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Studying the Interaction of Waves to Determine the Impact Response of a Layered Elastic Medium

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ABSTRACT

When an impactor strikes a layered target, both the impactor and the target experience waves. The waves produced travel and engage in interactions with other waves as well as the interfaces in the impactor-target system. For the impact problems on a layered mediumwith periodic properties and layered elastic media of Goupillaud-type (each layer has the same wave travel time), researchers have presented an analytical solution for stress variation with position and time within the target. However, the solution for an elastic media not satisfying the above conditions is not available in the literature. The present study fills this gap and finds the behaviour of a generalized layered medium to an impact problem. The response of the material at any position inside the layered medium is found by solving the interaction between waves, interfaces, and boundaries. The mass, momentum balance and constitutive relationship are solved to get the exact analytical expressions for particle velocity and stress for each possible wave interaction happening in the impactor and the layered medium. The expressions are utilized in a computer program to study the impact behaviour of a layered media. The code tracks each wave as it travels through the system and identifies those interactions that occur in the shortest time, uses the stress and velocity expression for that interaction, and updates the state of the material. When stress produced at the impact surface is tensile in nature, the impactor and target can be separated. The work can be applied to both finite and semi-infinite impactors and targets, and the layered medium does not necessarily have to be a periodic layered media or a Goupillaud-type medium.

Keywords: One-dimensional impact; Layered media; Elastic waves; Exact solution; Riemann problem; Wave interactions

1. INTRODUCTION

Numerous research studies that offer computational or analytical solutions to impact problems apply a step in stress or velocity on the surface of the medium as a boundary condition without considering the impact[1-4]. In reality, whenever atleast one of the bodies between the impactor and target is finite, or has multiple layers or is inhomogeneous, reflected waves reach the impact surface and change its velocity and stress. As a result, the velocity and stress profiles generally vary with time, making it difficult to predict them before the problem is solved. Meyers[5] investigated the interaction between elastic waves and the interface between two layers with varying properties. Meyers[5] used equilibrium and continuity equations at the interface to calculate the strength of reflected and transmitted waves. Chen[6-7] presented an analytical solution for an impactor's impact on a target with periodic layers and obtained the expression of asymptotic stress using Floquet's theory. Their conclusions show that the target's heterogeneities raise the average stress relative to the stress in the homogeneous target.The behaviour of a layered medium to shock loading was observed by implementing the nonlinearities through the shock and particle speed relation. One of the limitations of the studies is that it was restricted to semi-infinite target-impactor systems and periodic laminates. Agrawal and Bhattacharya[8] studied the shock propagation in a layered media using the jump conditions along with maximal dissipation criteria and assumed the medium was homogeneous to elastic waves. However, the materials are not homogeneous to elastic waves because of different impedances.

Singh[9], *et al.* modelled the interaction of two shock waves, two rarefaction waves, and a shock wave with a material interface between two layers, a free and fixed boundary. Singh[10], *et al.* examined how elastic waves, which are typically neglected during shock propagation, affect the behaviour of a multilayered media to an impact loading. The effect of wave propagation and debonding in a metallic system with multiple layers with continuously varying impedance under impact loading was investigated by Fernando[11], *et al.* and Kevadiya and Singh[12]. Singh[13] numerically found the shock response by approximating the non-linear Hugoniot with a multi-linear behaviour. The present study is not explicitly correlated to recent studies on the shock response of layered medium[8-13], which concentrate on layered media that are shocked with stress above the Hugoniot Elastic Limit (HEL) level.

In order to obtain the formulas for the velocity and stress in the layered medium for the impact problem for shocks below

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the HEL, Gazonas[14] combined d'Alembert's wave equation solution with that of the Laplace transform technique. The study of Gazonas[14] was only applicable to a single target, but Gazonas[15] expanded it to include targets with multiple layers of the Goupillaud type (each layer has the same wave travel time). They examined a one-dimensional impact in which a semi-infinite Goupillaud-layered elastic target collides with a semi-infinite impactor. Using the Riemann invariants method, they created a set of recurrence relations for the elastic impactor and the target's one-dimensional impact. They showed that the type of heterogeneity in the target does not affect the asymptotic velocity and stress. Gazonas[14-15] made the study for a semi-infinite target and impactor. They did not permit the detachment of the target and impactor when the stress of a tensile nature is produced at the impact interface.

