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Post-flight Analysis and Design Improvement in Command Guidance System for a Short-range Surface-to-air Missile System

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ABSTRACT

A short-range missile with command-to line-of-sight and three-beam guidance has been considered in this paper. The earlier command guidance system (CGS) design shows unacceptably high-low-frequency weave-mode oscillations, leading to high latax and body rate oscillations, even for benign, low-speed non-manoeuvring target engagements. For successful target engagements with the three-beam guidance, missile is to be handed over from wide-to-medium receiver beam, and finally, from medium-to-the most accurate narrow receiver beam, depending on the angular error wrt line-of-sight as early as possible. Due to large amplitude oscillation in the earlier CGS design, the handing over of the missile to narrow receiver beam, and in many cases, to the medium receiver beam, itself could not take place, leading to failure of guidance. In this paper, the cause for this undesirable high magnitude weave-mode oscillation has been analysed in detail. After establishing this, saturation aspects of the earlier CGS design; a simple implementable CGS re-design was carried out to reduce this saturation aspect drastically for preserving almost full-phase advance effects of the linear new analogue compensators designed to give the required stability margins of guidance loop.

Keywords: Command-to line-of-sight, CLOS, comand guidance system, surface-to-air missile system, closed-loop system, linear error channel, dynamic error channel, fast Fourier transformation, weave-mode oscillations

1. INTRODUCTION

The existing command guidance system (CGS) design uses analog compensators (with operational amplifiers output voltage limit of 10*v*) in the guidance loop to maintain its stability margin. Detailed investigations with receiver noises of different channels, ie, wide, medium, and narrow, has been carried out to establish how those noises get heavily amplified in the existing CGS phase-advance networks, leading to saturation of analogue compensators. With saturation, phase-advance effects of these compensators get very much reduced, leading to reduced stability margin of the guidance loop, and thereby, leading to weave-mode oscillations as noticed during flights. The comparison of earlier and the new command guidance system design with flight-extracted noise of several earlier flights on a high fidelity 6-DOFs model of the missile, has been carried out, showing the great performance improvement of the new design in terms of drastic reduction of weave-mode oscillations, leading to planned changeover to narrow beam and miss-distance within a specified limit. Finally, the new design has been fully established through several guided flights recently carried out,

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where the angle error channels profiles were found to be quite smooth and of low magnitude, leading to changeover to narrow beam as per the plan and giving miss-distance within the specification.

2. MISSILE GUIDANCE SYSTEM

A short-range, low altitude point defence surface-to-air missile system has been considered. The guidance used is command-to line-of-sight (CLOS). Figure 1 shows the basic functional schematic of the missile guidance system. The narrow beam of the fly catcher radar was tracking the target. The missile was interrogated in the transpoder mode and the missile angular error from the line-of-sight (LOS) was estimated by the three receivers on the ground, aligned and slaved to the track beam of the target. The missile was initially gathered and guided in the wide beam. Depending on the angular error from the LOS, missile was handed over from wide-to-medium, and later, from medium-to-the most accurate narrow beam. The missile was canard control agile with high latax capability. A complete guidance loop is shown in Fig. 2. The ground CGS generates latax command using linear error channel (LEC), computed positional separation from LOS ΔY error. In addition, latax demand was computed in dynamic error compensation (DEC) to compensate for the moving LOS arising out of target motion, which is added as a feed-forward command to the command guidance loop (Fig. 2). The present CGS has been realised as an analog system.

The linear error channel is the closed-loop system where the latax command generated is proportional to the linear displacement of the missile from the target LOS. The angular error from LOS ($\Delta\beta$ in azimuth), as measured from the ground receiver is multiplied by the missile range, R_m to get the position separation from the LOS, ie, ΔY error. This error was further passed through different compensators, giving sufficient lead for achieving the required stability margin of the guidance loop.

