Defence Science Journal, Vol. 55, No. 2, July 2005, pp. 265-280

REVIEW PAPER

Current Trends in Tactical Missile Guidance

S. Vathsal

Directorate of ER & IPR, New Delhi–110 066

and

A.K. Sarkar

Defence Research & Development Laboratory, Hyderbad–500 058

ABSTRACT

The problem of tactical missile guidance is very challenging and has been treated using several basic methodologies in the past four decades. Major techniques can be grouped under classical guidance laws, modern guidance laws, laws for manoeuvring targets, predictive guidance for endgame scenario, and guidance laws based on intelligent control methods. Each technique has some advantages and disadvantages while implementing in a practical system. Guidance law selection is dictated by nature of flight profile like boost, midcourse, terminal homing, etc, and also miss-distance and a single-shot kill probability. This paper presents a brief survey of the existing techniques and current trends in tactical missile guidance.

Keywords: Tactical missile guidance, proportional navigation, optimal guidance law, true proportional navigation, extended Kalman filter, line-of-sight, LOS

1. INTRODUCTION

The guidance law, as part of the guidance loop, represents an essential component in the design of guided missile system[1,2] (Fig. 1). The information needed to perform the guidance task of the missile-target intercept determines basically the configuration of necessary sensors and information processing. The demands on the missile acceleration capability, as an important system parameter, depends strongly on the guidance law. The proportional navigation

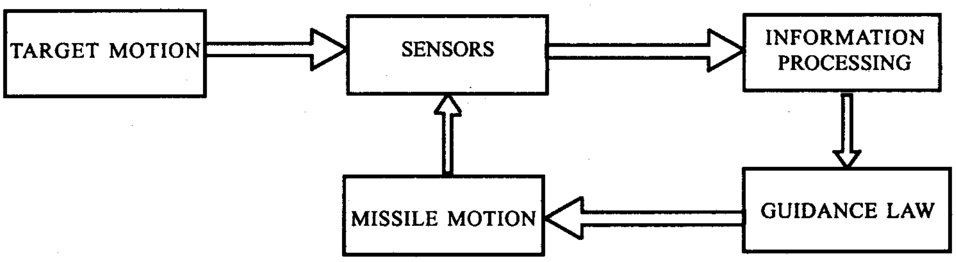

Figure 1. Structure in information flow in missile guidance loop

Received 15 November 2004

(PN) is probably the most commonly used method for guidance of homing missiles. Fossier[3,4] in his review paper describes in details the origination of proportional navigation along with its practical application in a missile system. Many variants like pure proportional navigation (PPN), true proportional navigation (TPN), generalised true proportional navigation (GTPN) are also discussed depending on how the lateral acceleration is applied to the missile, perpendicular to the velocity vector or thrust axis, or at an angle to the thrust axis, of the missile.

Many investigators have attempted to derive analytical solutions which have provided insight into the performance of the proportional navigation guidance. Rao[5-6] and Vathsal have derived a closed-form solution of proportional navigation guidance. Ghose has also derived analytically capture region of realistic TPN guidance and also generalised TPN in two dimensions[8-10]. Shukla and Mahapatra[11-12] and Bryson and Ho[13], have brought out clearly the salient differences between the true proportional navigation and the pure proportional navigation.

The authors have focused their research on the theoretical development of proportional navigation guidance. They have studied planar 2-D engagement, mainly with constant pursuer and evader velocity. Recently, Sarkar[14], *et al.* have revisited all different proportional navigation guidance laws and evolved a generalised formulation of different proportional navigation laws in three dimensions for practical application in a pursuer evader engagement. They have simulated realistic engagements with sensor noise, guidance, and autopilot lag along with pursuer and evader velocity variation at different altitudes. In their paper, they have also surveyed all application-oriented papers on proportional navigation guidance. The proportional navigation is known to yield reasonable miss-distance when applied against highly manoeuvring or moderately manoeuvring target. If target acceleration is available, then proportional navigation can be modified as augmented proportional navigation (APN). It is however impossible to obtain zero miss-distance (ZMD) using proportional navigation or augmented proportional navigation. Modern control theory is useful for obtaining zero miss-distance.

In subsequent sections, current state-of-the-art on research and practical application of all guidance laws based on modern control, nonlinear control, and artificial intelligence, have been discussed along with future research on missile guidance.

2. GUIDANCE LAWS BASED ON MODERN CONTROL

Modern guidance techniques have borrowed ideas from optimal control theory. If all the n-states are physically available noise-free, it can be fedback through an optimal control gain matrix by solving a matrix Riccati equation backwards. For cases where all the n states are not physically available and only r measurements where $r < n$, then it is essential to reconstruct n states using optimal observer. If the r measurements are corrupted by noise, it is essential to estimate all the n states using optimal filtering techniques. These concepts are explained here.

2.1 Information Processing by Optimal Filtering Techniques

Information processing represents a substantial link between the information needs of the guidance law and the possible information offer of the feasible sensor equipment of a guided missile system. The filtering theory provides the tools of information processing on noisy measurements. It is based on the reasonable idea to separate the measurement signals on time-correlated signals and time-uncorrelated disturbances. These are purely random. Therefore, filtering techniques aim at estimation of the complete time-correlated informations. The correlated portion of measurement signals include the information signal as well as time-correlated disturbances, ie, coloured noise. To describe their dynamical behaviour, mathematically, differential equations can be used[15]. From the physics point of view, uncorrelated disturbances represent noise with negligible time-correlation relative to the correlated signals. Mathematically these can be modelled by white noise[16]. The block diagram in Fig. 2 shows the solution structure of information processing by optimal filtering. For linear Gaussian case, the filtering theory is based on the mathematical (real world) model.