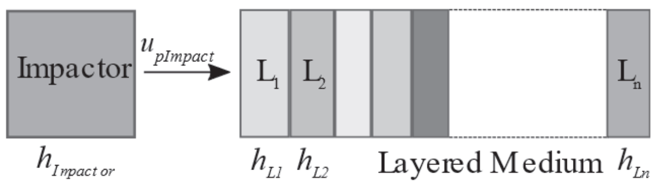


Figure 1. Abstract representation of an impactor moving with velocity $u_{pImpact}$ strikes the layered medium with layers L_1, L_2 … L_n.

The primary objective of this research is to study the behaviour of an elastic medium with multiple layers to a low-velocity one-dimensional impact by observing interactions happening in the layered medium. A schematic representation of an impact between an impactor flying at velocity $u_{pImpact}$ and a target made up of layers L_1, L_2 … L_n is shown in Fig. 1. The method performs well for finite, non-periodic, non-Goupillaud targets because it is applicable to arbitrary impactor and target systems and, therefore, is more general. The remainder of the article is structured as defined. The governing equations and the method used for determiningthe impact response of the layered medium are covered in Section 2. The various wave interactions happening in the impactor-target system are discussed in Section 3 of the article. Section 4 validates the proposed model using the results reported in the literature and discusses the behaviour of the layered medium to low and high-velocity impacts that result in elastic waves in the medium. The finite element analysis results and the comparison of the stress and particle velocity variation in the layered target are also done in Section 4. The conclusions of the current work are outlined in Section 5.

2. GOVERNING EQUATIONS AND METHODOLOGY

The particle velocity, stress, and strainat a time t for a particle at position X in the reference configuration are denoted as $u_p(X,t)$, $\sigma(X,t)$, and $\epsilon(X,t)$, respectively. Eqns (1) and (2) describe the Lagrangian form of the jump equations derived from the mass and momentum.[10-13]

$$[[\sigma]] + \rho U[[u_p]] = 0 \tag{1}$$

$$[[u_p]] + U[[\epsilon]] = 0 \tag{2}$$

Superscripts 0 and 1 represent the states prior to and following wave propagation, respectively. The following symbols stand for particle velocity, stress, and strain: u_p, σ, and ϵ. The change from state 0 to state 1 in strain value is represented by the expression $[[\epsilon]] = \epsilon_1 - \epsilon_0$. In a similar way, the variation in velocity and stress is defined. The wave propagation velocity is represented by U. The wave propagation speed for elastic waves (c) is calculated to be $U = +/-c = \sqrt{(E/\rho)}$ where ρ and E represent the density and elastic modulus of the layer, respectively.

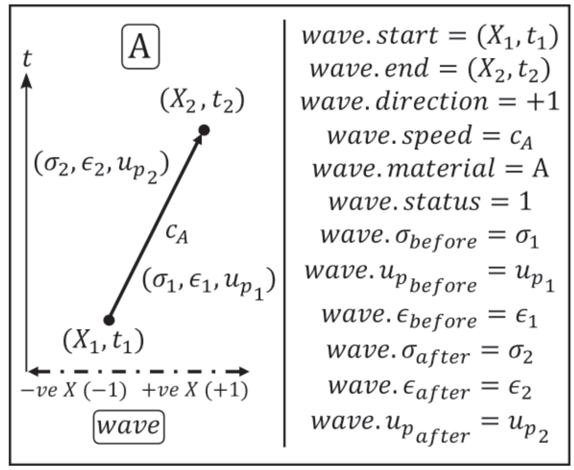


Figure 2. Schematic diagram representing the wave propagation and the different characteristics attributedto each wave in the impactor and the target.