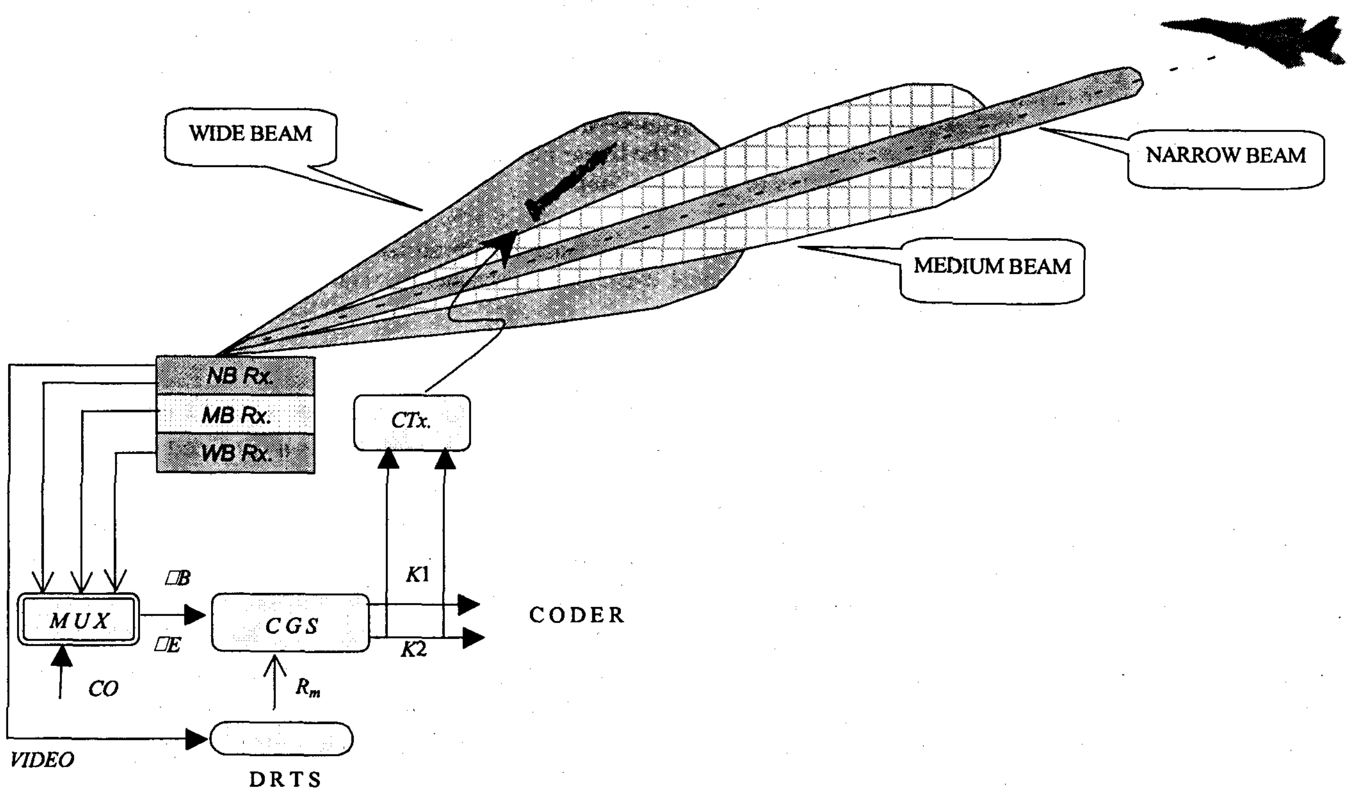

Figure 1. Missiles command guidance system

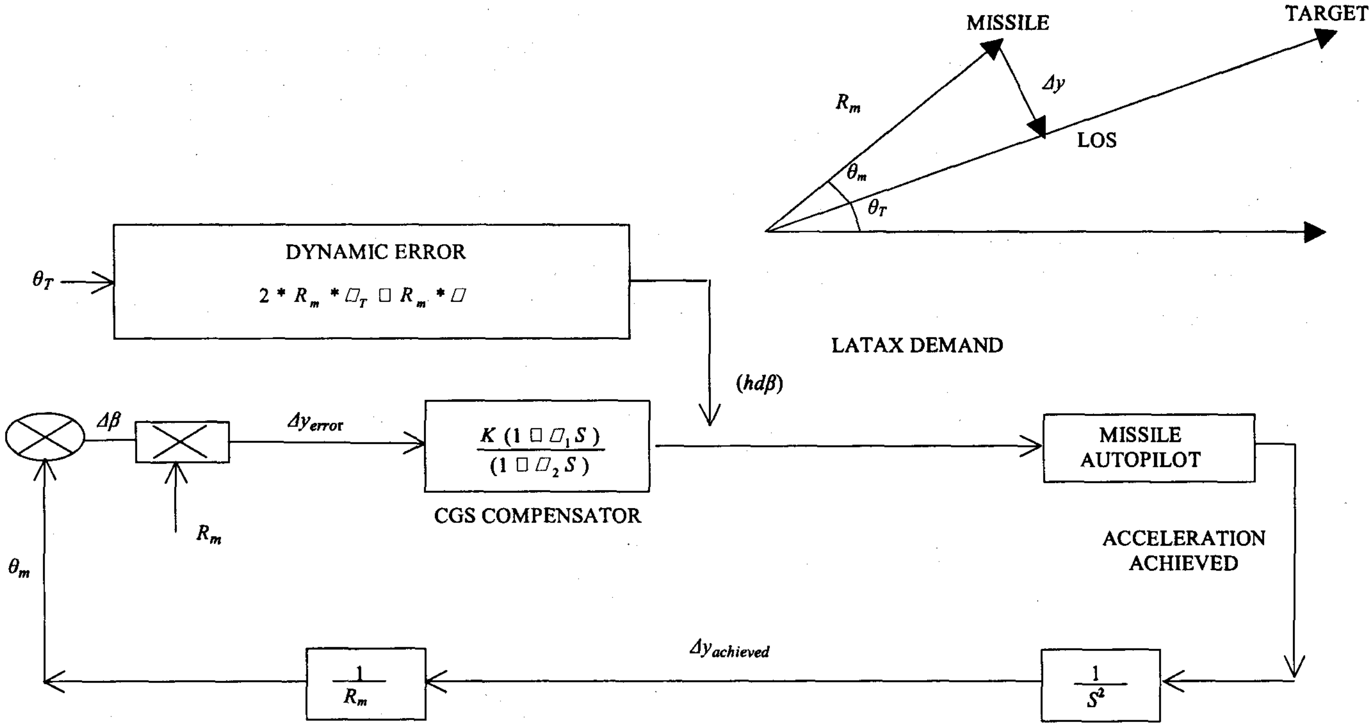

Figure 2. Command guidance loop

Latax due to linear error channel = $K_G \times \Delta\beta \times R_m$, where K_G is the command guidance gain and compensator transfer function.

Latax of dynamic error compensation is computed to keep the missile on the moving LOS to target, based on the measurement of LOS angular rate $(\dot{\theta}_T)$, angular acceleration $(\ddot{\theta}_T)$, and missile range (R_m), range rate $(\dot{R}_m)$ as per

Latax due to dynamic error compensation

$$=2\times \dot{R}_m \times \dot{\theta}_T + R_m \times \ddot{\theta}$$

Latax commands generated in both the elevation and azimuth planes in the ground command guidance system were transmitted to the missile, which was achieved by missile autopilot reducing positional separation, ΔY and closing guidance loop (Fig. 2).

3. RE-DESIGN OF COMMAND GUIDANCE SYSTEM

In many flights, it was observed that both the missile angle channel errors, $\Delta\beta$ and $\Delta\varepsilon$ oscillate with large amplitude at the guidance loop frequency of 0.5 Hz/0.6 Hz. These weave-mode oscillations also generates undesirable large amplitude of latax and body rate oscillations, even for a benign target engagement. This, along with error originating due to tracker oscillations lead to unacceptably high oscillating angular error and miss-distance in different flights and the changeover from medium beam to narrow beam, and in some case, even from wide beam to medium beam, was not happening as per the plan. A detailed investigation of the missile guidance loop design has been presented in Section 3 to justify the reason for this undesirable and high amplitude weave-mode oscillation. This is followed by a remedial measure, which turns out to be a re-design of the command guidance system.

4. SYSTEM ANALYSIS

A detailed system analysis has been carried out including guidance loop design studies, old flight data analysis, and correlation with simulation for bringing out the reason for weave-mode oscillations.

4.1 Guidance Loop Studies

Ground command guidance system and missile autopilot are the main elements in the complete guidance loop. The command guidance system and the missile autopilot transfer functions have been investigated. Gain and phase plot of the command guidance system in Figs 3(a) and 3(b) show that the command guidance system has been designed to give maximum phase lead of 60° near guidance loop gain crossover frequency (GCF) at 0.7Hz with a penalty of large relative noise amplification greater than 8 for frequencies between 2Hz to 5Hz. The onboard autopilot is implemented in analog and its gains are not scheduled as a function of flight parameters. The missile velocity reaches maximum at the sustainer cutoff of 7.2 s and reduces in the coast phase, as shown in Fig. 4(a). This gives varying autopilot closed-loop gain with time as shown in Fig. 4(b). Due to this, gain/phase of guidance loop, open-loop transfer function vary with time. The open-loop gain and the phase plot of guidance loop at 7.2 s and 15.2 s are shown in Figs 5 and 6. The guidance loop gain and phase margin variation, as a function of flight time, are shown in Figs 7(a) and 7(b). The command guidance system-lead network is designed to give maximum phase margin of 48° at the highest dynamic pressure condition (ie, 7.2 s) and this maintains margin of 30° till 18 s. The gain