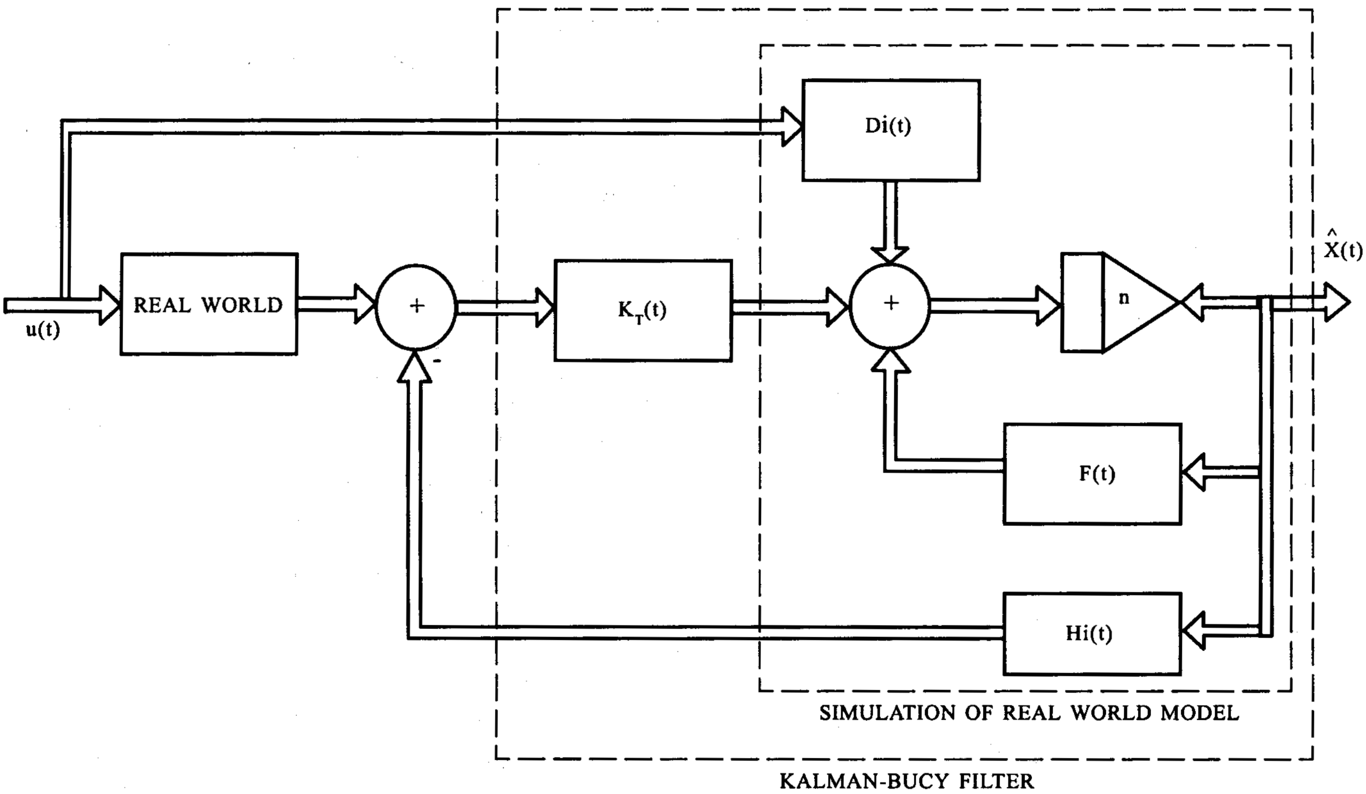

Figure 2. Structure of information processing by linear filtering techniques

2.1.1 *Measurement Model*

$$Z = HX + v \tag{1}$$

where $Z(t)$ is the m dimensional measurement vector, $v(t)$ is the m dimensional measurement noise vector with white Gaussian noise $v(t) \sim N(0, R(t))$, and $X(t)$ is the n dimensional state vector for correlated signal modelling.

2.1.2 *State Space Model*

$$X = FX + Gw + Du \quad X(t_0) \sim N(X_0, P_0) \tag{2}$$

where $w(t)$ is the s dimensional input noise vector with white Gaussian noise $w(t) \sim N(0, Q(t))$ and $u(t)$ is the dimensional deterministic input vector.

The matrices $F(t)$, $G(t)$, $D(t)$ and $H(t)$ are of appropriate dimensions. If $X(t)$ is the state estimate and $\hat{x}(t)$ is the estimation error, the error-covariance matrix is given by

$$P(t) = E\{\hat{x}(t) \quad \hat{x}^T(t)\} \tag{3}$$

The estimation problem is defined as follows:

Given measurements $Z(\tau)$, $t_0 \leq \tau < t$ based on Eqns (1) and (2), find a state estimate $X(t)$ of the actual state $X(t)$ such that a quadratic performance criteria J which is given by $J = Trace\ (P(t))$, is minimised.

The solution of the optimal filtering problem is given by the well-known Kalman-Bucy filter, which consists of

- A linear vector diffrential equation for the state estimate $\dot{X}(t)$ as

$$\dot{X}(t) = F\dot{X} + K_t(Z - H\dot{X}) + Du$$
$$\dot{X}(t_0) = \dot{X}_0 \tag{4}$$

- A nonlinear matrix riccati differential equation for the error-covariance matrix $P(t)$ to be integrated forward on time

$$\dot{\hat{P}} = FP + PF^T - PH^T R^{-1} HP + GQG^T$$

$$\dot{\hat{P}}(t_0) = \dot{\hat{P}}_0 \quad (5)$$

- And a computational rule for the filter feedback matrix, $K_t(t)$

$$K_t(t) = PH^T R^{-1} \quad (6)$$

Tools of filtering are very useful in guidance in the following aspects:

- Estimation of missile and target states using onboard seeker measurements or ground-based radar measurements.
- Estimation of target acceleration for its utility in augmented proportional navigation.

2.2 Specification-oriented Optimal Control Technique

To treat the guidance design problem systematically wrt the guidance specification of most accurate system performance and of low realisation effort, it is always advantageous to

- Use physically meaningful performance measure in terms of variances of state variables
- Put constraints on the structure of admissible solution for the guidance problem
- Improve the performance of line-of-sight (LOS) guidance law by applying optimal control techniques.

The design procedure is based on a state space model similar to filtering problems and an optimality condition J is formulated as follows:

$$J = E\left\{x^T(t_f)S_t x(t_f)\right\} + \int_{t_0}^{t_f}\left(x^T L x + u^T M u\right)dt \quad (7)$$

It assumes a fixed control interval $t_0 \le t \le t_f$. The symmetric weighting matrices S_t, $L(t) \ge 0$ and $M(t) > 0$ represent the free parameters of the design procedure. By variation of these coefficients of the weighting matrices, the final and transition behaviour of the state vector x (t) as well as control input behaviour u (t) can be influenced.

Generally, tracking radar or seeker is used to get evader information wrt pursuer. These sensor measurements, which are contaminated by different noise sources, are processed by an optimal estimator as observer (Section 2.1) to estimate the relative position, velocity, and evader acceleration. Based on these estimated states, the guidance law is used to generate pursuer lateral acceleration command (Section 2.2) in close loop to steer it towards the evader for interception during terminal guidance (Fig.3). Based on the specification of performance

