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Effect of Shear Correction Factor on Response of Cross-ply Laminated Plates using FSDT

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ABSTRACT

The results of the first-order shear deformation theory (FSDT) depend on the choice of the shear correction factor. In a companion paper, the authors presented a unified general formulation of all higher-order theories for geometrically nonlinear response of cross-ply plates, based on a single polynomial expansion of displacements in the thickness coordinate, z. It includes 10 models available in the literature as special cases. In the present paper, numerical results for the static, vibration, and buckling of the FSDT are compared with the elasticity solution and /or a higher-order theory solution, to investigate the effect of choice of the shear correction factor on the response of composite and simply supported rectangular plates.

Keywords: FSDT, shear correction factor, cross-ply laminate, first-order shear deformation theory, cross-ply plates, models, buckling loads, composite laminated plates, higher-order shear deformation theory

NOMENCLATURE

a, b, h	Sides and thickness of the plate
h_f	Face sheet thickness
L	Number of layers
E_i, G_{ij}, v_{ij}	Young's moduli, shear moduli, Poisson's ratios
E, E_f, ρ_f	Moduli and density for non-dimensionalisation
k_{sx}^2, k_{sy}^2	Shear correction factors
M_x, M_y, M_{xy}	Moments
$\overline{M}_x, \overline{M}_y, \overline{M}_{xy}$	Nondimensionalised moments
N_x, N_y, N_{xy}	Inplane forces
$\overline{N}_x, \overline{N}_y, \overline{N}_{xy}$	Nondimensionalised inplane forces
p_0	Transverse load
Q_x, Q_y	Transverse shear forces
$\overline{Q}_x, \overline{Q}_y$	Nondimensionalised transverse shear forces
$w; \overline{w}$	Deflection; dimensionless deflection
x, y, z, t	Cartesian coordinates, time
$\omega; \overline{\omega}$	Natural frequency; nondimensionalised natural frequency
$\sigma_x, \sigma_y, \tau_{xy}$; $\overline{\sigma}_x, \overline{\sigma}_y, \overline{\tau}_{xy}$	Stresses; nondimensionalised stresses

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1. INTRODUCTION

Laminated composites and sandwich plates are used extensively in the aeronautical, aerospace, and other fields. For their efficient design, a good understanding of their deformation characteristics, stress distribution, natural frequencies, and buckling loads under various load conditions is needed. Pagano[1], and Pagano and Hatfield[2] gave exact solutions for rectangular bidirectional composites and sandwich plates. Srinivas[3], et al., Srinivas and Rao[4], and Noor[5] presented exact three-dimensional elasticity solutions for the free vibration of isotropic, orthotropic and anisotropic composite laminated plates. Noor[6] gave elasticity solutions for buckling loads of multilayered composite plates. Pandya and Kant[7] and Kant and Mallikarjuna[8] have presented solutions both for the laminated composites and sandwich plates.

Swaminathan[9], and Kant and Swaminathan[10] have recently compared five non-classical plate theories for deflection and stresses under transverse load, natural frequencies of free vibration, and buckling loads under inplane static load, for cross-ply composite and sandwich simply supported rectangular plates. One of their findings is that the first-order shear deformation theory (FSDT) yields poor results for thick sandwich plates using a shear correction factor of 5/6.

In the companion paper, Dube[11], *et al.* presented a unified general formulation of all higher-order shear deformation theories (HSDTs) for geometrically nonlinear response of cross-ply plates, based on a single polynomial expansion of displacements in the thickness coordinate, z. It includes 10 models studied by Swaminathan[9] as the special cases. The objective of this study is to investigate the effect of choice of the shear correction factor on the response of composite and sandwich simply supported rectangular plates. The static, vibration, and buckling load results of the FSDT (models 5, 10, and 11, 12 of Swaminathan[9]) for two shear correction factors are compared with the available elasticity solutions[1,2,5,6] and / or with the HSDT (models 1, 6 of Swaminathan[9]).

2. DISPLACEMENT APPROXIMATION OF VARIOUS THEORIES

A laminated cross-ply composite or sandwich plate of sides a, b along axes x, y and thickness h with its mid-plane at $z = 0$ has been considered. Summation convention was used with the summation indices i ranging from 0 to p and j ranging from 0 to q. The displacements were expanded" as polynomials in the thickness coordinate z:

$$u(x,y,z,t) = z^i u_i(x,y,t)$$

$$v(x,y,z,t) = z^i v_i(x,y,t)$$

$$w(x,y,z,t) = z^j w_j(x,y,t) \tag{1}$$

The number of terms $p + 1$ in the inplane displacement can be different from the number of terms $q + 1$ in the transverse displacement.