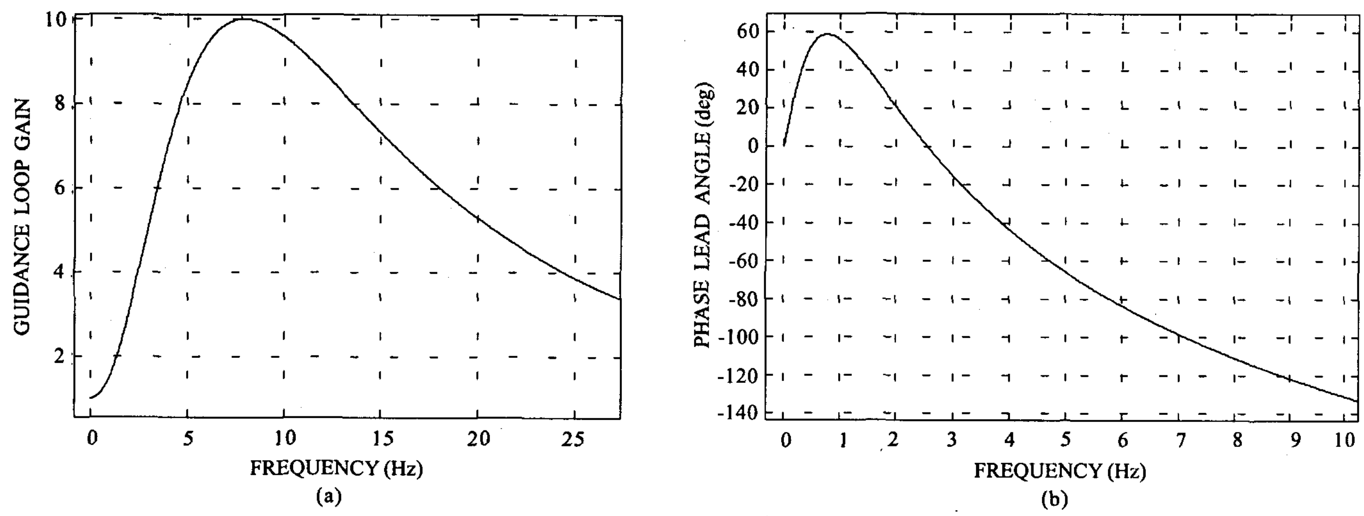

Figure 3. Frequency response of old CGS: (a) CGS gain and (b) CGS phase lead

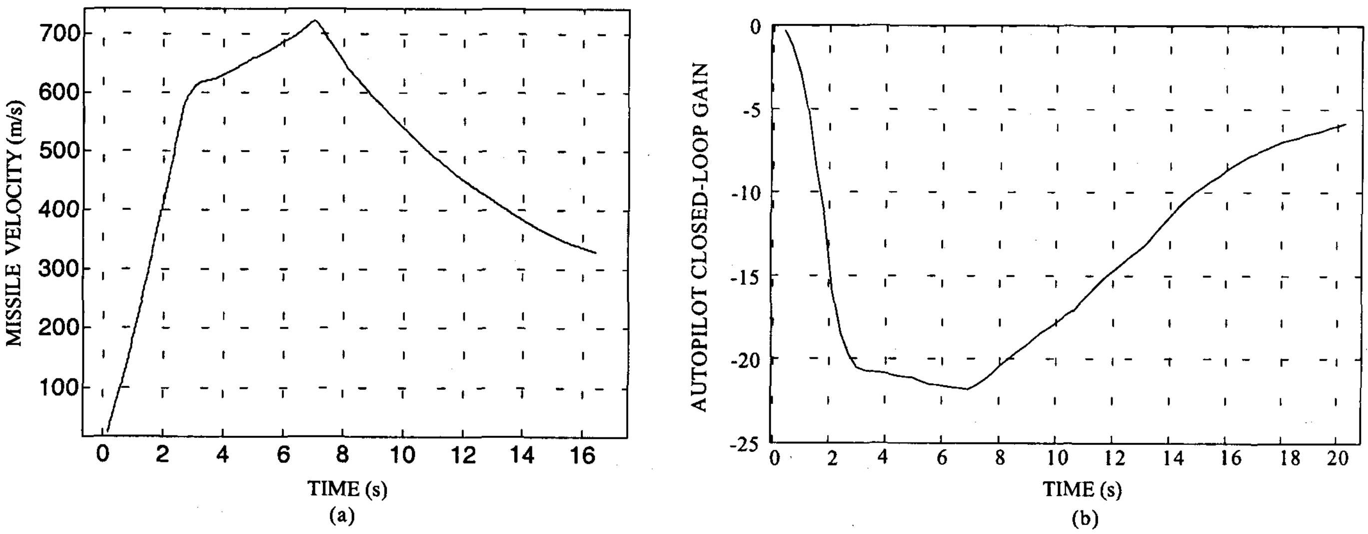

Figure 4.Guidance loop (a) Missile velocity and (b) autopilot gain

margin is reasonably high (7 dB) during initial phase and goes to 15 dB at the end. From the control margin point of view, the above system should have good damping and the weave-mode oscillation is not expected to occur.

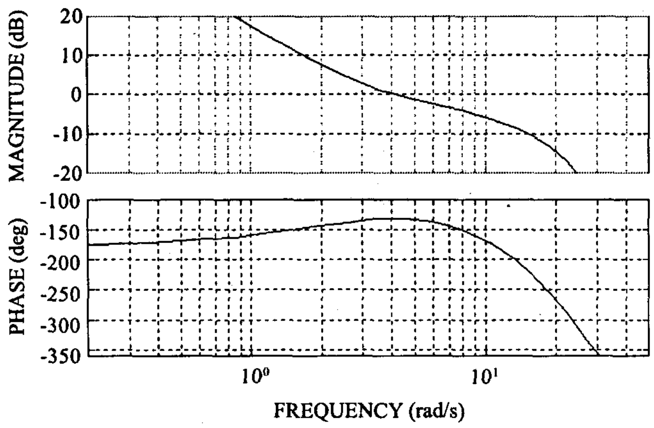

Figure 5. Bode diagram of complete loop with old CGS at t=7.2 s

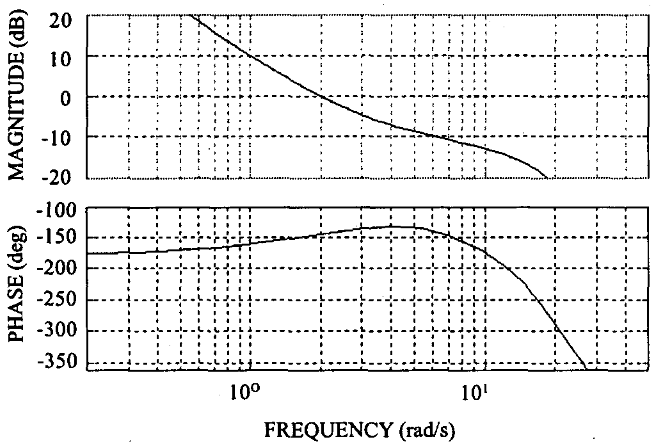

Figure 6. Bode diagram of complete loop with old CGS at t=15.2 s

4.2 Flight Data Analysis

Flight data shows 0.5-0.7 Hz weave-mode oscillation even for a very benign target, as shown in Figs 8(a) and 8(b) for FT-44 flight against micro-light aircraft moving with a speed of 20 m/s and medium-to-narrow changeover could not take place. Figure 9 shows a large amplitude latax oscillation of 20 g in guidance command, giving high missile body rates oscillation, leading to unacceptable performance. Fast fourier transformation (FFT) of flight records of angle channel errors clearly shows appreciable low frequency noise from 1 Hz to 5 Hz besides guidance weave mode of 0.5 Hz–0.7 Hz