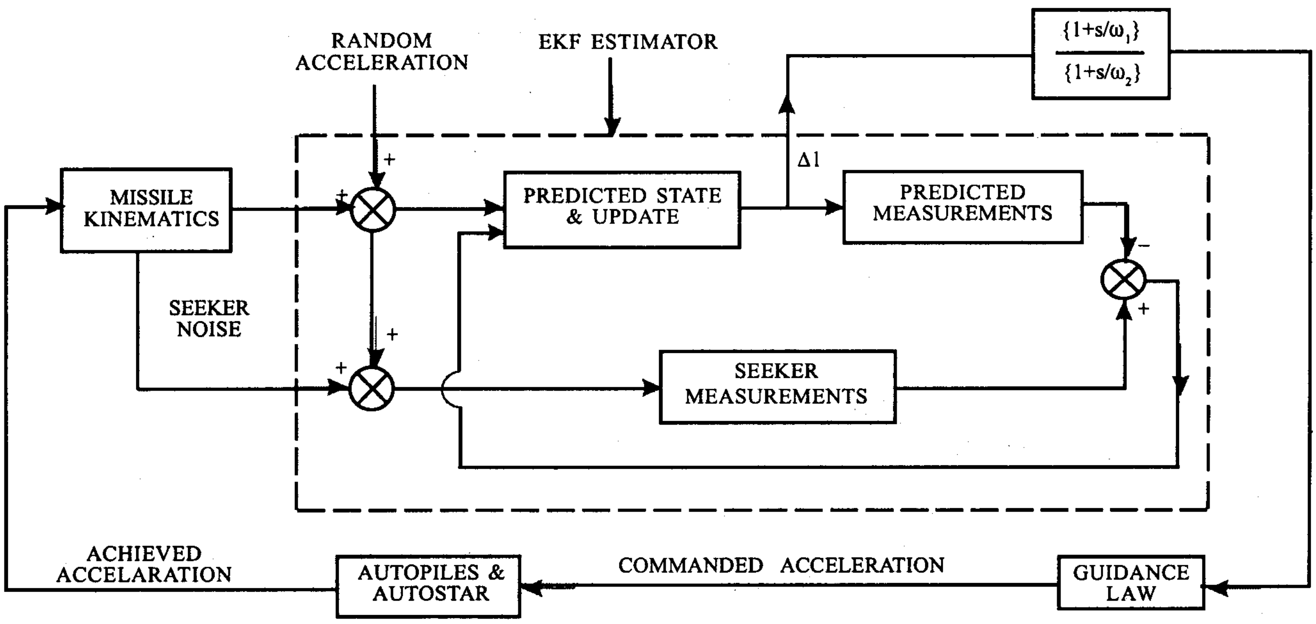

Figure 3. Engagement simulation in close loop

index of optimal control law, different guidance laws have been evolved. These are:

- Optimal guidance
- Predictive guidance
- Game theory-based guidance.

2.2.1 *Optimal Guidance*

In the past, guidance problem has been solved using the well-known proportional navigation technique, in which LOS rate between the evader and the interceptor is used for interception[17]. However, in noisy scenario in which the target initiates an evasive manoeuvre during endgame, the PN guidance laws are not capable of providing the necessary guidance commands to ensure intercept[17,19]. So as to improve the performance of the interceptor during manoeuvre of evader, it is necessary to

- Maximally utilise all sensor measurements to accurately estimate the target position, velocity and acceleration.
- Generate meaningful guidance commands from the estimates so that the final miss-distance is minimised in the presence of target manoeuvre.

The augmented proportional navigation and optimal guidance law (OGL) are popular guidance laws which perform better over the proportional navigation law because these make the use of target acceleration levels to generate guidance command[17,18]. So, for using augmented proportional navigation and optimal guidance law, the estimation of target manoeuvre is mandatory. Open literatures on target manoeuvre are very scanty. Dowdle[19], *et al.* had estimated target manoeuvre using estimator based on extended Kalman filter (EKF) where the measurements were obtained by onboard active radar seeker with measurements as relative range, range rate, LOS angles and LOS rates. Uhrmeister[20] estimated acceleration of manoeuvring aircraft as target from the available radar and imaging sensors as primary sources of measurements. He estimated LOS rates from radar measurements using Kalman filter and target acceleration from imaging sensor. Kim[21], *et al.* had used active radar seeker with range and LOS angle alone measurements. They used extended Kalman filter estimator to estimate the relative position, velocity, and target acceleration, which were used for augmented proportional navigation-based guidance law generation.

Vergez[22], *et al.* in their pioneering work had solved the target manoeuvre estimation problem using passive seeker where LOS angles and LOS rates were the available measurements. Balakrishnan[23,24] estimated LOS rates from passive measurements of LOS angles alone using modified polar coordinate. Stallard[25] estimated the LOS rates based on LOS angles measurements only. Based on Stallard's research, Robinson[26], *et al.* estimated the LOS rates based on relative range, range rate and LOS angles as measurements. Hablani[27,28] solved the similar problem by reconstructing the LOS rates for exo-atmospheric interceptor from the LOS angle measurements only.

Very recently, Waldman[29] discussed modelling of an imaging seeker and the formulation of an extended Kalman filter for estimating the LOS rates from measurements of relative angular displacement between the seeker gimbals and the strapdown inertial unit. Based on the estimation of states based on these measurements, the augmented proportional navigation guidance law[21] or optimal guidance law can be used[22] as guidance laws. An excellent description of optimal guidance law according to current state-of-the-art has been given by Ben-Asher and Yaesh[30]. Robust guidance laws based on H_∞ control theory can be evolved to reduce the sensitivity of optimal guidance law on time to go.

In Defence Research & Development Laboratory, Hyderabad, pioneering work on optimal estimation and guidance was taken up by Sharma and Swamy in MAMTHA and PRAGNA design[31,32]. Due to nonavailability of seeker,these techniques could not be implemented in practice. However, their work became a starting point of applying optimal adaptive filter and control techniques for homing guidance problem. In this direction, seeker based guidance using estimation of target acceleration during endgame scenario has been reported[33]. Based on Gurfil's work[34,35], estimated relative position and velocity states as output of Kalman filter were passed through a lead lag compensator.

$$\frac{1+\dfrac{s}{\omega_1}}{1+\dfrac{s}{\omega_2}}(\omega_1 < \omega_2)$$

The estimated target acceleration states were passed through low-pass filter $\omega_2 / s+\omega_2$. The compensator and low-pass filter outputs were used to generate commanded accelerations for guidance which reduced miss-distance considerably in the simulated environment (Fig. 3). Here the authors have implemented both augmented proportional navigation and optimal guidance law along with extended Kalman filter observer in 6-DOF simulation model for intercepting manoeuvring target in the presence of sensor noise, estimator and autopilot lag. The sensor noise was assumed to be Gaussian in nature. But in real-life, the noises in seeker are time-correlated due to eclipsing, RCS fluctuation and glint[18]. Currently, pursual is going on to estimate the states from non-Gaussian measurements.

2.2.2 *Predictive Guidance*

In optimal control approach to guide a missile approaching a target, the computation of the missile lateral acceleration sequence is based on the minimum variance unbiased estimates of target state. Optimality of the scheme relies on linearity of the motion model and the observation process. The two factors which make this approach difficult are:

- The target motion model during manoeuvre is usually nonlinear in the preferred coordinate system, and so, linearisation errors are incurred.
- Target's future position is uncertain due to manoeuvre. Predictive guidance is based on predicting the probability density function (PDF) of the target position at interception for a range of possible manoeuvres using the nonlinear model (Fig. 4).

To sum up, predictive guidance has been a current trend for guidance of a missile to intercept a target, considering the uncertainty in the present state and future manoeuvres of the target. This framework has much in common with the model-predictive control approach popular in process control[36,37]. Recently, Talole and Banavar have modified the proportional navigation guidance law using predictive control and found it to be superior to the proportional navigation law from the total control effort point of view during an engagement[38]. Talole[39], *et al.* have also estimated the target state during point mass planar engagement based on seeker measurements to estimate the states[39]. In both the papers[38,39], the method is in the conceptual level and can be tried in 6-DOF platform to substantiate their claims in a real-life situation.

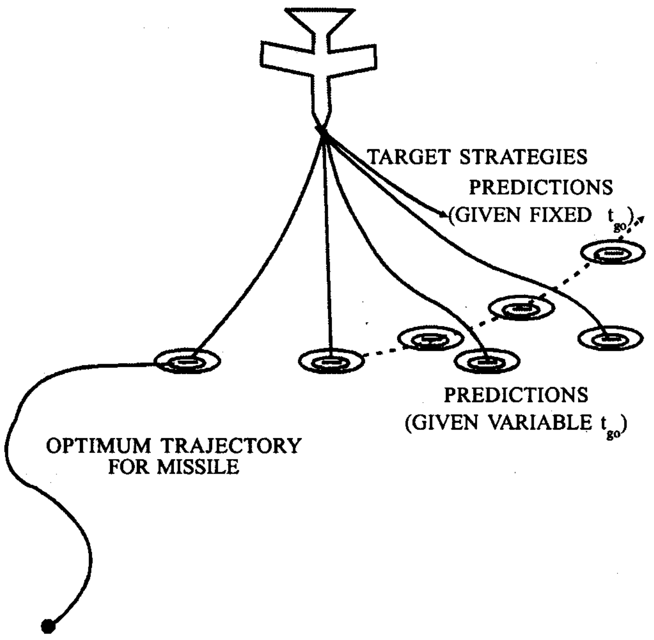

Figure 4. Effect of non uniform time to go (t_{go}) on optimal interception traectory.

