations as material, Eulerian, Lagrangian, and updated-Lagrangian. All the four formulations are, however, equivalent in the case of smooth motion of a deformable body. In finite element analysis of problems of sheet metal forming processes, only the last two formulations have been found useful. In the so-called material approach, the independent variables are the current position (X)of a point in the body relative to a fixed reference, and the time t. (This description is mainly used in analytical dynamics of a rigid body). In the Eulerian (or spatial) formulation, the independent variables are the current position $\bar{X}$ of a particle P, and the time t. In the Eulerian description of the motion of the body, one is concerned with what is happening in a fixed region of space as time goes on (which seems to be suited to the study of fluids where the flow in a fixed region in space is observed). It is also important to emphasise that the independent variables $\bar{x}$ is a function of the Lagrangian position $\bar{X}$ and the time t i.e. $\bar{x} = \bar{x}(\bar{X}, t)$. The material derivatives are therefore more difficult to handle in the Eulerian description.

In the Lagrangian (or referential) formulation, the independent variables are time t and the position $\bar{X}$ of a particle P relative to an arbitrary chosen reference configuration at time t_0. It is important to note that the choice of the reference configuration is arbitrary. In certain publications[10], a particular description is called Lagrangian when the position $\bar{X}$ of the body point X at the particular time $t_0 = 0$ (ie, in the undeformed state) is used to describe the motion. However, any other choice of the reference configuration at a specific time other than $t_0 = 0$ will still be Lagrangian in nature, in the sense that the independent variable $\bar{X}$ is considered at a fixed time instant. Lagrangian finite element formulation makes use of linear incremental methods".

As discussed above, it is a Lagrangian (or referential) formulation if the reference configuration corresponds to that at $t = 0$. On the other hand, if the reference configuration is taken at a variable time t, then it is called an updated Lagrangian (or relative) formulation. In other words, the future is described wrt the present. The independent variables are $\bar{x}$ and t, where $\bar{x}$ is the position occupied by the material point P at time t. This means that $\bar{x}$ is independent of time t. One of the earliest updated Lagrangian formulation is that proposed by Murray and Wilson[12]. Yoghmai and Popov[13] discuss an updated Lagrangian approach based upon an incremental variational principle and a moving reference configuration. Alturi[14] describes an updated Lagrangian formulation based upon the principle of virtual work expressed in terms of a current configuration. Another updated Lagrangian formulation for problems involving large strain and large rotation is due to Boisse[15], et al. An arbitrary Lagrangian-Eulerian (ALE) formulation was introduced by Panthot[16] to overcome the problem of free boundaries (encountered when using a purely Eulerian formulation) and of mesh distortion (encountered when using a purely Lagrangian formulation). An arbitrary Lagrangian-Eulerian formulation was made use of, among others, by Haaren[17], *et al.* and Gadala[18], *et al.*

Three types of approaches have been described in the published literature for dealing with problems of material nonlinearity[19]. These are called deformation theory, solid approach, and flow formulation. A survey of available formulations dealing with problems involving geometric and material nonlinearity has been done by Gadala[20], et al. According to the deformation (total strain) theory[21], the plastic strains are assumed as a function of the current state and independent of the history of loading. In other words, the material can be assumed to deform directly from an initial configuration to a final configuration without any intermediate steps. Majlessi and Lee[22] have proposed a finite element formulation based upon the deformation theory of plasticity. A finite element formulation for planar anisotropic sheet materials based upon an incremental deformation theory was developed by Yoon[23], *et al.* Considerable saving in computational time can be achieved using the deformation theory. However, the method applies to exceptional loading cases only. In a general case of loading, the method can be used merely to obtain rough estimates of strain distribution, thickness variation, wrinkling, necking, etc.

In the solid approach, the work material is treated as either an elasto-plastic or an elasto-viscoplastic solid. The generalised rate-independent Prandtl-Reuss constitutive model is often used. Either the displacement or displacement increment is considered as the dependent variable. One of the earliest elasto-plastic formulations for large strain and large displacement are due to Hibbit[11], *et al.* Alternative formulations and implementations of the solid approach are due to Huang and Chen[24]. Another solid approach, called initial stress approach, was proposed by Zienkiewicz[25], *et al.* Chi and Shein[26] presented an elasto-plastic model of sheet forming processes that includes the bending effects.