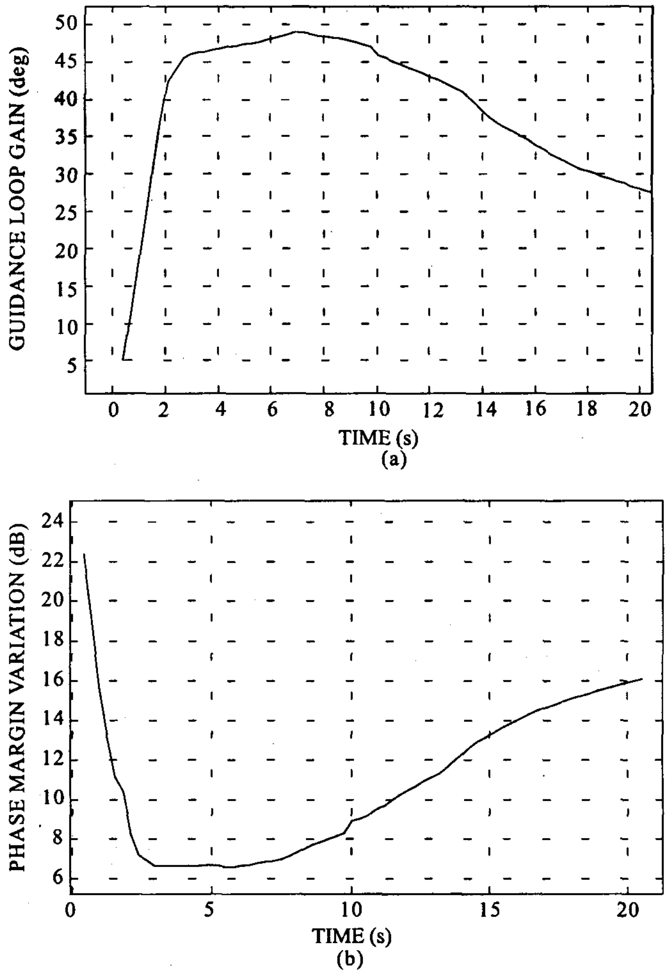

Figure 7. Guidance loop: (a) gain margin and (b) phase margin.

(Table 1). The command guidance system is having high relative noise amplification for frequencies from 1 Hz to 5 Hz as seen in Fig. 3(a), which amplifies the angle channel error noise appreciably. This low frequency noise from 1 Hz to 4 Hz is also passed by the autopilot since nominal bandwidth of autopilot is around 4 Hz, leading to large latax/ rate oscillations.

4.3 Correlation with 6-DOFs Simulation

A detailed 6-DOFs simulation package has been developed for the complete missile system performance analysis and predictions. Nominal 6-DOFs simulations without receiver noise show good damping of missile angular error in both the elevation and azimuth planes with smooth guidance commands as shown in Figs 10 and 11. Flight noise was extracted from

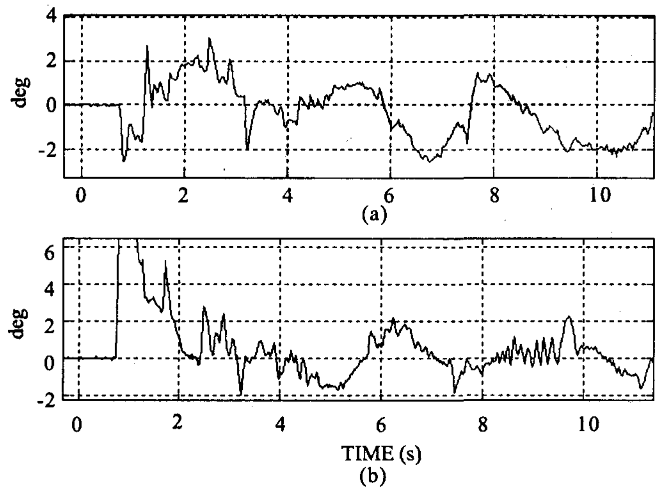

Figure 8. Flight (FT-44) total error in (a) azimuth (b) elevation

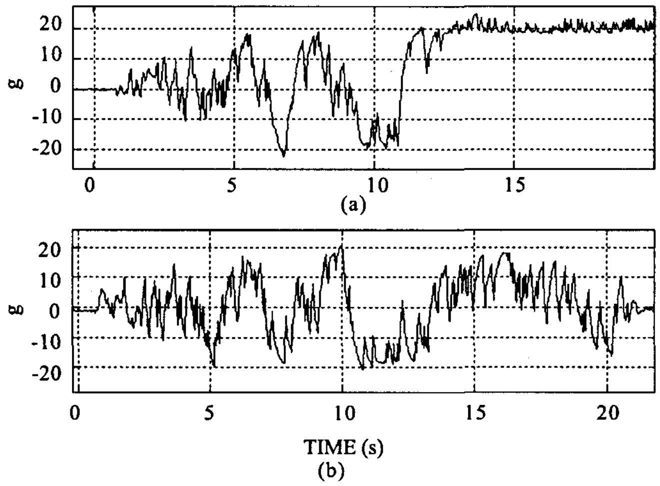

Figure 9. Acceleration demand (FT-44 flight) in (a) yaw and (b) pitch.

the flight data by removing the frequency component below 1 Hz in FFT and again doing inverse FFT. Six-DOFs simulation study was repeated by adding above flight-extracted noise to the angular errors. Simulation results show weave-mode oscillation of 0.5 Hz as shown in Figs 12(a) and 12(b). The magnitude of oscillation was high and settling characteristics were poor, and guidance commands were oscillatory as shown in Fig. 13, similar to flight command. A detailed monitoring of the data in simulation shows that the phase advance networks peaky responses get clipped or saturated due to high amplification of low-frequency noise and analogue phase advance circuits limit of 10 v. Linear system latax requirement in the absence of above analog

Table 1. Fast Fourier transformation of flight noise (FT-44)

Wide azimuth error

Frequency (Hz)	Power	Relative Signal (%)
0.4	41.00	100.0
0.8	3.20	28.0
1.4	5.50	36.6
2.5	1.25	17.5
3.2	2.25	23.0
3.8	2.95	26.8

Wide elevation error

Frequency (Hz)	Power	Relative signal (%)
0.2	266	100.0
1.2	60	47.5
2.4	10	19.4
3.0	3	10.6
3.6	7	11.0

Medium azimuth error

Frequency (Hz)	Power	Relative signal (%)
0.4	41	100
0.8	3.2	28
1.4	5.5	36.6
2.5	1.25	17.5
3.2	2.25	23
3.8	2.95	26.8

limits also goes to a very high value for the comparative small error input signal [Fig. 13(a)]. Though it may give reasonable damping, such high commands

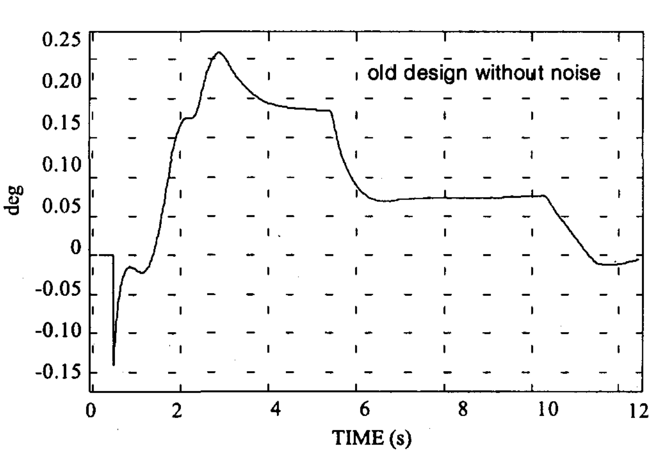

Figure 10. Total azimuth error (actual-deg)

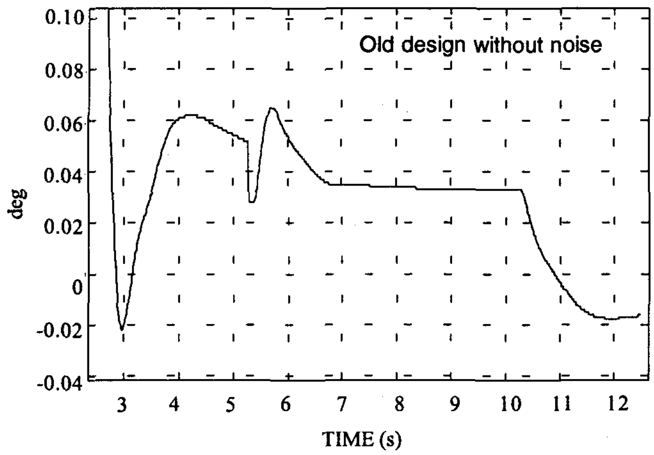

Figure 11. Total elevation error (actual-deg)

are not implementable due to flight structures and actuator limitation.