Waves are produced in the impactor and first layer of the medium as a result of the impactor hitting the layered medium. The waves produced in the impactor and the first layer of the medium during the impact propagate and interact in various ways. Exact analytical expressions for particle velocity and stress are obtained using the mass and momentum balance for each possible wave interaction happening in the impactor and the layered medium (discussed in section 3). The expressions are utilized in a computer program to study the behaviour of a layered medium subjected to an impact. Every interaction, including the repeated interactions, in the impactor and the layered medium system is tracked and solved by the created program. The code is written to solve the repeated interactions in the flyer-target system. After every interaction, the final state of the material is evaluated in terms of velocity and particle velocity.

The wave is stored in the code with the material state as particle velocity, stress, and strainbefore and after the wave, and wave attributes as its direction of travel, the origin, and the end coordinates of the wave from the *X-t* graph, the layer in which the wave is moving, its status, and its propagation speed. The status of the wave informs whether the wave has already interacted (active, status=1) or is yet to undergo the interaction (passive, status=0). It helps in wave tracking as only the active waves have to be traced for the wave interactions in the algorithm. Figure 2 represents an example of an active wave which starts from (X_1, t_1), propagates in material A with speed c_A, moves in $+veX$ direction and ends at (X_2, t_2). It shows that the material state is changed from (σ_1, u_{p1}, ϵ_1) to state (σ_2, u_{p2}, ϵ_2) due to the propagation of the wave.

The impactor-target system's interactions have all been solved by a computer program that analyses each wave as it travels through the system. The code initially solves the wave interaction that happens at the earliest. The wave interaction causes the system to produce additional waves and changes the material's condition. After each interaction, the parameters are changed, and the procedure is repeated until all the interactions in the system have occurred. Fig. 3(a) represents the input variables fed into the computer program and the output sought from it. In order to specify the steps followed in the developed code to get the impact response, a flow chart representing all the tasks performed inside the program in a stepwise manner is shown in Fig. 3(b).

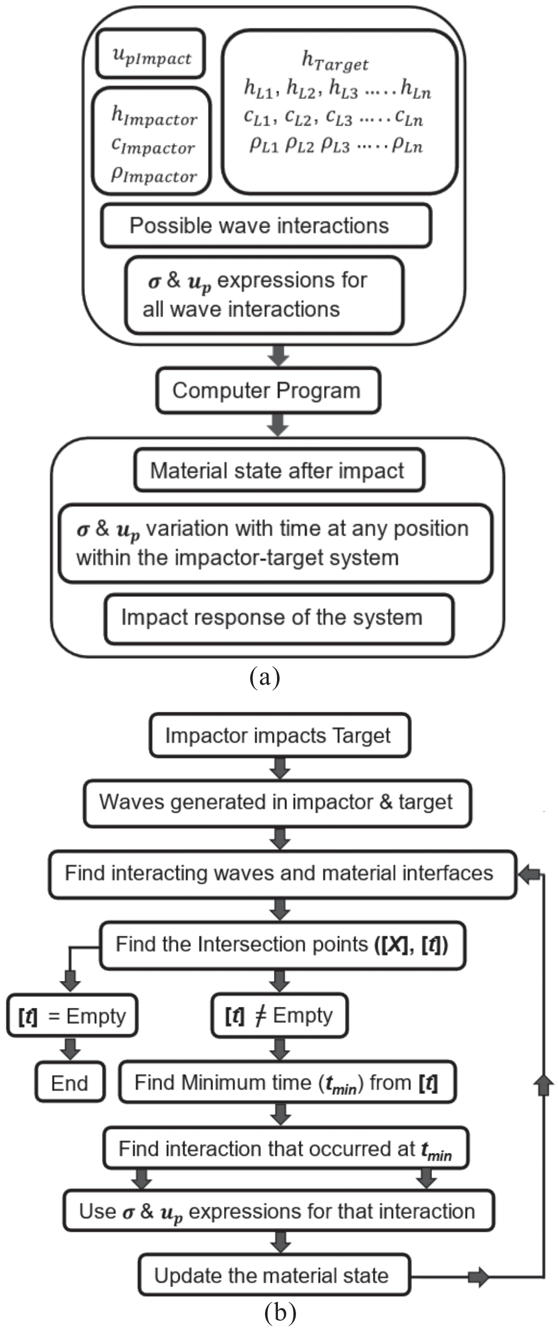


Figure 3. Schematic diagram showing the working of the computer program by means of (a) Input and Output diagram for the code, (b) Flow chart mentioning the various steps followed in the code to get the material response.