The 10 theories (model 1 to model 10) studied by Swaminathan[9] are the particular cases of the unified formulation" as shown in Table 1. The model 1 to model 5 are for symmetric laminates and the model 6 to model 10 are for the unsymmetric laminates. The models 5 and 10 are the FSDT models with shear correction factors $k_{sx}^2 = k_{sy}^2 = 5/6$.

The FSDT models with the more general values of k_{sx}^2, k_{sy}^2 , based on the quadratic variation of shear stress across the thickness are called models 11 and 12 for the symmetric and unsymmetric laminates, respectively with

$$k_{sx}^2 = \frac{4\sum_{i=1}^{L}[z_u(i)-z_l(i)]/Q_{55}(i)}{9\sum_{i=1}^{L}\left[\begin{array}{l} z_u(i)-z_l(i) \\ -\frac{8}{3h^2}\{z_u^3(i)-z_l^3(i)\} \\ +\frac{16}{5h^4}\{z_u^5(i)-z_l^5(i)\}\end{array}\right]/Q_{55}(i)} \tag{2}$$

Similarly k_{sy}^2 is defined with Q_{55} replaced by Q_{44}.

Table 1. Identification of the elements of the displacement vector u^* and the variables used in various theories

z^i	i^{th} element of u^*	Variable	Model number									
			1	6	2	7	3	8	4	9	5	10
z^0	1	u_0		u_0		u_0		u_0		u_0		u_0
z^0	2	v_0		v_0		v_0		v_0		v_0		v_0
z^0	3	w_0	w_0	w_0	w_0	w_0	w_0	w_0	$w_0^b + w_0^s$	$w_0^b + w_0^s$	w_0	w_0
z	4	u_1	θ_x	θ_x	θ_x	θ_x	θ_x	θ_x	$-w_{0,x}^b$	$-w_{0,x}^b$	θ_x	θ_x
z	5	v_1	θ_y	θ_y	θ_y	θ_y	θ_y	θ_y	$-w_{0,y}^b$	$-w_{0,y}^b$	θ_y	θ_y
z	6	w_1		θ_z								
z^2	7	u_2		u_0^*		u_0^*						
z^2	8	v_2		v_0^*		v_0						
z^2	9	w_2	w_0^*	w_0^*								
z^3	10	u_3	θ_x^*	θ_x^*	θ_x^*	$-\theta_x^*$	$-\frac{4}{3h^2}(\theta_x + w_{0,x})$	$-\frac{4}{3h^2}(\theta_x + w_{0,x})$	$-\frac{4}{3h^2} w_{0,x}^s$	$-\frac{4}{3h^2} w_{0,x}^s$		
z^3	11	v_3	θ_y^*	θ_y^*	θ_y^*	$-\theta_y^*$	$-\frac{4}{3h^2}(\theta_y + w_{0,y})$	$-\frac{4}{3h^2}(\theta_y + w_{0,y})$	$-\frac{4}{3h^2} w_{0,y}^s$	$-\frac{4}{3h^2} w_{0,y}^s$		
z^3	12	w_3	θ_z^*									
size			6	12	5	9	5	7	5	7	3	5

3. NUMERICAL RESULTS

3.1 Static Analysis

The following material property sets were used for obtaining the numerical results:

Material Set 1

$E_1/E_2 = 25$, $E_2 = E_3 = 7$GPa, $G_{12} = G_{13} = 0.5E_2$, $G_{23} = 0.2E_2$, $\nu_{12} = \nu_{23} = \nu_{13} = 0.25$

Material Set 2

E_1/E_2=Open, $E_2 = E_3 = 7$GPa, $G_{12} = G_{13} = 0.6E_2$, $G_{23} = 0.5E_2$, $\nu_{12} = \nu_{23} = \nu_{13} = 0.25$

Material Set 3

Faces: $E_1/E_2 = 25$, $E_2 = E_3 = 7$GPa, $G_{12} = G_{13} = 0.5E_2$, $G_{23} = 0.2E_2$, $\nu_{12} = \nu_{23} = \nu_{13} = 0.25$

Core: $E_1/E_2 = E_2/E_3 = 0.8$, $E_3 = 3.5$GPa, $G_{13} = G_{23} = 0.12E_3$, $G_{12} = 0.032E_3$, $\nu_{12} = 0.25$, $\nu_{23} = \nu_{13} = 0.2$

Material Set 4

Faces: $E_1/E_2 = 19$, $E_2 = E_3 = 1$GPa, $G_{12} = G_{13} = 0.52E_2$, $G_{23} = 0.338E_2$, $\nu_{12} = \nu_{23} = 0.32$, $\nu_{13} = 0.25$.