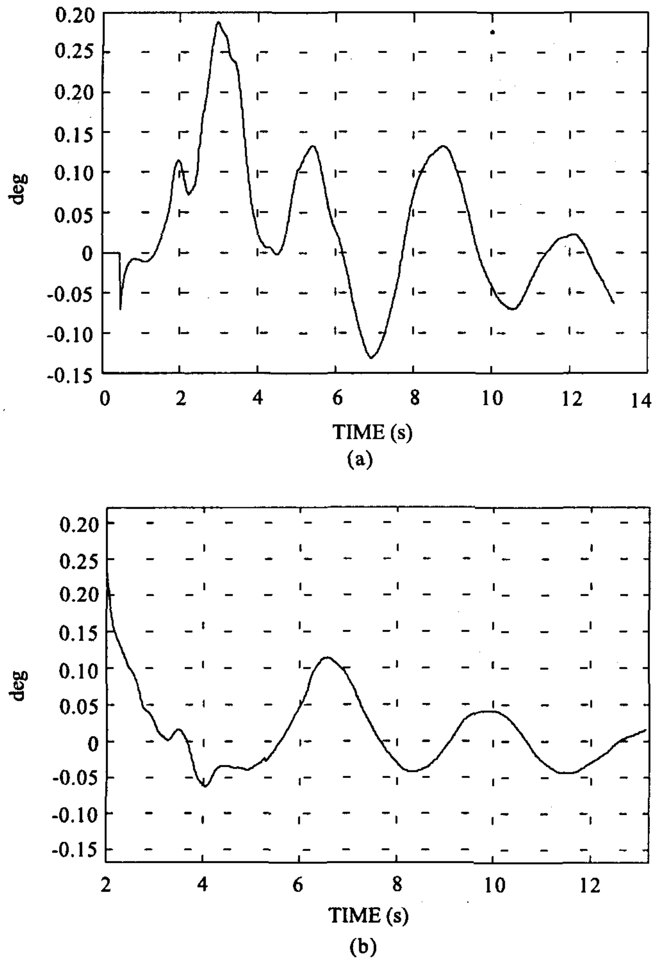

Figure 12. (a) Total azimuthal error (actual-deg) with FT-44 flight noise (b) total elevation error (actual-deg) with FT-44 flight noise.

4.4 Guidance Loop Analysis

Thus, missile performance expected as per design stability margin is good but the performance in the flight degrades beyond the acceptable limit in the presence of flight noises as is shown in the simulation above. High gain amplification for the noise frequency from 1 Hz to 5 Hz, wrt low guidance frequency is found to be the main reason for the degraded performance. This amplified noise component affects the guidance loop as missile reacts to this noise due to high autopilot bandwidth and also the required phase lead, as per the command guidance system design, is not actually available to the guidance loop because the phase advance networks peaky responses are clipped, rendering these almost ineffective. This leads to near-undamped, low-frequency

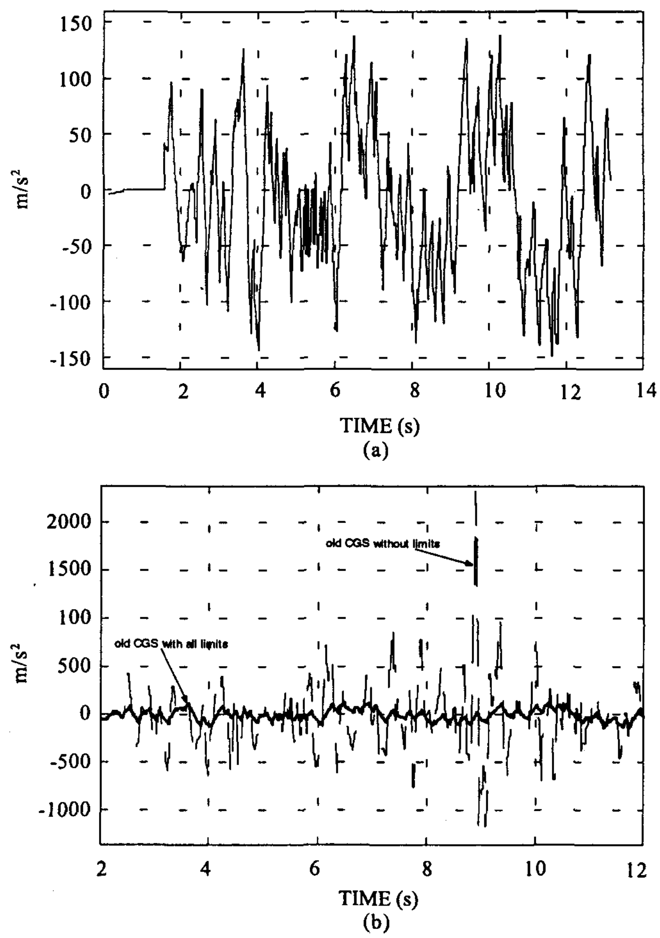

Figure 13. (a) Simulation acceleration demand (yaw) with flight noise (b) simulation acceleration demand (pitch) with and without voltage limit (with flight noise).

oscillations in angle error channels. Thus, re-design in command guidance system is required to reduce the low-frequency noise amplification from 1 Hz to 5 Hz adequately, which in turn, would overcome the weave-mode oscillation problem.

5. COMMAND GUIDANCE SYSTEM RE-DESIGN

It has been seen that the present command guidance system higher frequency relative noise amplification wrt signal frequency of 0.5 Hz is very high, and with considerable low-frequency noise in angle errors between 1 Hz to 5 Hz (as obtained in FFT's of flight angular error signals), undesirable latax oscillations of high magnitudes are obtained. This gets severely limited due to analogue phase advance network voltage limit, rendering phase advance of network almost ineffective, and thus, eroding stability margin, leading to rather undamped weave-mode angle error oscillation. Therefore, to improve the effective stability margin and to reduce weave-mode oscillation and miss-distance, appreciably filtering of low-frequency noise between 1 Hz to 5 Hz is essential. Appreciable filtering of low-frequency noise of 1 Hz to 5 Hz through low-pass lag networks would erode the stability margin at gain crossover frequency (GCF) $\approx$ 0.4 Hz to 0.5 Hz. In fact, about 60° phase advance has to be given at GCF of 0.4 Hz for the stability of guidance loop in the presence of 180° phase lag due to kinematics and also $\approx$ 10° phase lag due to autopilot dynamics with its natural frequency ω_{ap} = 4 Hz (autopilot bandwidth). The solution of the above is described in the subsequent section.