The advantage the present model offers is that it provides the exact solutions for the impact problem in terms of stress and particle velocity. The algorithm finds the solution by studying the wave interactions in the layered medium. The model doesn't have to find the solution by time marching, as it uses the exact expressions for the stress and velocity and only has to follow the wave interactions occurring in the layered medium. The other advantage is its good computational efficiency,as this algorithm does not require the mesh requirement over the whole domain. Only the heterogeneity in terms of material interfaces and boundaries needs to be modelled; only wave tracking is required, and the analytical expressions are used to get the updated material state.

3. VARIOUS INTERACTIONS OCCURRING IN THE LAYEREDMEDIUM

The generated waves in the impactor and the layered medium during the impact spread out and interact with the boundaries, interfaces, and other waves in the system. Elastic waves are produced when an elastic impactor collides with an elastically layered object. These elastic waves experience many interactions, and resolving them results in the target's impact response. These interactions have been thoroughly discussed and are included below.

3.1 Elastic Waves in the Impactor and the Layered Media Generated after the Impact

Elastic waves begin to propagate in the impactor and layered target media when an elastic impactor travelling at a velocity $u_{pImpact}$ impacts an elastic layered media. The *X-t* schematic of the generation of an elastic wave inside the first material of the layered mediumdue to impact is shown in Fig. 4(a), and the σ- ϵ state of the layer is shown in Fig. 4(b). The mass and momentum balance equations described in Eqns (3) and (4) are used to compute the final output in the impactor and the first layer of the elastic layered media.

$$\sigma_1 - \sigma_0 - \rho_{Impactor} c_{Impactor} (u_{p1} - u_{pImpact}) = 0 \tag{3}$$

$$\sigma_1 - \sigma_0 + \rho_A c_A (u_{p1} - u_{p0}) = 0 \tag{4}$$

where, $\sigma_0 = u_{p0} = 0$. σ_0 and u_{p0} are zero as the target is initially at rest and stress-free. The above equations form a set of two equations in two variables where σ_1 and u_{p1} are variables. Solving Eqns (3) and (4), we get the expression for the final stress and velocity state after impact.

3.2 Elastic Wave Interaction with an Interface Between Two Layers of the Medium

At the moment of impact, the wave created in the first layer moves toward the material interface and interacts with it. Elastic waves are reflected and transmitted when they interact with a material's interface. The type of the reflected wave is dependent on the elastic impedance difference between the two layers of the medium forming the interface. The reflected wave may be of a loading or unloading type. Mayers[5] determined the reflection and transmission ratios based on the impedances of the material composing the interface to determine the strength of transmitted and reflected waves. The material gets loaded by the reflected elastic wave when the impedance of layer B