2.2.3 *Game Theory-based Guidance*

In differential game formulation, the cost function is the miss-distance whereas in the previous formulation, the cost function was integral of total pursuer acceleration or total control effort. Here, the information available is imperfect with noise contamination. Also, the target can manoeuvre randomly. Probably, the first cited references are by Gutman[40] and Anderson[41]. In both the case studies, the engagement is planar, point mass model. In Anderson's work, the seeker dynamics was ideal, Autopilot was rst order. The two players were pursuer and evader during endgame.

The most notable work on this formulation was carried out by Shinar,[42-45] *et al.* Forte and Shinner[42,43] used mixed guidance strategy based on both the optimal and the diffrential game approaches for the missile aircraft interception. The radar was used for guidance. Point mass 2-DOF model was used for endgame simulation. Green[44], *et al.* have solved more practical problem of missile-target engagement based on the game theory. Later, they used the same game theoretic formulation to evolve the guidance law for intercepting incoming antiship missile in ballistic missile defence scenario[45].

Recently, a new guidance law[46] has been derived using diffrential game concepts for the interception of highly manoeuvring targets. This guidance law is implemented using a suitable 3-D estimator. Shima[46], *et al.* used linear time varying kinematic gain model and compensated for inherent estimation delay of target acceleration. Because of errors in estimating t_{go} and the maximum acceleration of both players as a result of noisy measurements, the bang-bang strategy is used. This can be adopted for any of the existing or current development missile defence system because its implementation does not require any hardware modification.

3. GUIDANCE LAWS BASED ON NONLINEAR CONTROL

The proportional navigation is a well-known guidance law[17] that performs very well in a large variety of engagement geometries. Later, several variants of proportional navigation such as augmented proportional navigation, optimal guidance law have been derived optimising some suitable cost functions.

Recently, application of diffrential geometric control methods[47] has led to a new class of control techniques for nonlinear systems that are linear (or affine)in the control input. The basic technique known as feedback linearisation (FL) has been used to cancel nonlinearities in the input-output behaviour of the plant. Ha and Chong[48] were the first to use feedback linearisation technique in 3-D guidance problem to derive an exact command to the LOS guidance scheme. They minimised tracking error in position in both the yaw and the pitch plane along the LOS. Later, Bezick[49], *et al.* applied this technique to a 2-D engagement problem where they minimised the LOS rate to zero for interception. They constructed feedback linearising guidance law (FLGL) to steer the flight vehicle to the predicted interception point (PIP). The problem addressed being 2-D in nature, was solved as a single-input single-output (SISO) problem. Also, they proposed the novel idea of a composite nonlinear guidance law (CNGL) with the feedback linearisation being used initially and the proportional navigation being used prior to the endgame, to minimise the miss-distance. They also solved the problem as a regulator problem with demand as range rate derivative. The difference between the actual and the desired closing velocities was used as output feedback. So, whenever this difference becomes zero, range rate derivative also becomes close to zero and ensures interception. To avoid zero-dynamics[47], they also proposed a composite nonlinear guidance law (CNGL) which switches between the feedback linearisation guidance law when the flight vehicle is far from the intercept heading, to the proportional navigation guidance law (PNGL) when the flight vehicle is near to the intercept heading.

Subsequently, Leng[50] proposed a guidance law that annuls relative heading error (RHE) to minimise the miss-distance using a nonlinear control approach. He validated his algorithm through limited simulation in a 2-D engagement scenario. His research contribution is more of a fundamental and conceptual nature. Based on his idea, Taur[51] developed a nonlinear guidance law (NGL) by driving relative heading error in both the yaw and the pitch plane to zero in a regulator problem. These simulation results are in 3-Ds. He also demonstrated that output elements for feedback can be estimated through a Kalman filter as observer by processing the LOS angles, the LOS rates, range, and range rate measured by the seeker. These simulation results correspond to 2-D engagement scenario. He has reported that composite nonlinear guidance law based on NGL and extended proportional navigation (EPN) gives minimum miss-distance, and, in certain manoeuvres with very high heading error (HE), outperforms the proportional navigation. Also, Cloutier[52], *et al.*

have developed a new guidance law by solving state-dependent Riccati equation, which, in combination with any homing guidance law at terminal phase, was used for interception. Shneydor[53] has also discussed the concept of mixing of different types of guidance laws.

Recently, Srivastava[54,55], *et al.* have developed two nonlinear guidance laws. The first is based on nonlinear inverse dynamics (NID) with nonlinear tracking of line-of-sight angle with pursuer flight-path angle and heading angle as output feedback. Application of nonlinear inverse dynamics on control system design of aircraft and reentry vehicle are available in Snell[56], *et al.* and Valasek[57], *et al.* respectively. However, to the best of the author's knowledge, application of nonlinear inverse dynamics on guidance law design has not been attempted till date. The second guidance law has been developed as a regulator problem where line-of-sight rate derivative is made zero by taking line-of-sight rate as output feedback. The feedback linearisation technique has been used here. In fact, nonlinear inverse dynamics is feedback linearisation technique without zero-dynamics. It has been demonstrated through point mass 3-D kinematic simulation with standard third-order autopilot lag that the proposed guidance law is a viable option for intercepting a highly manoeuvring target with high heading error over the well-known proportional navigation guidance law. The schematic diagram of feedback linearisation guidance is shown in Fig. 5.

Sliding mode control (SMC) has also been used in tactical guidance. Babu[58,59], *et al.* evolved two new approaches for target manoeuvre estimation and guidance of short-range homing missiles based on sliding mode control. Like other variants of proportional navigation, this is structured around basic proportional navigation with an additional bias term to compensate for target acceleration and other unmodelled dynamics. This guidance law is switched bias proportional navigation (SBPN). They validated these algorithms through 6-DOF simulation and showed SBPN to be robust against a wide class of intelligent target manoeuvres. Later, Zhou[60], *et al.* developed a adaptive sliding-mode guidance law of homing missiles for achieving a robust guidance law. They generated sliding surface based on the heuristic that zeroing the LOS rate must eventually lead to intercept. They validated the algorithm through a 6-DOF simulation model of air-to-air engagement and it scored better over proportional navigation by giving a lower miss-distance.

4. GUIDANCE LAW BASED ON ARTIFICIAL INTELLIGENCE

The basic tools of artificial intelligence are the artificial neural network (ANN), and fuzzy logic. The advantages of artificial neural network are learning, good interpolation and extrapolation properties, and fast parallel hardware implementation.