6. ASYMMETRIC NOTCH FILTER IN COMMAND GUIDANCE SYSTEM

To satisfy the requirement of guidance loop stability margins without amplifying low-frequency noise of 1 Hz to 5 Hz (present in angle error channel), a 4 Hz/2 Hz asymmetric notch filter with lag at $\omega_2 = 2*(2*\pi)$ rad, frequency, damping $\varepsilon_2 = 1$, and lead at $\omega_1 = 4*(2*\pi)$ rad, damping $\varepsilon_1 = 0.2$ has been finally selected based on design studies. Figures 14 (a) and 14 (b) show notch filter gain and phase plot.

The notch filter transfer function is

$$\frac{S^2 + 2\varepsilon_1\omega_1 S + \omega_1^2}{S^2 + 2\varepsilon_2\omega_2 S + \omega_2^2} \times \frac{\omega_2^2}{\omega_1^2}$$

$\varepsilon_1 = 0.2,\ \omega_1 = 4 *(2*\pi)$ rad

$\varepsilon_2 = 1,\ \omega_2 = 2 *(2*\pi)$ rad

With high-frequency gain = 0.25 and large undershoot corresponding to zero at 4 Hz with low damping: $\varepsilon_1 = 0.2$; $\omega_1 = 4$ Hz: gain from 2.5 Hz onwards are heavily attenuated [Figs 14(a) and 14(b)]. Again, with highly damped pole at $\omega_2 = 2$ Hz

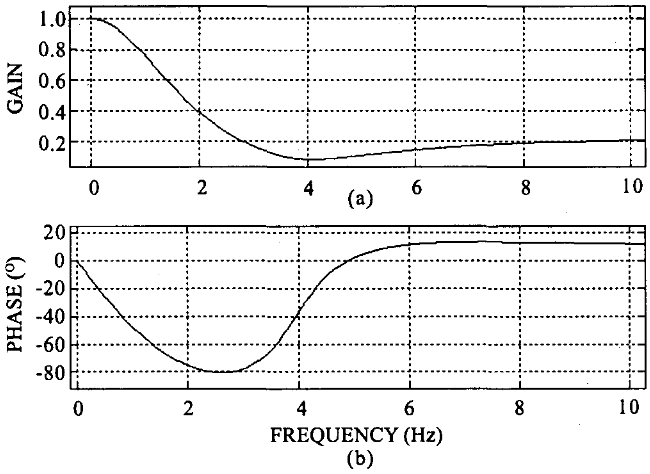

Figure 14. Notch filter response

and $\varepsilon_2 = 1$: notch filter gain at 2 Hz and below also gets sufficiently attenuated [Fig. 14(a)]. Again with those pole-zero arrangement, notch filter phase lag at the guidance loop gain crossover frequency (ie, 0.4 Hz) is $\approx$ 20. The phase lag due to notch filter is compensated by a new compensator designed whose transfer function is

$$\left(\frac{S+2.2}{S+4.4}\right)$$

Figures 15(a) and 15(b) show the guidance loop step response comparison of old and new designs, which shows similar response in the absence of noise. When 10 per cent 2 Hz noise is incorporated along with step input, continuous weave-mode

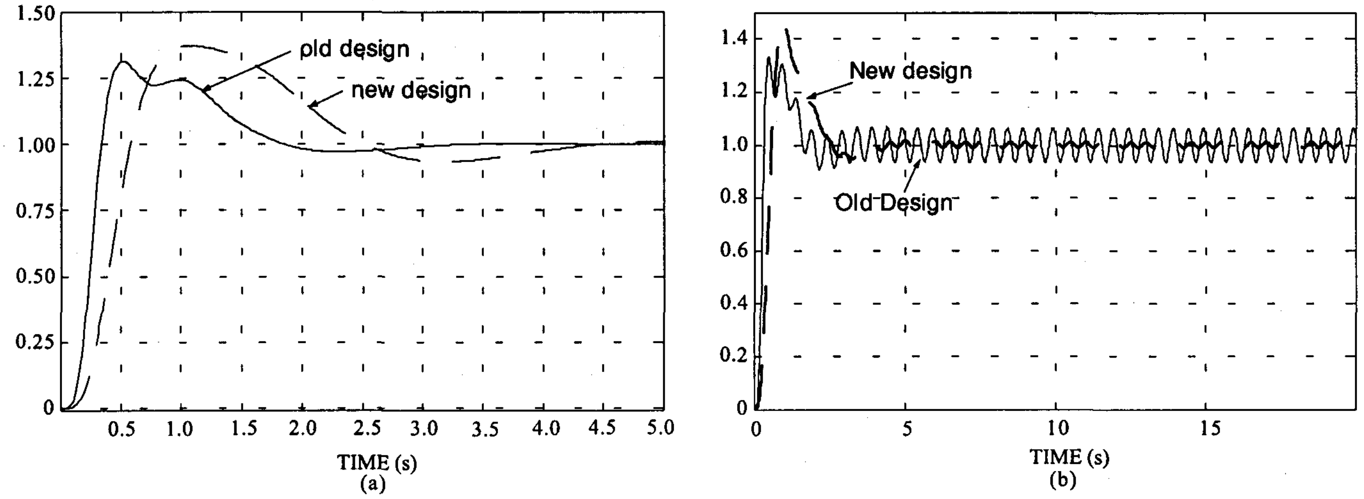

Figure 15. (a) Comparison of guidance loop in time response (step response) at t =7.2 s and (b) comparison of guidance loop in time response at t=7.2 s (disturbance 2 Hz and 0.1 magnitude).

oscillation of 2 Hz is seen in the response of old design, whereas new design shows quite smooth response [Fig. 15(b)]. This shows the adequacy of the new command guidance system design.

7. TRADE-OFF DESIGN FOR COMMAND GUIDANCE SYSTEM & GUIDANCE LOOP GAIN

In this design based on ground-based command generation scheme, feed-forward latax command is computed in the dynamic error channel to generate the latax required to keep the missile on the LOS to target, which is given to autopilot. Therefore, dynamic error compensation channel is designed to generate major portion of guidance latax catering for manoeuvring and crossing targets. The linear error channel is to be designed only for errors and lag in the dynamic error compensation latax computation besides trajectory errors at guidance start and disturbances. Therefore, gain of linear error channel is designed based on the following three considerations:

(a) Maximum error in latax computation of dynamic error compensation channel and miss-distance in linear error channel to provide maximum error latax.

(b) Settling of error in the wide beam under worst case perturbation so that medium can takeover.

(c) Settling of error in the medium beam so that narrow can takeover.

Based on the design study, guidance loop gain of $3 m/s^2/m$ is finally chosen trading-off between the noise-induced miss and miss-distance due to kinematics latax based on the following:

- With a gain $3 m/s^2/m$ and with new compensators [Fig. 16(a)] and dynamic range profiling: gain crossover frequency can be made ≈ 0.4 Hz constant throughout the flight [Fig. 16(b)]. This ensures that the phase advance compensators of CGS are always optimally tuned to the designed grain crossover frequency, giving uniform phase margin and gain margin [Figs 16(c) and 16(d)]. In the present command guidance system design, guidance loop gain varies widely from 6.8 at 7 s to 1.8 s at the end, leading to wide phase margin and gain margin variation, particularly phase margin going a low value at the end-phase.