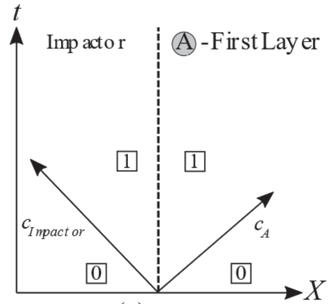
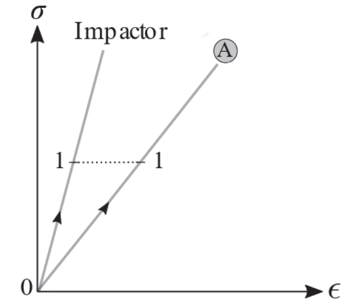
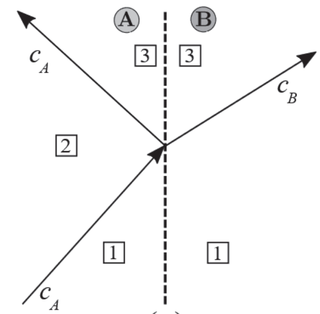
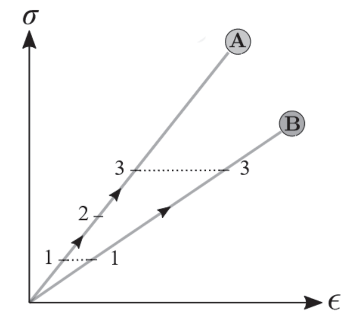
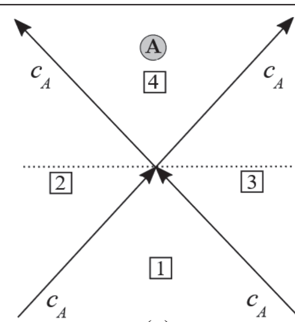
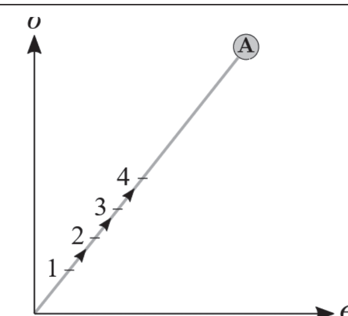
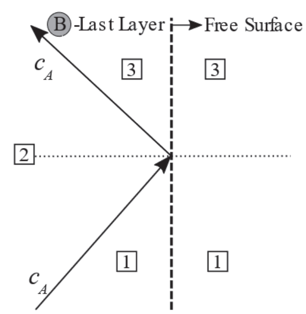
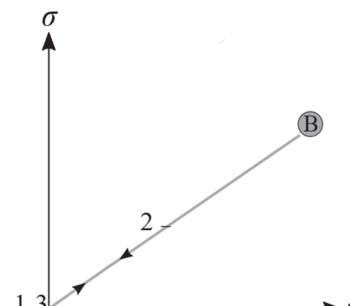
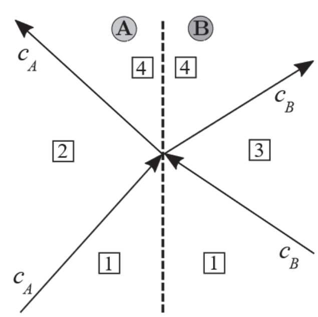
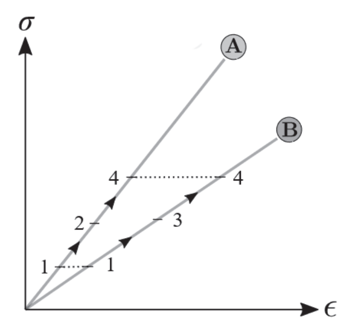
Type of interaction	*X-t* diagram	σ-ϵ curve
Impactor hits the layered medium	 (a)	 (b)
Elastic wave interacts with the interface between the layers of the medium	 (c)	 (d)
Two elastic waves interact with each other inside a layer of the medium	 (e)	 (f)
Elastic wave interacts with the free surface of the medium	 (g)	 (h)
Two elastic waves interact with each other at an interface between the layers of the medium	 (i)	 (j)

Figure 4. The various interactions occurring in the layered medium, the update in the impactor, and the different layers of the medium are depicted using X-t and σ- ϵ graphs.

is more than that of layer A, and it gets unloaded when the impedance of material B is lower. The material state is depicted in the *X-t* and σ- ϵgraphs, respectively, in Fig. 4(c) and 4(d) for elastic wave interaction with an interface between two layers, A and B, of the medium. Eqns (5) and (6), respectively, provide the material's final value of stress and particle velocity.

$$\sigma_3 = (\rho_A c_A \sigma_1 + \rho_B c_B \sigma_2 + \rho_A \rho_B c_A c_B (u_{p1} - u_{p2})) / (\rho_A c_A + \rho_B c_B) \quad (5)$$

$$u_{p3} = (\sigma_1 - \sigma_2 + \rho_A c_A u_{p2} + \rho_B c_B u_{p1}) / ((\rho_A c_A + \rho_B c_B)) \quad (6)$$