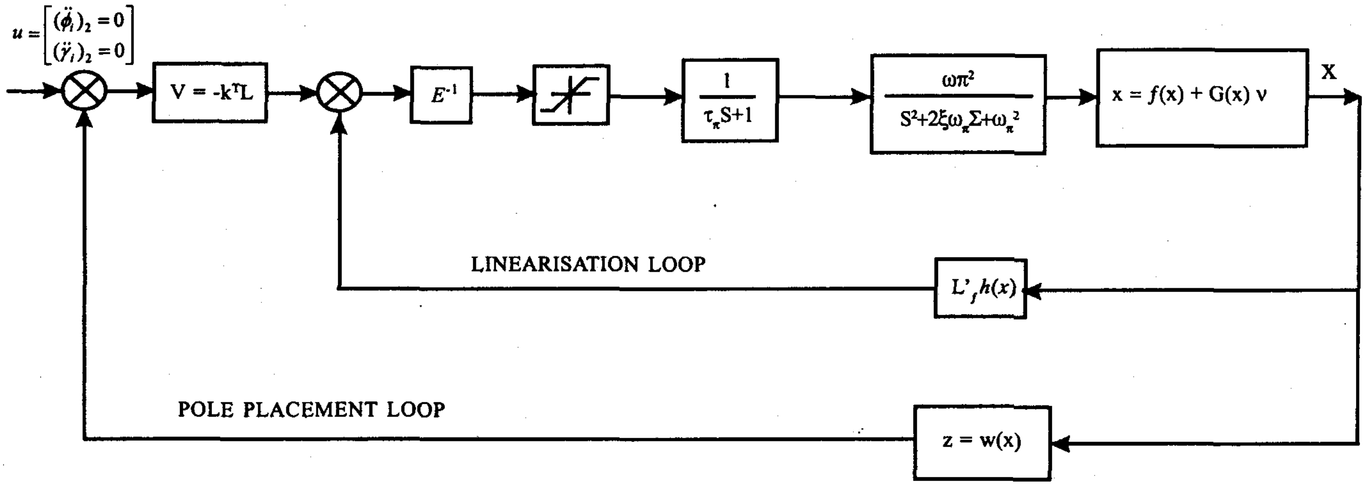

Figure 5. Feedback linearisation guidance law (Schematic)

4.1 Offline Artificial Neural Network Training

The trained artificial neural network constitutes a feedback guidance law which allows the pursuer to adapt to changes in target states and in its own trajectory. From these parameters, acknowledge base consisting of a set of rules, to select among feasible control laws, will be generated for online implementation. There are many popular guidance laws like proportional navigation and its variants available for tactical missile homing guidance. In tactical guidance scenario, miss-distance is an important factor which increases when the target starts manoeuvring. There is not a single guidance law available in the literature to solve this problem. The concept of intelligent agent is a current trend in missile guidance (Fig. 6). Currently, there is a trend to propose the use of an artificial neural network for intelligent agent to learn a set of counter moves[61] (Fig. 7). Recently, Sarkar[62], *et al.* estimated target acceleration using artificial neural network from noisy radar track data. In the last few years, Geng and Mc Cullough[63] have used a novel fuzzy cerebellar model arithmetic computer (CMAC)for controlling a HEAVE DASH II bank to turn (BTT) missile[63].

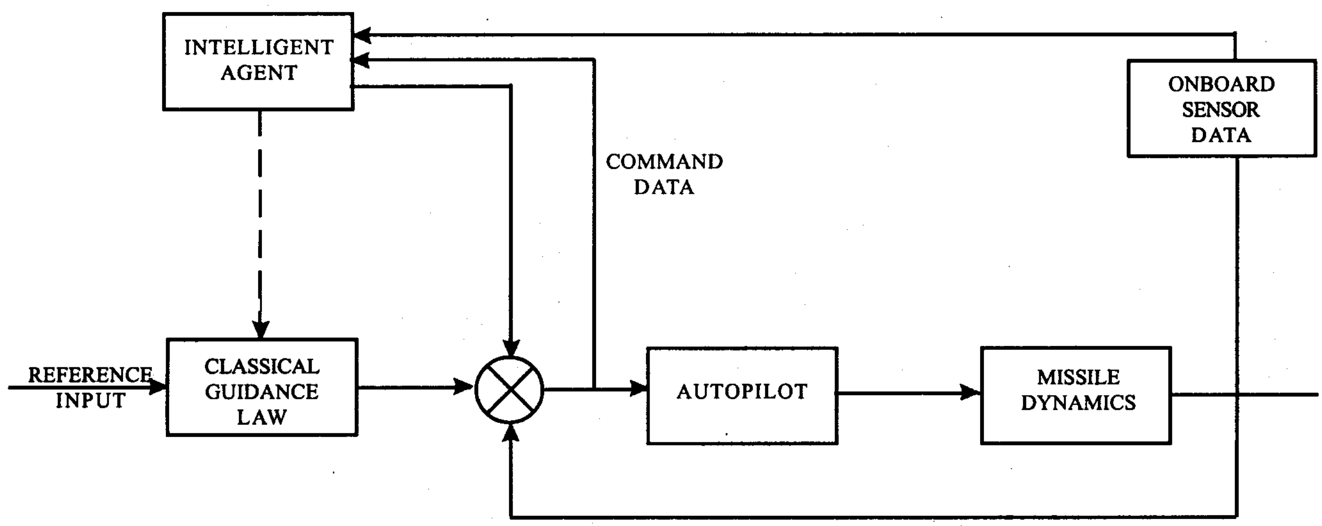

Figure 6. Intelligent agent for missile guidance

4.2 Fuzzy Logic-based Guidance

Fuzzy control is a practical alternative for a variety of challenging control applications since it provides a convenient method for constructing nonlinear

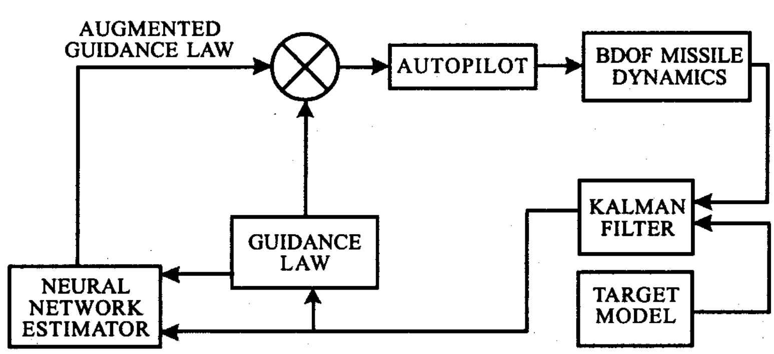

Figure 7. Schematic diagram of neural network-based guidance law

controllers using rule-based heuristic informations. Fuzzy controller consists of a rule base, an inference mechanism, a fuzzification interface, and a defuzzification interface[64]. Though application of fuzzy logic on real-life control problem has matured to some extent, its application on guidance problem is an open area of research at present. Probably, first work on fuzzy logic-based guidance was carried out by Mishra[65], *et al.* They had evolved homing guidance scheme based on fuzzy logic for planar engagement with random jinking target manoeuvre and LOS measurement corrupted by glint noise. They had developed two versions of fuzzy guidance corresponding to the both proportional navigation and the augmented proportional navigation and had shown through MC simulation that miss-distance using fuzzy guidance is less than the corresponding proportional navigation as well as the augmented proportional navigation laws.