- With lower gain and with new compensator, weave-mode oscillations are less.

- Gathering and error settling consideration: With the worst case, 40 m error at the start of guidance, latax produced with gain of 3 is 12 g for the maximum error. Therefore,

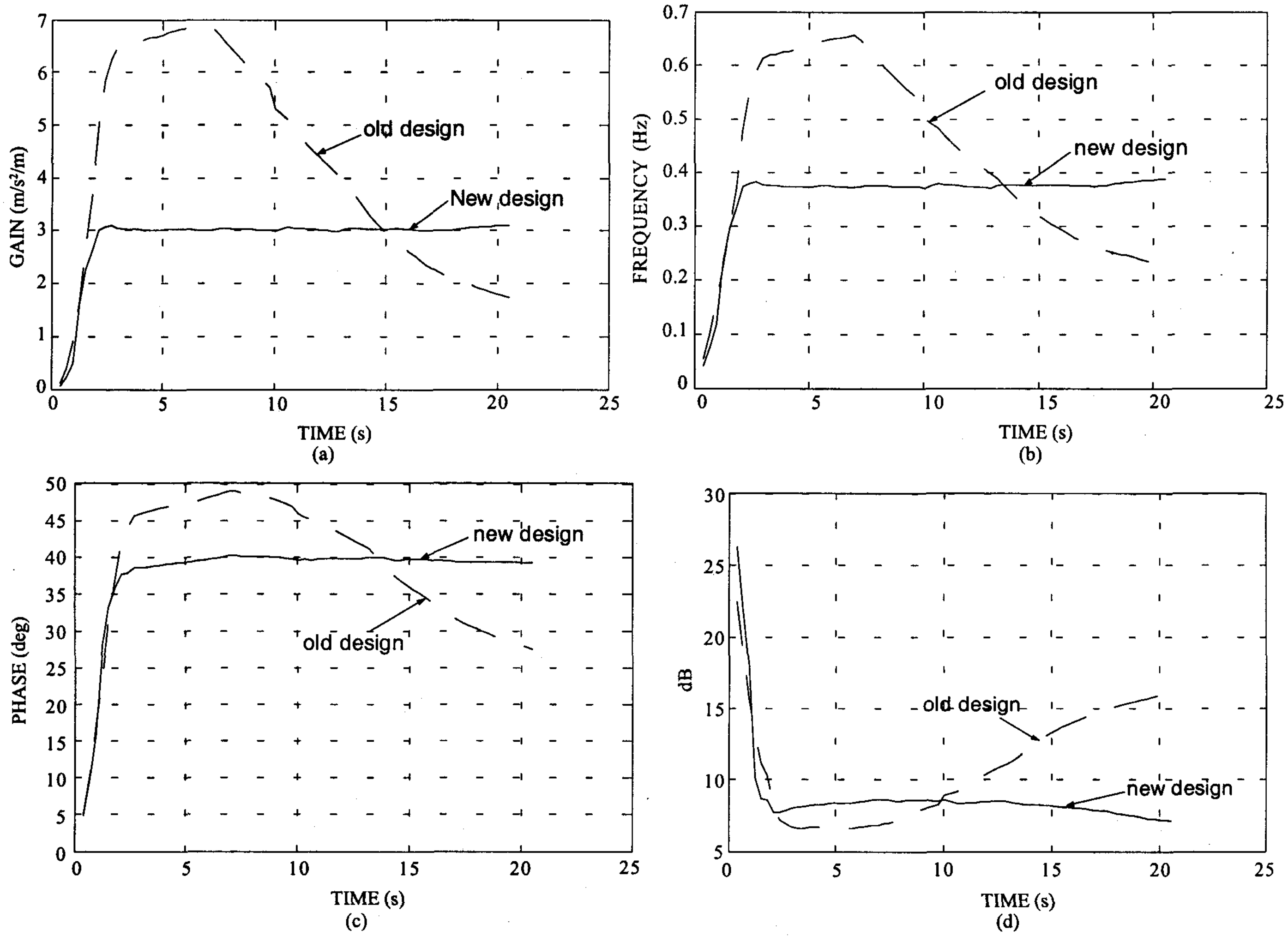

Figure 16. Dynamic range proofing: (a) Total system gain (b) guidance loop gain crossover frequency (c) guidance loop phase margin, and (d) guidance loop gain margin.

taking 6 g average acceleration, time required to correct 40 m error is given by

$$\frac{1}{2} \times 60 \times t^2 = 40 \text{m}$$

$$t = 1.15 \text{s}$$

With the worst case damping possible due to nonlinearity, etc (once required is provided), 40 m would be corrected to low errors well within one cycle. With gain of 3 and gain crossover frequency = 0.4 Hz, one cycle time corresponds to 2.5 s. Missile would be close to LOS well within 4 s. Therefore, missile will be within medium beam error at 4 s and the wide-to-medium handover condition would be satisfied.

8. DYNAMIC RANGE PROFILING

Missile guidance loop gain and gain crossover frequency in flight changes with the missile velocity and command guidance system compensators get detuned. This changes the loop stability margin for the old design as seen in Figs 16(a) to 16(d). In the new design, it is proposed to adjust the guidance loop gain to a constant value by dynamically changing the pre-stored range profile. The range profiling is done by multiplying the present range profile with the additional gain profile. Figures 16(a) to 16(d) show that for the new design, gain crossover frequency, gain margin and phase margin are maintained constant throughout the flight and the margins are reasonably good.

9. VALIDATION OF THE NEW COMMAND GUIDANCE SYSTEM DESIGN

The new design is validated through 6-DOFs simulation and finally implemented in hardware and validated through flight test.

9.1 Design Validation through Simulation

A 6-DOFs simulation with old command guidance system design and with FT-44 flight noise showed large amplitude weave-mode oscillations in angular error channels, leading to large latax oscillations and miss-distance much higher than the acceptable limits.

When the new command guidance system design was incorporated in the above 6-DOFs simulation with the same noise, the oscillations in angular error channels were reduced drastically, and thus, miss-distance was kept well within the permissible limit [Figs 17(a) to 17(d)]. This establishes the adequacy of the new command guidance system design.

9.2 Design Validation through Flight Tests

The new command guidance system design was validated and fully established consistently through 11 flight tests (FT-46 to FT-56), where