3.3 Interaction of Two Elastic Waves Within a Layer of the Medium

In a linearly elastic material, two elastic waves interact to create two elastic waves. Depending on how the interacting elastic waves behave, the material's final condition may be more or less stressed. The material gets unloaded as a result of the interaction of two elastic waves. Two loading elastic waves interact with each other to load the material. The strength of the loading and unloading elastic waves determines the final state following an interaction between loading and unloading elastic waves. Eqns (7) and (8), respectively, provide the material's final state of stress and particle velocity. The material state when two elastic waves interact with one another inside a layer of the medium is depicted in the *X-t* and σ- ϵgraphs, respectively, in Fig. 4(e) and 4(f).

$$\sigma_4 = (\sigma_2 + \sigma_3 - \rho_A c_A (u_{p2} - u_{p3})) / 2 \quad (7)$$

$$u_{p4} = (\sigma_3 - \sigma_2 + \rho_A c_A (u_{p2} + u_{p3})) / (2\rho_A c_A) \quad (8)$$

3.4 Elastic Wave Interaction with the Free Surface of the Medium

A wave on interaction with a free surface gets completely reflected, and no part of the incident wave is transmitted. The stress level at the free surface must remain zero. The stress on the free surface remains zero because the reflected elastic wave is of tensile form and has the same strength as that of the initial elastic wave that interacts with the free surface of the medium. The particle velocities produced by the incident and the reflected waves are equal (magnitude and direction). Two particle velocities added together provide twice as much initial particle velocity. The final particle velocity increases by two times only when the free surface is initially at rest. It is noted that the particle velocity varies upon interaction with the free surface by a factor of two times that produced by the elastic wave. The stress at the free surface is always zero, and the final state of zero stress is shown in Eqn (9). Eqn (10) provides the resultant particle velocity, represented by u_{p3}, which also accounts for the velocity of the free surface.

$$\sigma_3 = 0 \quad (9)$$

$$u_{p3} = u_{p1} - 2(u_{p1} - u_{p2}) \quad (10)$$

where, u_{p3} is the resultant particle velocity after an elastic wave undergoes interaction with a free surface. The particle velocities prior to and following the incident elastic wave are represented by u_{p1} and u_{p2}, respectively. The change in particle velocity caused by the incident elastic wave is given by u_{p1} - u_{p2}. The expression for resultant velocity mentioned in Eqn. (10) is valid for both the cases of initially at rest and moving free surface. When the free surface is at rest,$u_{p1}=0$, and when it is moving, $u_{p1}=u_{pf}$, where u_{pf} is the velocity of the free surface. Figures 4(g) and 4(h) represent the material state in *X-t* and σ- ϵ for the interaction, respectively.

3.5 Interaction of Two Elastic Waves With Each Other at an Interface Between the Layers

Two elastic waves can also interact with each other at the interface between layers A and B of the layered medium. The interacting elastic waves and the elastic waves generated after interaction propagate in different layers of the target, resulting in different wave properties of the interacting and the resulting waves after the interaction. Eqns (11) and (12) provide the material's final state of stress and particle velocity after the interaction. The material state for the case when elastic waves interact with one another at the interface between layers A and B is depicted in the *X-t* and σ- ϵ graphs, respectively, in Fig. 4(i) and 4(j). Material A is moved from state 2 and changed to state 4 in layer A by means of an elastic wave travelling back into layer A, and the state of layer B is moved from state 3 to state 4 due to an elastic wave moving forward in layer B.