In the flow formulation*‘, the velocity is a dependent variable. Within the confines of the flow approach, the metal can be modelled in various ways, eg, as a rate-dependent rigid plastic material, as in the work of Osakada[28], *et al.*, a purely viscous material, as in the work of Zienkiewicz and Godbole[29], as a Maxwell visco-elastic material, as in the works of Thompson[30], and as an elasto-plastic and elasto-viscoplastic material, as in the work of Chenot[31]. Crochet[32] pointed out that the flow approach is less successful for elasto-viscoplastic materials due to numerical instabilities, which occur at an increased elastic response. In many forming operations, the plastic strains far outweigh the elastic component of strain. The flow approach that neglects the elastic strain is quite often appropriate to metal forming problems and has become quite popular. The flow formulation was used by Zienkiewicz[33], *et al.* to study the large deformation of a thin sheet of metal (or shell). An analogy between the equations of pure plastic and the viscoplastic flow theory for void-containing metal and those of standard nonlinear elasticity is presented by Onate[34], *et al.* Onate and Zienkiewicz[35] adopted this analogy by replacing displacement, strain, and shear modulus by velocity, strain rate and nonlinear viscosity, respectively. Ziekiewicz[27] discusses the use of flow formulation for numerical solution of forming processes at some length. The implementation of the above approach is due, among others to Sekhon and Chenot[36]. Flow formulation-based finite element analysis of metal forming processes was generalised by Damir[37] to include history sensitive material and boundary friction. A mixed velocity-pressure formulation based upon a rigid-viscoplastic material model for the simulation of transient and stationary metal forming processes was presented by Horocio[38]. Saran[39] investigated the influence of elasto-plastic and rigid-plastic material models on finite element analysis. A mixed elasto-plastic/rigid-plastic material model was proposed for forming processes by Huetink[40], *et al.* It degenerates to elasto-plastic model for small strain increments, and to rigid-plastic model for large strain increments. Lazar[41] has discussed the merits and demerits of many of the above formulations.

3. CONTINUUM & SHELL APPROACHES

The discretisation of a sheet metal blank can be done on the basis of continuum elements or else by shell elements. Continuum elements discretise the sheet not only along its mid-surface but also across it, ie, along the direction of its thickness. Contact conditions are modelled independently on’ both sides of the metal blank. More than one layer of elements is used for considering variation of strains through the sheet thickness caused by bending and shear. Continuum elements were employed for analysis of sheet forming operation by Wifi[42] using a rigid viscoplastic formulation, and by Makinouchi[43] using an elasto-plastic formulation. Other works of this nature are those of Oh and Kobayashi[44], and Cao and

Teodosiu[45]. Oh and Kobayashi[44] studied the sheet bending problem using six rows of linear isoparametric rectangular elements for rigid-plastic analysis and six rows of linear triangular elements for elasto-plastic analysis. Cao and Teodosiu[45] simulated axisymmetric deep drawing using two rows of quadrangular elements. They found performance of continuum elements satisfactory in describing processes that involve two-sided control of the sheet, severe bending/unbending and localised necking. Lee and Yang[46] used a rigid-plastic finite element formulation employing geometric nonlinearity during an incremental time step.

An adequate approach for dealing with sheet metal forming problem must take into account both membrane and bending effects. The shell elements can be too simplistic if bending effects are neglected. The resulting membrane formulation can be used to model sheet forming problems only when stretching effects are dominant. Several investigators have attempted to introduce the bending effect in the membrane approach in an approximate manner, as in the work of Batoz[47], et al. or in a selective manner, as in the work of Onate and Sarasibar[48]. Zienkiewicz[49], *et al.* introduced axisymmetric shell element that takes into account memebrane, bending, and shear effect. Some of the important applications of shell elements to nonlinear deformation are due to Wang and Tang[50], and Quoirin[51]. A comparison of finite element models employing different discretisation schemes has been presented by Giovanni[52], *et al.* and Papazian[53], *et* al. An axisymmetric shell element was developed by Lee and Cao[54] for the multi-step inverse finite element analysis. Hembrecht[55], *et al.* concluded that material modelling had a more profound impact on the computational economy than the type of element an individual program uses. Huh and Choi[56] derived a modified membrane finite element formulation for sheet metal forming analysis. The formulation incorporates membrane elements and the bending effects are taken into account explicitly. The strain energy term in the formulation is decomposed into a membrane energy term for mean stretching, and a bending energy term for pure bending.