Gonsalves[66], *et al.* have also applied fuzzy guidance based on the proportional integral derivative (PID) controller during homing phase of a surface-to-surface missile. Here, the guidance nullified heading and flight-path angle errors in 3-D by generating angular rate commands based on position information available from a passive sensor like TV seeker through image processing. Rajasekhar and Srinatha[67] also used fuzzy logic-based proportional navigation guidance with noisy LOS rate measurements in purser evader planar engagement scenario. They reported that total control effort required by pursuer is less in fuzzy guidance than the conventional proportional navigation guidance.

Later, Lin and Chen[68] used fuzzy guidance law during midcourse phase for trajectory shaping and PID controller-based fuzzy guidance in terminal phase for intercepting a high-speed target. They reported that proposed fuzzy guidance law resulted in smaller miss-distance and greater kill zone over the conventional guidance law. Their simulation was based on planar engagement. Subsequently they extended their research for 3-D engagement[69] where it was shown that integrated guidance scheme had better tracking performance as well as better performance sensitivity wrt some parametric variations over classical guidance laws.

Recently, Chen[70], *et al.* proposed fuzzy H_∞ guidance (F H_∞ G)law for intercepting manoeuvring target in 3-D engagement. Their results show that pursuer integral control effort, lateral relative velocities, and total engagement time using F H_∞ G is less than that of the augmented proportional navigation. Lin[71], *et al.* developed a new self-organising fuzzy logic control for CLOS guidance law. They have used a learning algorithm where rules get modified adaptively and claim its better performance over other conventional guidance laws.

So, based on the research carried out by the above researchers, can be definitely concluded that fuzzy logic-based guidance laws are viable alternatives over the other guidance laws for intercepting incoming high-speed ballistic target or highly manoeuvring target.

5. SINGULAR PERTURBATION GUIDANCE

In an air-to-air engagement situation, the evader is normally located at a distance beyond the visual range of pilot of the mother aircraft. This long initial distance of the evader implies that the pursuer cannot be used in autonomous mode from the beginning of launch from the mother aircraft using the onboard seeker in pursuer having limited range capability. Hence, the evader has to be tracked by the radar on the mother aircraft, and the information has to be communicated to the pursuer through data link for generating guidance command. This happens during midcourse phase of flight of the pursuer when the main objective of guidance scheme is to reduce the LOS separation between the pursuer and the evader and bring the evader within the seeker lock on range. An equally important objective is to place the pursuer in a position to allow easy acquision of the evader by pursuer seeker and provides favourable initial condition for terminal homing phase.

So, design of a good midcourse guidance law is very crucial for an air-to-air engagement. The proportional navigation law performs reasonably well for short- and medium-range air-to-air interception. But one more guidance law for midcourse navigation developed by Sridhar and Gupta[72] is a singular perturbation (SP) guidance law. This guidance law is based on a singular perturbation control . Here, a two-point boundary-value problem is solved in real-time with some engineering approximations. The state variables are partitioned in the following three classes based on time-scale separation as

Slowest: Position, specific energy (x,y,E)

Slow: Altitude h

Fast: Flight path and heading angle (γ,ϕ).

The problem is solved in closed-form on each time scale separately. The resulting guidance law is a singular perturbation (SP)guidance law which is near optimal and found to be simple for real-time implementation.

Cheng and Gupta[73] and Sheu[74], *et al.* have applied this guidance law for air-to-air application. However, their study concluded that the pursuer must pitch up and rise to some high altitude to conserve its energy and gain higher speed as it finally pitches down to intercept the evader. Recently, Raikwar and Ghose[75] have also compared the performance of a singular perturbation law as well as the proportional navigation during midcourse phase.

They concluded that though this manoeuvre does not improve the miss-distance performance to any significant extent, the major returns are in terms of larget launch envelope due to pursuer's high forward speed at the end of midcourse phase. But there are some difficulties in implementing a singular perturbation guidance law in practice. One of these is that, it is difficult to sent the evader information to the pursuer from the mother aircraft using data link when its attitude becomes large at high altitude due to hardware constraints. So to alleviate this problem, Raju and Ghose[76] have evolved a new virtual sliding target (VST) where the pursuer reaches the evader along a path similar to a singular perturbation trajectory where the flight-path angle is constrained by data link's hardware limitation. This concept works successfully in a 2-D engagement in terms of increase in launch boundary compared with the conventional guidance laws. But VST guidance law is yet to be realised in a 3-D engagement scenario.

6. SUMMARY & CONCLUSION

In Section 1, the proportional navigation guidance law and its variants like TPN, PPN, TPN, etc. have been discussed and which have been found to work well with reasonable miss-distance for non-manoeuvring targets. But for intercepting a highly manoeuvring target or an incoming object at high speed, advanced guidance laws based on modern control theory are required (Section 2). This calls for design of state observer from noisy seeker or radar measurements. The critical issues of observer design for a practical missile system based on non-Gaussian sensor measurement noise has also been addressed. If the observer output for guidance requirement becomes smooth, estimation lag becomes more and *vice versa.* This is a contradictory requirement for better guidance performance. So filter tuning is a critical issue in observer design. For using augmented proportional navigation or optimal guidance law, target acceleration has to be estimated accurately, which is an open area of research.

Simulation results using guidance laws based on nonlinear control in Section 3 also are very promising. In fact, minimum miss-distance calls for best guidance law as well as best estimator design. So, if estimator is suboptimal, guidance law has to be better.

In Section 4, important research contributions of recent papers based on artificial neural network and fuzzy logic also have been discussed. The researchers' study indicate that these are the viable alternatives to other guidance laws for intercepting a highly manoeuvring targets or incoming exoatmospheric objects at high speed.

Design and development of guidance laws based on diffrential game theory discussed in Section 2 has also potential avenue for further research. All these laws are for terminal guidance. In Section 5, the current research activities on midcourse guidance have also been discussed. In fact, artificial intelligence-based guidance laws (Section 4) can be used in principle for midcourse guidance also. This is also an open area for research.

One important point likely to be stressed is that guidance laws after design have to be validated through 6-DOF simulation. In fact, during the process of design of a guidance law, generally both the pursuer velocity and the evader velocity are assumed to be constant and kinematic equations are used to represent the system. Only after implementation in 6-DOF model, its robustness in the presence of autopilot lag, pursuer maximum latax capability, aerodynamic and inertial parametric variations, nonlinearities due to flight envelope can be studied. A lot of literatures exist on the guidance law design with its performance validation in a 2-D or a 3-D engagement scenario. But studies on its performance under complete 6-DOF platform, including practical constraints, are scanty. In fact, this is the most crucial study which should attract the attention of the research and development organisations.

At last, it is felt that if contents of this review paper create interest of the budding scientists and students in academic institutes to carry out research on this fascinating field of applied science, ie, missile guidance, their effort is justified.

ACKNOWLEDGEMENT

The authors thank Mr N.V. Kadam, Emuritus Scientist of Defence Research & Development Laboratory, Hyderabad, for motivating them to prepare this review paper.

REFERENCES

1. Goodstein, R. Guidance law applicability for missile closing. Guidance and control of tactical missiles. Report of the Advisory Group on Aerospace Research and Development (AGARD), France. Report No. AGARD-LS-52, 1972.