Core: $E_1 = 3.2\times10^{-5}$GPa, $E_2 = 2.9\times10^{-5}$, $E_3 = 0.4$GPa, $G_{23} = 6.6\times10^{-2}$GPa, $G_{13} = 7.9\times10^{-2}$GPa, $G_{12} = 2.4\times10^{-3}$GPa, $\nu_{12} = 0.99$, $\nu_{23} = \nu_{13} = 3\times10^{-5}$.

Material Set 5

Faces: $E_1 = 131$GPa, $E_2 = E_3 = 10.34$GPa, $G_{12} = 6.895$GPa, $G_{13} = 6205$GPa, $G_{23} = 6.895$GPa, $\nu_{12} = \nu_{13} = 0.22$, $\nu_{23} = 0.49$, $\rho_f = 1627$ kg/m^3.

Core: $E_1 = E_2 = E_3 = 6.9\times10^{-3}$GPa, $G_{12} = G_{13} = G_{23} = 3.45\times10^{-3}$GPa, $\nu_{12} = \nu_{23} = \nu_{13} = 0$, $\rho_c = 97$ kg/m^3

The static results under transverse load $p_0\sin(\pi x/a)\sin(\pi y/b)$, are nondimensionalised as

$$\overline{w} = \frac{100h^3E}{p_0a^4}w$$

$$(\bar{\sigma}_x, \bar{\sigma}_y, \bar{\tau}_{xy}) = \frac{h^2}{p_0 a^2}(\sigma_x, \sigma_y, \tau_{xy})$$

$$(\bar{M}_x, \bar{M}_y, \bar{M}_{xy}) = (M_x, M_y, M_{xy}) / p_0 a^2$$

$$(\bar{Q}_x, \bar{Q}_y) = (Q_x, Q_y) / p_0 a$$

with $E = E_2$. Unless otherwise specified within the table(s), the locations for the maximum values of the displacements, stresses and stress resultants

Table 2. Comparison of static response of 2-layered (0°/90°), j-layered (0°/90°/0°), and 9-layered (0°/90°/0°/90°/0°) simply supported square laminates under sinusoidal transverse load

L	a/h	Model No.	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\sigma}_y$	$\bar{\tau}_{xy}$	$\bar{M}_x$	$\bar{M}_y$	$-\bar{M}_{xy}$	$\bar{Q}_x$	$\bar{Q}_y$
2	5	Elas[1]	1.7287	0.7723	0.8036	0.0586	•				
		6	1.6800	0.7615	0.7615	0.0559	0.0420	0.0420	0.0087	0.1592	0.1592
		10	1.7584	0.7158	0.7158	0.0525	0.0419	0.0419	0.0087	0.1592	0.1592
		12	1.7584	0.7158	0.7158	0.0525	0.0419	0.0419	0.0087	0.1592	0.1592
	10	Elas[1]	1.2318	0.7317	0.7353	0.0540			•		
		6	1.2192	0.7271	0.7271	0.0534	0.0419	0.0419	0.0087	0.1592	0.1592
		10	1.2373	0.7158	0.7158	0.0525	0.0419	0.0419	0.0087	0.1592	0.1592
		12	1.2373	0.7158	0.7158	0.0525	0.0419	0.0419	0.0087	0.1592	0.1592
3	4	Elas[1]	•	0.755	0.556	0.0505					
		1	1.8948	0.7648	0.4939	0.0487	0.0667	0.0215	0.0065	0.2302	0.0881
		5	1.7758	0.4370	0.4775	0.0369	0.0703	0.0187	0.0062	0.2403	0.0780
		11	1.9938	0.4032	0.5764	0.0421	0.0649	0.0224	0.0070	0.2260	0.0923
	10	Elas[1]	•	0.590	0.285	0.0289	•				
		1	0.7151	0.5836	0.2705	0.0279	0.0813	0.0113	0.0044	0.2692	0.0491
		5	0.6693	0.5134	0.2536	0.0252	0.0826	0.0104	0.0042	0.2726	0.0457
		11	0.7220	0.5050	0.2782	0.0265	0.0812	0.0113	0.0044	0.2690	0.0493
	20	Elas[1]		0.552	0.210	0.0289		•			
		1	0.5053	0.5504	0.2049	0.0231	0.0852	0.0086	0.0038	0.2794	0.0389
		5	0.4921	0.5318	0.1997	0.0223	0.0855	0.0084	0.0037	0.2804	0.0379
		11	0.5064	0.5296	0.2064	0.0227	0.0852	0.0086	0.0038	0.2794	0.0389
9	4	Elas[2]		0.6840	0.6280	0.0337					
		1	1.5065	0.6311	0.5710	0.0306	0.0518	0.0422	0.0036	0.1743	0.1440
		5	1.5292	0.4744	0.5070	0.0219	0.0500	0.0440	0.0036	0.1687	0.1496
		11	1.5338	0.4573	0.5273	0.0221	0.0483	0.0457	0.0037	0.1632	0.1551
	10	Elas[2]		0.5510	0.4720	0.0233				•	
		1	0.6095	0.5466	0.4659	0.0229	0.0548	0.0393	0.0036	0.1835	0.1348
		5	0.6096	0.5145	0.4595	0.0215	0.0542	0.0399	0.0036	0.1816	0.1368
		11	0.6119	0.5081	0.4672	0.0216	0.0535	0.0406	0.0036	0.1795	0.1388
	20	1	0.4762	0.5398	0.4410	0.0217	0.0561	0.0381	0.0036	0.1875	0.1308
		5	0.4761	0.5312	0.4398	0.0214	0.0560	0.0383	0.0036	0.1869	0.1314
		11	0.4769	0.5292	0.4422	0.0214	0.0557	0.0385	0.0036	0.1863	0.1320