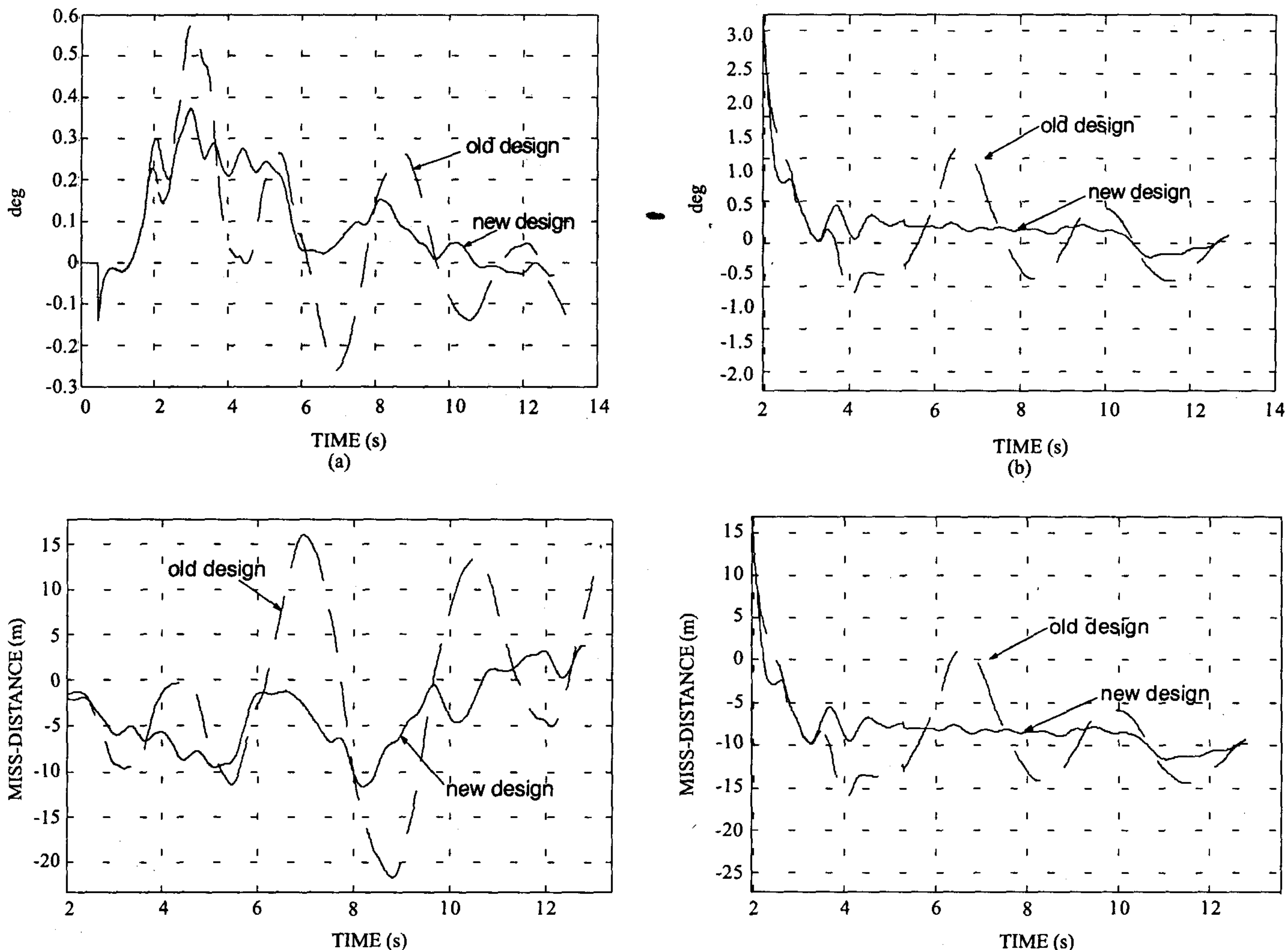

Figure 17. Design validation through simulation: (a) Total error in azimuth with flight noise (comparison of old and new design) (b) total error in elevation with flight noise (comparison of old and new design) (c) miss-distance in azimuth with flight noise (comparison of old and new design) and (d) miss-distance in elevation with flight noise (comparision of old and new design).

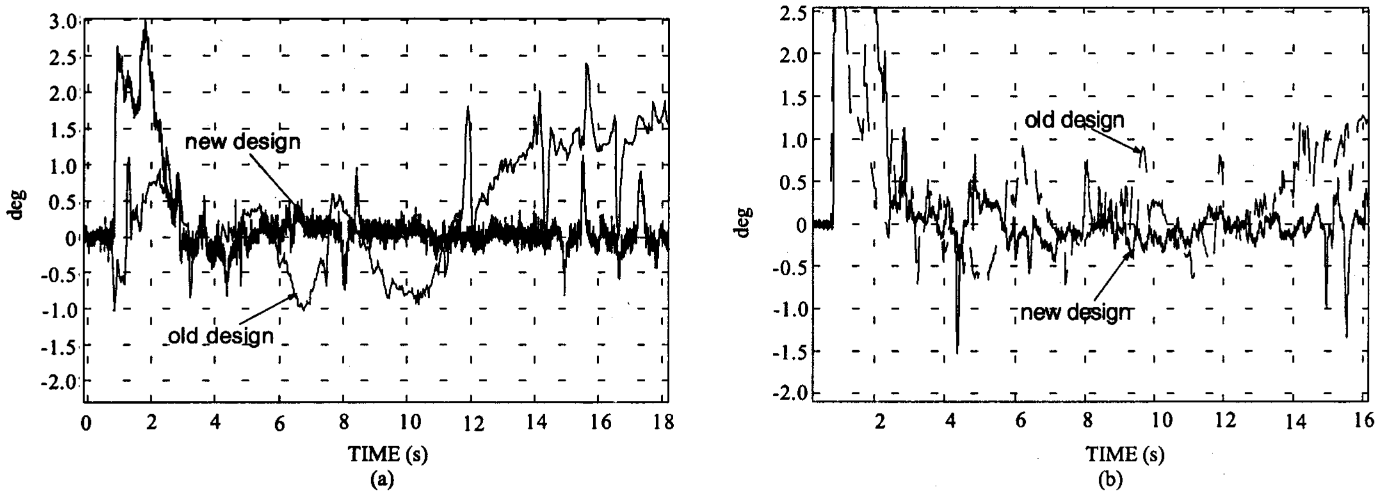

Figure 18. (a) Total error in azimuth (comparison of FT-44 and FT-48 flight errors) (b) total error in elevation (comparison of FT-44 and FT-48 flight errors).

weave-mode oscillations in angular error channels were drastically reduced wrt the old design. This leads to low angular error and changeover narrow beam to finally giving low miss-distance, meeting missile system requirements. FT-55 has demonstrated the physical target destruction with complete functioning of the radio proximity fuse and warhead chain.

Figures 18(a) and 18(b) show comparison of total angular error for flight FT-44 carried out with the old command guidance system design and flight FT-48 carried out with the new command guidance system design. From the figures it was seen that for FT-48 flight with the new design, angular errors were much smoother compared to FT-44 flight with the old design, and the magnitude of the errors was quite low. Due to this, changeover to medium beam and narrow beam took place as per the plan in FT-48 with the new design (and also in flight FT-46 to FT-53), whereas in FT-44 and FT-45, which where carried out with the old command guidance system design (also in earlier flights), changeover to narrow could not take place, leading to large beam miss-distance and failure of guidance.

10. CONCLUSION

A new command guidance system design has been evolved to alleviate low-frequency weave-mode oscillation of large amplitude noticed in several flights of a short-range surface-to-air missile, leading to the failure of guidance. The causes for the above undamped oscillation have been identified based on flight data analysis and these are the low-frequency noise in angle error, and high relative noise amplification of the old command guidance design wrt signal, leading to clipping of the phase advance peaky response, rendering these almost ineffective. With phase advance network being ineffective; guidance loop linear phase margins are eroded almost fully, leading to undamped weave-mode oscillations as was observed in almost all flights up to FT-45. The new command guidance system design restores the linear phase margin of the guidance loop by allowing normal peaky response of the phase advance networks. This is achieved through drastic reduction of low-frequency noise amplification of the new command guidance system. An asymmetric notch filter and well trade-off guidance loop gain selection are the major features of the new command guidance system design, which have been successfully tested in several guided flights of the short range surface-to-air missile, establishing the new design completely.

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