2. Heap, E. Methodology of research into command, line-of-sight (LOS) and homing guidance. Guidance and control of tactical missiles. Report of the Advisory Group on Aerospace Research and Development (AGARD), France. Report No. AGARD-LS-52, 1972.

3. Fossier, M.W. The development of radar homing missiles. *J. Guid. Control Dyn.*, 1972, **7**(6). 641-51.

4. Fossier, M.W. Development of low altitude air defence System. *J. Guid. Control Dyn.*, 1984, **11**(2), 96-102.

5. Rao, M.N. Analytical solution of optimal trajectory shaping guidance. *J. Guid. Control Dyn.*, 1989, **12**(4). 600-01.

6. Rao, M.N. New analytical solution of proportional navigation. *J. Guid. Control Dyn.*, 1993, **16**(2), pp.-

7. Vathsal, S. & Rao, M.N. Analysis of generalised and optimal guidance laws for homing missiles. *IEEE Trans. Aerospace Elects. Syst.*, 1995, **AES-31**(2).

8. Dhar, A. & Ghose, D. Capture region for a realistic TPN guidance law. *IEEE Trans. Aerospace Elects. Syst.*, 1993, **29**(3), 995-1003.

9. Ghose, D. True proportional navigation with manoeuvring target. *IEEE Trans. Aerospace Elects. Syst* 1994, **AES-30**(1), 229-37.

10. Ghose, D. On the generalisation of true proportional navigation. *IEEE Trans. Aerospace Elect. Syst.*, 1994, **AES-30**(2), 545-55.

11. Shukla, U.S. & Mahapatra, P.R. Generalised. *IEEE Trans. Aerospace Elect. Syst.*, 2003, **AES-24**(3), 231-38.

12. Shukla, U.S. & Mahapatra, P.R. True proportional navigation dilemma- pure or true. *IEEE Trans. Aerospace Elect. Syst.*,1990, **AES-26**(2), 382-92.

13. Bryson, A.E. & Ho, Y.C. Applied optimal control. Hemisphere Publications, Washington DC, 1969.

14. Sarkar, A.K.; Tiwari, P.K.; Srinivasan, S.; Bhattacharjee, R.N. & Ghose, D. Generalised PN guidance law for a practical pursuer evader engagement. American Institute of Aeronautics & Astronautics (AIAA) Paper No AIAA-2003-5651-CP, 2003.

15. Bryson, A.E. & Ho. Y.C. Applied optimal control. Hemisphere Publications, Washington DC, 1969.

16. Gelb, A. Applied optimal estimation. MIT Press, Cambridge, MA, 1974.

17. Zarchan, P. Tactical and strategic missile guidance. Progress in Aeronautics and Astronautics, Ed. 3, 1997, **176**.

18. Lin, C.F. Modern navigation, guidance and control processing. Prentice Hall, 1991.

19. Dowdle, J.R.; Athans, M. & Gully, S. Willasky. An optimal control and estimation algorithm for missile endgame guidance. *In* Proceedings of the IEEE Conference on Decision and Control, 1982.

20. Uhrmeister, B. Kalman filter for a missile with radar and/or imaging sensor. *J. Guid. Control Dyn.*, 1994, **17**(6). 1339-344.

21. Kim, Y. & Seo, J.H. The realisation of the three dimensional guidance law using augmented proportional navigation. *In* Proceedings of 35th IEEE Conference on Decision and Control.

22. Vergez, P. & Liefer, R.K. Target acceleration modelling for tactical missile guidance. *J. Guid. Control Dyn.*, 1984, **7**(3).

23. Balakrishnan, S.N. & Speuer, J.L. Coordinate transformation-based filter for improved target tracking. *J. Guid. Control Dyn.*, 1986, **9**(6), 704-09.

24. Balakrishnan, S.N. Extension to modified polar coordinates and applications with passive measurements. *J. Guid. Control Dyn.*, 1989, **12**(6), 906-12.

25. Stallard, D.V. An angle only tracking target for manoeuvring target. American Institute of Aeronautics and Astronautics (AIAA). Paper No. AIAA-90-3343-CP, 1990.

26. Robinson, P.N. & Yin, M.R. Modified spherical coordinates for radar. American Institute of Aeronautics and Astronautics (AIAA). Paper No. AIAA-94-3546-CP, 1994.

27. Hablani, H.B. Gaussian second-order filter for proportional navigation of exoatmospheric interceptors with angle only measurements. American Institute of Aeronautics and Astronautics (AIAA). Paper No. AIAA-98-4217-CP, 1998.

28. Hablani, H.B. Pulse guidance of exoatmospheric interceptor with image processing delays in angle measurements. American Institute of Aeronautics and Astronautics (AIAA). Paper No. AIAA-2000-4272-CP, 2000.

29. Waldmann, J. Line-of-sight (LOS), rate estimation and linearising control of an imaging seeker in a tactical missile guided by proportional navigation. *IEEE Trans. Control Sys. Techn.*, 2002, **10**(4), 556-67.

30. Ben-Asher, J. & Yaesh, I. Advances in missile guidance theory. American Institute of Aeronautics and Astronautics (AIAA). AIAA Publication, 1998, **180.**

31. Sarma, I.G. & Swamy, K.N. Maximum acceleration model tracking terminal homing guidance of *Akash*-MAMTHA. Report No. JATP-86-005, 1988.

32. Sarma, I.G. & Swamy, K.N. Proportional acceleration-augmented guidance for *Akash*-PRAGNA. Report No. JATP-90-001 to Report No. JATP-90-005, 1990.

33. Vathsal, S.; Sarkar, A.K. & Bhattacharjee, R.N. Seeker-based guidance using estimation of target. Acceleration in endgame scenario. Aeronautical Society of India (ASI), Calcutta Chapter, 2003.

34. Gurfil, P.; Jodorkovsky, M. & Guelman, M. Neo classical guidance law for homing missiles. *J. Guid., Control Dyn.*, 2001, **24**(3), 452-59.

35. Gurfil, P. Zero miss-distance guidance law based on line-of-sight rate measurements only. American Institute of Aeronautics and Astronautics (AIAA). Paper No. AIAA-2001-4377-CP, 2001.

36. Barron, R. Reduced computational endgame steering laws for predictive guidance. *J. Guid. Control Dyn.*, 1995, **18**(2), 295-03.

37. Best, R.A. & Norton, J.P. *Predictive Missile Guid.*, 2000, **23**(3), 539-46.

38. Talole, S.E. & Banavar., R.N. Proportional navigation through predictive control. *J. Guid. Control Dyn.*, 1998, **21**(6), 1004-06.

39. Talole, S. & Phadke, S.B. Nonlinear state estimation in homing guidance. American Institute of Aeronautics and Astronautics (AIAA). Paper No. AIAA-2002-4772-CP, 2002.

40. Gutman, S. Optimal guidance of homing missiles. *J. Guid. Control Dyn.*, 1979, **2**(4).

41. Anderson, G.M. Optimal control and differential games interception missile guidance law. *J. Guid. Control Dyn.*, 1981, **4**(2).

42. Forte, I. & Shinar, J. Can a mixed guidance strategy improve missile performance? *J. Guid. Control Dyn.*, 1988, **11**(1).

43. Forte, I. & Shinar, J. Improved guidance law design based on mixed strategy concept. *J. Guid. Control Dyn.*, 1989, **12**(5).