Maximum value of $\bar{\sigma}_y$ occurs at $z = \pm h/6$ for $L = 3$ and at $z = \pm 0.4h$ for $L = 9$.

for the present evaluations are as follows:

$w(a/2,b/2,0)$, $\sigma_x(a/2,b/2,\pm h/2)$, $\sigma_y(a/2,b/2,\pm h/2)$, $\tau_{xy}(0,0,\pm h/2)$

$M_x(a/2,b/2)$, $M_y(a/2,b/2)$, $M_{xy}(0,0)$

$Q_x(0,b/2)$, $Q_x(a/2,0)$.

Static response of the following simply supported square laminates for material set 1 under transverse load is compared in Table 2:

Two-layered antisymmetric **(0°/90°)** with each ply of thickness *h/2*

Three-layered symmetric **(0°/90°/0°)** with each ply of thickness *h/3*

Table 3. Comparison of static response of 3-layered (0°/90°/0°) simply supported square laminates under sinusoidal transverse load for different E_1/E_2 ratios

E_1/E_2	a/h	Model No	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\sigma}_y$	$\bar{\tau}_{xy}$	$\bar{M}_x$	$\bar{M}_y$	$-\bar{M}_{xy}$	$\bar{Q}_x$	$\bar{Q}_y$
3	4	1	2.4351	**0.2984**	0.1327	0.1095	**0.0449**	0.0221	0.0172	0.1949	0.1234
		5	2.4858	0.2751	0.1187	0.1066	0.0448	0.0210	0.0178	0.1966	0.1217
		11	2.5002	0.2726	0.1202	0.1070	**0..0444**	0.0213	0.0178	0.1955	0.1228
	10	1	1.8719	**0.2897**	0.1146	0.1052	0.0466	0.0200	0.0174	0.2008	0.1175
		5	1.8777	0.2860	0.1124	0.1047	0.0466	0.0198	0.0174	0.2012	0.1172
		11	1.8803	0.2855	0.1126	0.1048	0.0465	0.0199	0.0175	0.2008	0.1173
	20	1	1.7888	0.2887	0.1119	0.1045	0.0469	0.0197	0.0174	0.2018	0.1165
		5	1.7902	0.2878	0.1113	0.1044	0.0469	0.0197	0.0174	0.2019	0.1164
		11	1.7908	0.2877	0.1114	0.1044	0.0469	0.0197	0.0174	0.2019	0.1164
10	4	1	1.7201	**0.4729**	0.2142	**0.0680**	0.0633	0.0177	0.0102	0.2309	0.0875
		5	1.7438	**0.3951**	0.2287	0.0629	0.0637	0.0166	0.0105	0.2332	0.0851
		11	1.7724	**0.3908**	0.2350	0.0637	0.0631	0.0170	0.0106	0.2315	0.0868
	10	1	1.2064	0.4515	0.1642	0.0552	0.0705	0.0128	0.0090	0.2498	0.0685
		5	1.0264	0.4380	0.1654	0.0543	0.0706	0.0126	0.0091	0.2504	0.0679
		11	1.0321	0.4372	0.1667	0.0545	0.0705	0.0126	0.0091	0.2500	0.0683
	20	1	**0.9183**	**0.4494**	0.1534	0.0530	0.0719	0.0119	0.0088	0.2534	0.0649
		5	0.9182	0.4460	0.1537	0.0528	0.0719	0.0118	0.0088	0.2536	0.0647
		11	0.9197	0.4458	0.1540	0.0528	0.0719	0.0118	0.0088	0.2535	0.0648
40	4	1	1.2201	**0.7309**	0.5211	**0.0390**	0.0721	0.0185	0.0054	0.2433	0.0750
		5	1.2412	0.4456	0.5951	0.0332	0.0716	0.0186	0.0055	0.2425	0.0758
		11	1.2732	0.4392	0.6183	0.0341	0.0706	0.0193	0.0057	0.2398	0.0785
	10	1	0.4564	0.5915	0.2776	0.0216	0.0852	0.0093	0.0034	0.2784	0.0399
		5	0.4541	0.5312	0.2817	0.0205	0.0854	0.0091	0.0034	0.2789	0.0394
		11	0.4615	0.5297	0.2817	0.0207	0.0851	0.0093	0.0035	0.2783	0.0400
	20	1	0.3257	0.5663	0.2111	0.0179	0.0884	0.0071	0.0029	0.2869	0.3139
		5	0.3248	0.5504	0.2115	0.0176	0.0884	0.0070	0.0029	0.2871	0.3122
		11	0.3268	0.5500	0.2130	0.0177	0.0884	0.0070	0.0029	0.2869	0.3138