4. TREATMENT OF FRICTIONAL CONTACT

In recent years, much attention has been paid to numerical analysis of the problems of frictional contact. Contact and friction appear because of interaction between different bodies. Kobayashi[57], *et al.* employed a trial and error formulation in which better approximation of contact velocity were found in successive trials. The contact nodes were constrained to lie in the tool surface and sustain the frictional forces. A similar formulation was followed by Keum[58], *et al.* Satisfaction of contact condition has been achieved through Lagrangian multipliers, penalty functions, or modified Lagrangian multipliers, Rebelo[3], *et al.* calculated the reaction force using Lagrangian multiplires along with a softening law. Eterovic and Bathe[59] also used the Lagrangian multiplier technique to model tool-work contact. Simo and Laursen[60] proposed the use of the Lagrangian multiplier method, in either direct or augmented form, in conjunction with implicit time integration. The penalty method has also been used by Karafills and Boyce[61], and Shimzu and Sanu[62]. The advantage of the penalty method is that the contact constraints can be enforced without changing the system equations when contact evolves. Once the contact has been established, the sliding or sticking contact status of the node is determined and corresponding friction reaction rule is assigned to the node. Oden and Pires[63] introduced an approximate model to mathematically describe the Coulomb's law. Another alternative approach for incorporating Coulomb friction has been proposed by Dalin and Onate[64]. A formulation based on a three-field Hu-Washizu-type functional has been proposed recently by Papadopoulus and Taylor[65] in the context of frictionless contact. Mahajan[66], *et al.* proposed an implicit scheme for treating the nodal contact conditions in non-steady state problems. They incorporated unilateral node-to-node condition (with possibility of nodes originally in contact, losing contact subsequently). Contact forces at the node were used to decide if the node was to be released. An implicit-explicit scheme designed to overcome problems of convergence caused by evolving contact in deep drawing was presented by Joundon[67], *et al.*

A concept based on the insertion of a fictive intermediate layer between the tool and the workpiece for the description of surface friction, was presented by Doege[68], *et al.* They used a compressible material law to model the constitutive behaviour of the intermediate layer.

A large number of authors have studied the problems of frictional contact between the workpiece and the die/tool during sheet metal forming process. Peterson[69] used an incremental theory to obtain a friction law. A finite element formulation for frictional contact based on variational inequalities was proposed by Fredriksson[70], *et al.* Eric and Doege[71] emphasised that for accuracy sake, the description of the frictional behaviour must be of the same order of accuracy as that of the material behaviour. Chenot[72] used the finite element method to simulate the deformation processes involving so-called unilateral and bilateral frictional conditions. Bohatier and Chenot[73] presented a finite element formulation for non-steady large deformation with sliding or evolving contact boundary conditions based upon an implicit integration scheme. Cao and Sio[74] carried out finite element simulation of contact stresses between the blank and the blank holder during axisymmetric deep drawing. Zhao and Wagoner[75] modified a rigid plastic membrane element program and used it for the simulation of sheet forming operations. They accounted for friction and contact of the deforming blank with irregularly shaped curved die surfaces.

5. ERROR ESTIMATION & ADAPTIVE MESH REFINEMENT

A recent trend in the development of the finite element technique is the use of adaptive procedures based upon error estimators. Presently, considerable research effort is underway for devising error estimators and adaptive mesh refinement procedures to improve the solution accuracy. Zienkiewicz[76] has listed some important achievements in the finite element method and presented an outline of some problems still requiring treatment.

An interpolation-based error estimation analysis is due to Demkowicz[77], *et al.* McNeice and Marcel[78] worked on the discretisation error and tried to minimise it, optimising by the total potential energy at the node.

The specific energy-difference method was developed by Melosh and Marcel[79] as a measure of the discretisation error. They studied the effect of increasing the number of degrees of freedom in a finite element model. Kelly[80], *et al.* used an incomplete set of higher-order hierarchical functions discussed by Zienkiewicz[81], *et al.* to express error distribution in an element. Grosse[82], *et al.* proposed an estimator based on a so-called generalised scalar energy density field. They suggested that the accuracy requirement should be made adaptable to the distribution of stress field across the domain.

Among the earliest researchers to present a residual based ***aposteriori*** error estimation technique were Szabo and Lee[83]. An ***aposteriori*** error estimator based on the computation of element residuals was proposed by Demkowicz[84], *et al.* Zienkiewicz and Zhu[8] proposed residual-based error estimation. They obtained approximation of the discontinuous finite element stress field by a global least-square fit of piecewise continuous field. They proposed that the difference between the continuous and the discontinuous field measured in terms of the L_2 or energy norm, represents an estimate of the discretisation error. Yang[85], *et al.* based their error estimator upon a set of resi'dual forces and complementary analysis that provides an upper bound estimate of the global energy of the error. Mucki and Whiteman[86] applied residual of the equilibrium conditions to predict the discretisation error of a finite element solution, both locally and globally.

The performance of the recovery method plays a predominant role in the post-processing-type error estimators. More recently, Zienkiewicz and Zhu[87] developed a local projection technique to estimate derivatives based on the least-square fit of the local polynomial to the super convergent value of the derivatives. This so-called super convergent patch recovery technique is believed to be a substantial improvement over the global projection method. Wiberg and Abdulwahab[88] modified the above method by incorporating least-square fit of equilibrium. Blacker and Belytschko[89] extended the Zienkiewicz and Zhu[87] technique by including the square of the residuals of the equilibrium equation and natural boundary conditions. They proposed a conjoint polynomial for interpolating the local patch stresses on the element. Zhu and Zienkiewicz[90], and Ainsworth[91], et *al.* demonstrated a simple method based on the concept of error in constitutive relations and suggested that the lack of fulfillment of constitutive relations provides a means of estimating the accuracy of the finite element solution.