44. Green, A.; Shinar, J. & Guelman, M. Game optimal guidance law synthesis for short-range missiles. *J. Guid. Control Dyn.*, 1992, **15**(1).

45. Lipman, Y.; Shinar, J. & Oshman, Y. Stochastic analysis of the interception of manoeuvring antiship missile. *J. Guid. Control Dyn.*, 1981, **4**(2).

46. Shima, T.; Shinar, J. & Weiss, H. New interceptor guidance law integrating time-varying and estimation-delay models. *J. Guid. Control Dyn.*, 2003, **26**(2). 295-03.

47. Slotine, J.E. & Li, Weiping. Applied nonlinear control. Prentice-Hall International Inc, 1991.

48. Ha, I.J. & Chong, S. Design of a CLOS guidance law via feedback linearisation. *IEEE Trans. Aerospace Elects. Syst.*, 1992, **AES-28**(4), 518-24.

49. Bezick, S.; Rusnak, I. & Gray, W.S. Guidance of a homing missile via nonlinear geometric control methods. *J. Guid. Control Dyn.*, 1995, **18**(3), 441-48.

50. Leng, G. Guidance algorithm design: A nonlinear inverse approach. *J. Guid. Control Dyn.*, 1998, **21**(5), 742-46.

51. Taur, D.R. Nonlinear guidance and navigation of a tactical missile with high heading error. American Institute of Aeronautics and Astronautics (AIAA). Paper No. AIAA-2002-4733-CP, 2002.

52. Cloutier, J.R. & Stansbery, D.T. All-aspect acceleration limited homing guidance. American Institute of Aeronautics and Astronautics (AIAA). Paper No. AIAA-99-4063-CP, 1999.

53. Shneydor, N.A. Missile guidance and pursuit kinematics, dynamics and control. Horwood Publishing Ltd, 1998.

54. Srivastava, R.; Sarkar, A.K.; Ghose, D. & Gollakota, S. Nonlinear three-dimensional composite guidance law based on feedback linearisation. American Institute of Aeronautics and Astronautics (AIAA). Paper No. AIAA-2004-4903-CP, 2004.

55. Srivastava, R.; Prabhakar, N.; Sarkar, A.K. & Ghose, D. Three-dimensional nonlinear inverse dynamics guidance law for parallel navigation. American Institute of Aeronautics and Astronautics (AIAA). Paper No. AIAA-2004-4904-CP, 2004.

56. Snell, S.A.; Enns, D.F. & Garrard, W. L. Nonlinear inversion flight control for a super manoeuvreable aircraft. *J. Guid. Control Dyn.*, 1992, **15**(4). 976-84.

57. Georgie, J. & Valasek, J. Evaluation of longitudinal desired dynamics for dynamic inversion-controlled generic reentry vehicle. *J. Guid. Control Dyn.*, 2003, **26**(5), 811-19.

58. Variable structure homing guidance schemes with and without target manoeuvre estimation. American Institute of Aeronautics and Astronautics (AIAA). Paper No. AIAA-94-3566-CP, 1994.

59. Babu, K.R.; Sarma, I.G. & Swamy, K.N. Switched bias proportional navigation for homing guidance against highly manoeuvring targets. *J. Guid. Control Dyn.*, 1994, **17**(6), 1357-363.

60. Zhou, D.; Mu, C. & Xu, W. Adaptive slidemode guidance of a homing missile. *J. Guid. Control Dyn.*, 1999, **22**(4), 589-94.

61. Pedar, A. Autonomous software agent for missiles. Newtech Software Pvt Ltd, Bangalore, India, 2003.

62. Sarkar, A.K.; Vathsal, S.; Suresh, S. & Mukhopadhay, S. Target acceleration estimation from tracking radar position data in real time using neural network. *In* NWTMG Conference, DRDL, Hyderabad, 2004.

63. Geng, J. & Mc Cullough. Missile control using fuzzy cerebellar model. *J. Guid. Control Dyn.*, 1997, **20**(3).

64. Passino, K.M. & Yurkovich, S. Fuzzy control. Addison-Wesley, 1998.

65. Mishra, S.K.; Sarma, I.G. & Swamy, K.N. Performance evaluation of two fuzzy logic-based homing guidance schemes. *J. Guid. Control Dyn.*, 1994, **17**(8), 1389-391.

66. Gons, Alves, P.G. & Caglayton, A.K. Fuzzy logic PID controller for missile terminal guidance. *In* Proceedings of 1995 IEEE International Symposium on Intelligent Control. NJ, USA. pp. 377-82.

67. Rajasekhar, V. & Srinatha, A.G. Fuzzy logic implementation of proportional navigation guidance. *Acta Astronautica*, 2000, **46**(1), 17-24.

68. Lin, C.L. & Chen, Y.Y. Design of fuzzy logic guidance law against high-speed target. *J. Guid. Control Dyn.*, 2000, **23**(1), 17-25.

69. Lin, C.L.; Hung, H.Z.; Chen, Y.Y. & Chen, B.S. Development of an integrated fuzzy logic-based missile guidance law against high-speed manoeuvring target. *IEEE Trans. Fuzzy Syst.*, 2004, **12**(2), 157-69.

70. Chen, B.S.; Chen, Y.Y. & Lin, C.L. Nonlinear fuzzy H_∞ guidance law with saturation of actuators against manoeuvring targets. *IEEE Trans. Control Syst. Tech.*, 2002, **10**(6), 769-79.

71. Lin, C.M.; Hsu, C.F. & Mon, Y.J. Self-organising fuzzy learning CLOS guidance law design. *IEEE Trans. Aerospace Elect. Systs.*, 2003, **AES-34**(4), 1144-151.

72. Sridhar, B. & Gupta. N.K. Missile guidance law based on singular perturbation methodology. *J. Guid. Control Dyn.*, 1985, **8**(3), 320-24.

73. Cheng, V.H.L. & Gupta, N.K. Advanced mid-course guidance for air-to-air missiles. *J. Guid. Control Dyn.*, 1986 **9**(2), 135-42.

74. Sheu, D.; Vinh, N.X. & Howe, R.M. Application of singular perturbation methods for three-dimensional minimum time interception. *J. Guid. Control Dyn.*, 1991, **14**(2), 360-67.

75. Raikwar, A.G.; Ghose, D.; Swamy, K.N. & Bhat, M.S. Design and evaluation of a mid-course guidance law for a BVRAAM. JATP. Report No. JATP-97-001, IISc, Bangalore, India, 1997.

76. Raju, P.A. & Ghose, D. Empirical virtual sliding target guidance law design. An aerodynamic approach. *IEEE Trans. Aerospace Elect. Systs.*, 2003, **AES-39**(4).

Contributors

Dr S. Vathsal obtained his ME (Control System) from the Birla Institute of Technology and Science, Pilani, in 1970 and PhD from the Indian Institute of Science (IISc), Bangalore, in 1974. His areas of research include: Control Systems, neural networks.

Mr A.K. Sarkar has been working as Scientist at the Defence Research & Development Laboratory (DRDL), Hyderabad, since 1984. He graduated from the Indian Institute of Technology (IIT), Kharagpur, during 1975-80 and did his postgraduation in Computer Science from the Indian Institute of Technology (IIT) Madras, Chennai, during 1982-84. His areas of research include: Flight mechanics, state and parameter estimation for offline and real-time applications, and guidance and control.