Maximum value of $\bar{\sigma}_y$ occurs at $z=\pm h/2$ for $E_1/E_2=3$ and at $z=\pm h/6$ for all other E_1/E_2 ratios.

Nine-layered symmetric (0°/90°/0°/90°/ $\bar{0}$ °) with each 0° ply of thickness $h/10$ and each 90° ply of' thickness $h/8$.

For antisymmetric composite plates, the value of improved shear correction factor for model 12 is also 5/6. Hence, the results of the FSDT using the models 10 and 12 are identical. For symmetric laminates, the FSDT model 11 using general shear correction factor yields improved results except for $\bar{\sigma}_x$ compared to the FSDT model 5 using $k_{sx}^2 = k_{sy}^2 = 5/6$. The results for 3-layered (0°/90°/0°)

Table 4. Comparison of static response of 5-layered (0°/90°/core/0°/90°), 3-layered (0°/core/0°), and 21-layered symmetric simply supported square sandwich plates under sinusoidal transverse load

L	a/h	Model No.	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\sigma}_y$	$\bar{\tau}_{xy}$	$\bar{M}_x$	$\bar{M}_y$	$-\bar{M}_{xy}$	$\bar{Q}_x$	$\bar{Q}_y$
3	4	Elas[1]	•	1.512	0.2533	0.1437	•	•		•	•
		1	7.0847	1.4982	0.2530	0.1375	0.0624	0.0194	0.0097	0.2266	0.0917
		5	4.7666	0.8918	0.1562	0.0907	0.0728	0.0133	0.0076	0.2527	0.0657
		11	5.4449	0.8665	0.1688	0.0966	0.0707	0.0143	0.0081	0.2477	0.0706
	10	Elas[1]	•	1.152	0.1099	0.0707				•	•
		1	2.0755	1.1470	0.1087	0.0683	0.0815	0.0089	0.0054	0.2732	0.0451
		5	1.5604	1.0457	0.0798	0.0552	0.0853	0.0068	0.0046	0.2824	0.0359
		11	1.6887	1.0364	0.0844	0.0573	0.0845	0.0072	0.0048	0.2806	0.0377
	20	Elas[1].	•	1.110	0.0700	0.0511			•	•	
		1	1.1897	1.1077	0.0708	0.0502	0.0871	0.0059	0.0041	0.2867	0.0316
		5	1.0524	1.0831	0.0612	0.0466	0.0883	0.0052	0.0039	0.2897	0.0286
		11	1.0860	1.0803	0.0626	0.0472	0.0881	0.0053	0.0040	0.2891	0.0292
	100	Elas[1]		1.098	0.550	0.0437	•	•	•		•
		1	0.8881	1.0964	0.0567	0.0435	0.0892	0.0048	0.0037	0.2918	0.0265
		5	0.8852	1.0964	0.0546	0:0435	0.0894	0.0046	0.0037	0.2923	0.026 1
		11	0.8866	1.0963	0.0547	0.0435	0.0894	0.0046	0.0037	0.2922	0.026 1
5	4	6	13.524	1.5353	1.5353	0.2065	0.0425	0.0425	0.0082	0.1592	0.1592
		10	2.8439	0.6207	0.6207	0.689	0.0426	0.0426	0.0081	0.1592	0.1592
		12	3.5507	0.6207	0.6207	0.689	0.0426	0.0426	0.0081	0.1592	0.1592
	10	6	3.1861	0.7790	0.7790	0.0926	0.0426	0.0426	0.0081	0.1592	0.1592
		10	1.3340	0.6207	0.6207	0.0689	0.0426	0.0426	0.0081	0.1592	0.1592
		12	1.4471	0.6207	0.6207	0.0689	0.0426	0.0426	0.0081	0.1592	0.1592
	20	6	1.5869	0.6610	0.6610	0.0749	0.0426	0.0426	0.008 1	0.1592	0.1592
		10	1.1183	0.6207	0.6207	0.0689	0.0426	0.0426	0.0081	0.1592	0.1592
		12	1.1466	0.6207	0.6207	0.0689	0.0426	0.0426	0.0081	0.1592	0.1592
	100	6	1.0681	0.6227	0.6227	0.0691	0.0426	0.0426	0.0081	0.1592	0.1592
		10	1.0493	0.6207	0.6207	0.0689	0.0426	0.0426	0.0081	0.1592	0.1592
		12	1.0504	0.6207	0.6207	0.0689	0.0426	0.0426	0.0081	0.1592	0.1592
21	5	1	3.6067	1.2937	1.1502	0.0658	0.0497	0.0427	0.0044	0.1702	0.1480
		5	2.7964	1.0718	1.0255	0.0567	0.0474	0.0446	0.0046	0.1636	0.1547
		11	3.1010	1.0751	1.0221	0.0567	0.0476	0.0445	0.0046	0.1641	0.1542
	10	1	1.7254	1.1283	1.0476	0.0584	0.0483	0.0439	0.0045	0.1661	0.1522
		5	1.5280	1.0647	1.0326	0.0567	0.0471	0.0449	0.0046	0.1627	0.1556
		11	1.6042	1.0666	1.0308	0.0567	0.0472	0.0448	0.0046	0.1629	0 . 1 5 5 4
	20	1	1.2498	1.0746	1.0338	0.0566	0.0474	0.0448	0.0046	0.1633	0.1550
		5	1.2108	1.0616	1.0365	0.0567	0.0470	0.0451	0.0046	0.1622	0.1561
		11	1.2299	1.0616	1.0358	0.0567	0.0470	0.0450	0.0046	0.1623	0.1561