Rank and Zienkiewicz[92] discussed the performance of the Zienkiewicz and Zhu (ZZ)-error estimator. Niu and Shephard[93] proposed an extraction technique for recovering stresses as well as displacements from the finite element solution with particular emphasis on the extraction of boundary stresses. The technique is super convergent in that the convergence of the recovered quantities is equal to that of the strain energy. Li and Wiberg[94] put forward an L_2 norm of displacement. They used an element-based patch recovery of displacement. Zienkiewicz[95], *et al.* discussed error estimation procedures based on recovery techniques and their effectiveness in linear problems. They also discussed the super convergent patch recovery technique and an alternative so-called recovery-by-equilibrium-in-patches technique.

Zienkiewicz and Zhu[96] showed that super convergent patch recovery procedures yield super convergent gradient values throughout the domain when based on sampling points with known super convergent properties. They put forth possible reasons of poor performance of other recovery procedures. Singh[97], *et al.* proposed a recovery technique based upon the least-square fitting of velocity field over an element patch. Gallimard[98], *et al.* used an incremental form of error estimation and a recovery based on achieving equilibrating stresses between elements. Fourment and Chenot[99] applied a finite difference smoothing method (so-called Orkisz method) to improve precision and efficiency of ZZ-type error estimator. Duarte and Carmo[100] investigated the validity of the convergent patch recovery technique in adaptive procedures involving independent local mesh refinement and polynomial of different degrees in neighbouring elements.

A mesh refinement criterion based on gradients (or curvature) of displacement has been introduced by Zienkiewicz[101], *et al.* Shephard[102] has proposed a mesh refinement technique based on a strain energy density function. Lee and Bathe[103] used pointwise error in strain to guide the mesh refinement. Ravindranath and Krishna[104] adopted mesh refinement criteria based on the equi-distribution of plastic power. Xing[105], *et al.* proposed a refinement criterion based on error in side curvature of the elements. Lo and Lee[106] developed the concept of selective regional refinement. The use of mesh refinement based on the error norm of state variable has been advocated by Buscaglia[107], *et al.*, and Mar and Hick[108], for elasticity problems.

Mucki and Whiteman[86] studied the convergence behaviour of discretisation errors in uniformly and adaptively refined finite element meshes in the context of finite elasticity problems. An rh-method that combines r-and h-adaptive procedures is proposed by Oh[109], *et al.* for linear elastic problems. Sandhu and Leibowitz[110] studied adaptivity in nonlinear finite element analysis. They used a simple linear error estimator based on stress averaging and a nonlinear error estimator based on effective stress and effective plastic strain for simple plasticity problems.

The ZZ-type error estimation and adaptive procedures were applied by Liu and Elemaraghy[111] for assessment of discretisation errors and for adaptive mesh refinement. Zienkiewicz[112,113], *et al.* employed adaptive meshing for problems involving porous and non-porous materials, and presented a method of adaptive mesh refinement analysis. based on error in the energy norm for plasticity problems. Mathisen[114], *et al.* have used a projection-type error estimator based on the L_2 norm of stress and the accumulated plastic strain to predict the discretisation error in quasi-static problems. Wiberg[115], *et al.* have discussed various error estimation and *h*-adaptive procedures for elasticity and plasticity problems. Huerta[116], *et al.* have discussed advantages and limitations of different alternative strategies of adaptive analysis. Boroomand and Zienkiewicz"' presented an adaptive procedure suitable for nonlinear elasto-plastic problems using recovery techniques.

A local *a posteriori* bending error indicator is developed by Han and Peter[118,119] for nonlinear h-adaptive analysis and applied to thin-walled structures incorporating membrane and bending elements.

6. CONCLUSIONS

The different methods of dealing with geometric nonlinearity, material nonlinearity, and frictional contact have been discussed in this paper. A survey of literature on geometric nonlinearity, material nonlinearity, and frictional contact in the context of finite element simulation of forming operations has been presented. Use of continuum and shell elements for analysing sheet forming operations is deliberated. Some of the topics of current research interest in the field of finite element analysis of forming operations, namely error estimation and adaptive mesh refinement are also outlined. Review of past works presented indicates that error estimation and adaptive mesh generation techniques are essential to control solution accuracy.

There appears to be a need for the development of alternative approaches to *apriori* error estimation and *a posteriori* error estimation, and adaptive mesh refinement. Further research is also clearly needed to investigate into the effectiveness and performance of adaptive procedures in finite element analysis of sheet forming operations.

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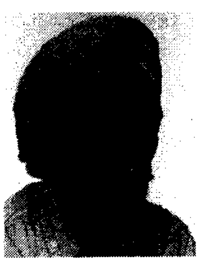

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