Maximum value of $\bar{\sigma}_y$ shown for 21-layers laminate occurs at the interface of layers 1 and 2 and at the interface of layers 20 and 21.

square plates of material set 2 for various modular ratios are compared in Table 3.

A comparison of static results of simply supported three-layered symmetric (0°/core/0°) square sandwich plate with thickness of each face sheet $h_f=h/10$ for material set 3. under sinusoidal static load is given in Table 4. Except for $\bar{\sigma}_x$, model 11 yields improved results compared to model 5. The results for five-layered square sandwich plate (0°/90°/core/0°/90°) with isotropic core of thickness $(2/3)h$ using material set 5, under static sinusoidal load are also compared in Table 5. The FSDT results of model 12 are identical to those of model 10 for all the variables except the deflection $\bar{w}$, for which the former yields slightly improved results.

A simply supported symmetric sandwich square plate composed of lo-layered cross-ply composite face sheets and a lightweight honeycomb core made of a titanium alloy is also considered for static analysis under sinusoidal load using material set 4. The fibre orientation of the layer of the bottom face sheet was $(0°/90°)_5$ with the fibres of the bottom layer making 0° with the x-coordinate. The thickness of each face sheet, h_f is taken as 0. lh. For such a 21-layered $[(0°/90°)_5/\text{core}]_s$ simply supported symmetric square sandwich plate, the static results under sinusoidal transverse load are compared in Table 4. The model 11 yields improved results, especially for $\bar{w}$ compared to model 6.

3.2 Natural Frequency

The natural frequency ω for (m, n)[th] spatial mode has been nondimensionalised as $\bar{\omega}=(\omega a^2/h)\sqrt{(\rho_f/E_f)}$, with ρ_f as the density of the face sheet and $E_f=E_2$ for the face sheet. The comparison of the fundamental natural frequency for cross-ply antisymmetric and symmetric laminates for material set 2 is presented in Tables 5 and 6. As remarked for the static case, the FSDT models 10 and 12 yield identical results for antisymmetric laminates, since the general shear correction factor is also 5/6 for such laminates. For symmetric

Table 5. Comparison of natural frequency ($m = n = 1$) of simply supported cross-ply laminated square plates with $a/h = 5$

Lamination	Model No.	E_1/E_2				
		3	10	20	30	100
(0" /90°)	Elas[5]	6.2578	6.9845	7.6745	8.1763	8.5625
	6	6.2336	6.974 1	7.7140	8.2776	8.7272
	10	6.2086	6.9392	7.7060	8.3211	8.8333
	12	6.2086	6.9392	7.7060	8.3211	8.8333
$(0"/90°)_s$	Elas[5]	6.6185	8.2103	9.5603	10.2723	10.7515
	6	6.5712	8.1697	9.2514	9.8595	10.2687
	10	6.5630	8.1848	9.2775	9.8851	10.2895
	12	6.5490	8.1291	9.1839	9.7690	10.1591
$(0"/90°/\bar{0}°)_s$	Elas[5]	6.6468	8.5223	9.948	10.785	11.3435
	6	6.6033	8.4382	9.8246	10.6437	11.1957
	10	6.5844	8.4201	9.8265	10.6785	11.2671
	12	6.5801	8.4030	9.7983	10.6447	11.2306
$(0°/90°/0°/90°/\bar{0}°)_s$	Elas[5]	6.66	8.608	10.1368	11.0525	11.6698
	6	6.6143	8.5422	10.0547	10.9644	11.5813
	10	6.5940	8.5197	10.0367	10.9545	11.5788
	12	6.5929	8.5149	10.0289	10.9453	11.5690

Table 6. Comparison of natural frequency ($m = n = 1$) of simply supported cross-ply laminated square plates with $E_1/E_2=40$, $G_{12}= G_{13}= 0.6E_2$, $G_{23}= 0.5E_2$, $\nu_{12}= \nu_{13}=\nu_{23}= 0.25$

Lamination	Model No.	a/h				
		4	10	20	50	100
(0° /90°)	6	7.9081	10.4319	11.0663	11.2688	11.2988
	10	8.0350	10.473 1	11.0779	11.2705	11.2990
	12	8.0350	10.473 1	11.0779	11.2705	11.2990
(0” /90°)$_s$	1	9.2870	15.1048	17.6470	18.6720	18.8357
	5	9.3949	15.1426	17.6596	18.6742	18.8362
	11	9.3026	15.0294	17.6083	18.6641	18.8336

laminates, model 11 yields marginally less accurate results than model 5.

The natural frequencies of a five-layered sandwich thin and thick square plates with antisymmetric and symmetric cross-ply faces using material set 5 with ratio of core thickness h_c to thickness h_f of each face sheet as 10, are compared in Table 7. Both the FSDT models 5 and 11 yield highly inaccurate results for thick sandwich plates with a/h = 10, though model 11 yields slightly improved results. Even for a thin sandwich plate with a/h = 10, the FSDT results have an error of 7 per cent for fundamental frequency and 35 per cent for mode $m = n = 3$. For thin plates also, the model 11 with general shear correction factor reduces the error by only a small amount. The exact results for these sandwich plates are not available. Hence, these

Table 7. Comparison of natural frequencies of square sandwich plates with h_c/h_f=10

a/h	m	n	Lamination					
			(0” /90° /core/90° /0°)			(0” /90° / core/0° / 90”)		
			Model 1	Model 5	Model 11	Model 6	Model 10	Model 12
10	1	1	4.9674	13.9986	13.6942	4.9618	13.9972	13.6936
	1	2	7.9796	**30.6940**	**29.4247**	8.1928	31.1131	**29.7829**
	1	3	11.5626	51.4107	48.5050	11.9857	51.9505	**48.9459**
	2	1	8.4196	31.5093	30.1185	8.1928	31.1131	**29.7829**
	2	2	10.5313	42.23 18	40.2382	10.5185	42.2444	**40.2509**
	2	3	13.5244	59.2501	55.8907	13.7520	59.5081	**56.0950**
	3	1	12.4993	51.9629	48.9560	11.9857	51.9505	**48.9459**
	3	2	14.0096	59.7252	56.2622	13.7520	59.508 1	**56.0950**
	3	3	16.4809	72.8329	68.3687	16.4599	72.8574	**68.3903**
100	1	1	15.5559	16.2828	16.2777	15.5455	16.2726	16.2676
	1	2	38.0613	43.5220	43.4818	39.2599	44.882 1	44.8375
	1	3	70.7908	91.9842	91.7962	73.4883	95.3282	95.1179
	2	1	40.493 1	46.2593	46.2098	39.2599	44.8821	44.8375
	2	2	55.1812	64.7778	64.6985	55.1396	64.7390	64.660 1
	2	3	84.5471	107.036	106.812	84.2766	109.346	109.103
	3	1	76.2389	98.6767	98.4425	73.4883	95.3282	95.1179
	3	2	86.1147	111.734	111.471	84.2766	109.346	109.103
	3	3	106.657	144.456	144.067	106.567	144.375	143.988

Table 8. Natural frequencies of square sandwich plates with h_c/h_f=10 using highly accurate zig-zag theory[12]

a/h	m	n	Lamination	
			$(0°/90°/\text{core}/90°/0°)$	$(0°/90°/\text{core}/0°/90°)$
	1	1	11.9541	11.9458
	1	2	22.7996	23.4145
	1	3	34.9949	36.1657
	2	1	24.0475	23.4145
100	2	2	30.9845	30.9612
	2	3	40.8329	4 1.4740
	3	1	37.3524	36.1657
	3	2	42.1676	41.4740
	3	3	49.8345	49.7963

are compared in Table 8 with the highly accurate zig-zag theory presented by Kapuria[12]. It is observed that the models 1, 5, 6, 10, 11, and 12 yield very inaccurate results. Therefore, a difference of 7 per cent for fundamental mode in these approximate theories is not unusual.

3.3 Buckling Load

Buckling results have been presented for uniform inplane loading N_x and the buckling load has been nondimensionalised as

$$\bar{N}_x = N_x b^2/(E_f h^3)$$

where E_f=E_2 for the face sheet. The comparison of buckling loads of cross-ply symmetrically laminated square plates of material set 2, is presented in Table 9. The FSDT models 5 and 11 yield quite accurate results with model 11 yielding slightly better results in majority of the cases. The effect of a/h on the bucking loads of 3-layered and 4-layered square plates are given in Table 10 for material set 2 with E_1/E_2= 40. The buckling load results of a five-layered square symmetric sandwich plate, using material set 5 with h_c/h_f=10, are also compared in Table 10. Both FSDT models 5 and 11 yield highly inaccurate results even for moderately thick sandwich plates with a/h=20, though model 11 yields slightly improved results. Even for thin sandwich plate with a/h=100, the FSDT results have an error of 10 per cent for buckling load. The error in buckling results for thin sandwich plates may be, for the same reason as listed above, for the natural frequency.

4. CONCLUSIONS

The effect of general shear correction factor dependent on the lay-up, instead of a factor of 5/6, in a FSDT on the static response, natural frequency,

Table 9. Comparison of buckling loads of simply supported cross-ply laminated square plates with a/h=10, E_2=E_3, G_{12}= G_{13}=0.6E_2, G_{23}= 0.5 E_2, ν_{12}= ν_{13}= ν_{23}= 0.25

Lamination	Model No.	E_1/E_2				
		3	10	20	30	40
$(0°/90°)_s$	Elas[6]	5.3044	9.762 1	15.0191	19.3040	22.8807
	1	5.3745	8.8066	14.8522	18.8314	22.0671
	5	5.3961	9.8711	14.9847	19.0266	22.3151
	11	5.3886	9.8167	14.8343	18.7703	21.9546
$(0°/90°/\bar{0}°)_s$	Elas[6]	5.3255	9.9603	15.6527	20.4663	24.5929
	1	5.3911	10.0552	15.7152	20.4584	24.5026
	5	5.4068	10.0762	15.7362	20.4847	24.5465
	11	5.4045	10.0593	15.6885	20.4022	24.4292
$(0°/90°/0°/90°/\bar{0}°)_s$	Elas[6]	5.3352	10.0417	15.9153	20.9614	25.3436
	1	5.3966	10.1452	16.0392	2 1.0824	25.4517
	5	5.4116	10.1682	16.0682	21.1171	25.4946
	11	5.4110	10.1635	16.0546	21.0935	25..4609

Table 10. Comparison of buckling load of simply supported square plates

Lamination	Model No.	a/h = 4	10	20	50	100
$(0^\circ/90^\circ/0^\circ)$	1	8.0554	22.067 1	31.0541	35.2248	35.9211
	5	8.1631	22.3151	31.1959	35.2552	35.9290
	11	7.9582	2 1.9546	3 1.0054	35.2153	35.9186
$(0^\circ/90^\circ/90^\circ/0^\circ)$	1	8.8148	23.2528	31.6278	35.3409	35.9511
	5	9.1138	23.4529	31.7071	35.3560	35.9550
	11	8.9414	23.1026	31.5226	35.3176	35.9450
$(0^\circ/90^\circ/core/90^\circ/0^\circ)$	1	0.1014	0.5404	1.6772	4.1730	5.3053
	5	1.9003	4.3803	5.3863	5.7567	5.8138
	11	1.6871	4.1848	5.3099	5.7426	5.8102

and buckling load of simply supported rectangular plates has been investigated. The comparison of results reveals that for symmetrically laminated cross-ply composite plates, the present model 11 using general shear correction factor generally yields improved results over model 5 using the shear correction factor of 5/6. However, for sandwich plates with a soft core, there is not much significant improvement. Hence, the FSDT models cannot be improved by changing the shear correction factor. It has been concluded that a layerwise theory with only five independent variables as in the FSDT, which ensures continuity of transverse shear stress at the layer interfaces, should be employed for the analysis of sandwich plates.

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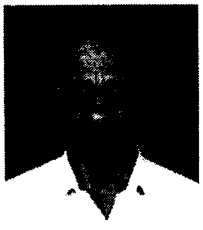

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