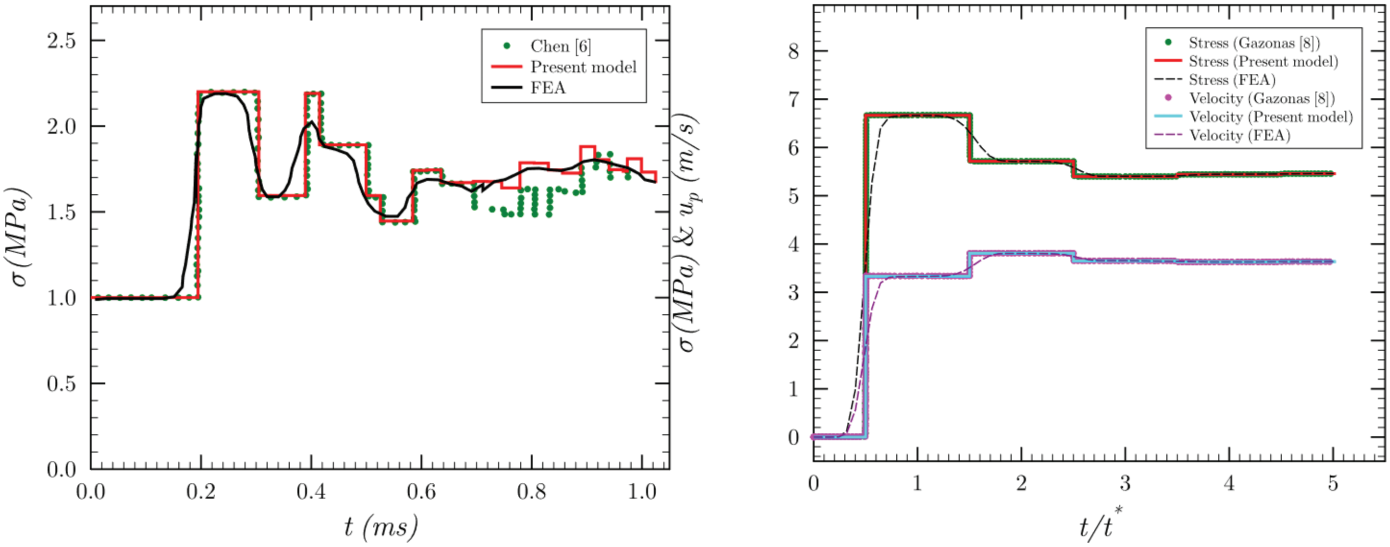
$$\sigma_4 = (\rho_A c_A \sigma_3 + \rho_B c_B \sigma_2 - \rho_A \rho_B c_A c_B (u_{p2} - u_{p3})) / (\rho_A c_A + \rho_B c_B) \quad (11)$$

$$u_{p4} = (\sigma_3 - \sigma_2 + \rho_A c_A u_{p2} + \rho_B c_B u_{p3}) / (\rho_A c_A + \rho_B c_B) \quad (12)$$

4. RESULTS AND DISCUSSIONS

The impact of an elastic impactor on a multilayered elastic target is studied. The results are first validated with the results present in the literature. Figure 5(a) shows the comparison of stress obtained by Chen[6], the present model, and FE analysis. Figure 5(b) compares stress and particle velocity obtained in the middle of a single-layered target backed by a semi-infinite backing obtained by Gazonas[14], the present model, and FEA analysis. However, the models reported by Chen[6-7] and Gazonas[14-15] are limited to periodically layered, single-layer targets and only Goupillaud-type elastic layered media. The method presented in this article solves those and is applicable to any arbitrary elastic impactor and layered elastic target media.

The following is a case study of an elastic impactor striking a six-layered elastic target. The impactor, moving at 5 *m/s*, collides with the target, causing the propagation of elastic waves in the impactor and the first layer of the target. The target's properties are chosen so that each layer has a different elastic impedance. The *X-t* graph for the case mentioned above study is shown in Fig. 6. The impactor is shown from *X*=0*m* to *X*=2*m*, while *X*=2*m* to *X*=4*m* represents the target. The impactor's free surface is represented by *X*=0*m*, and the target's free surface is shown by *X*=4*m*. The material states 0, 1, and 2 are marked in the *X-t* diagram shown in Fig. 6. State 0 is the initial state of the target and the impactor before the impact. It is observed that the impact generates the elastic waves in the impactor and the first layer of alayered medium (L_1). The generated waves take the impactor and the target from the material state 0 to state 1 and can be observed in Fig. 6 at



(a) (b)

Figure 5. (a) σ variation at the impact surface of the periodically layered target obtained by Chen[6] (Figure 5(b) of [6])), FEA and present model, (b) σ and u_p profiles at the middle of a single layer target obtained by Gazonas[14] (Figure 6 of [14]), FEA and present model.

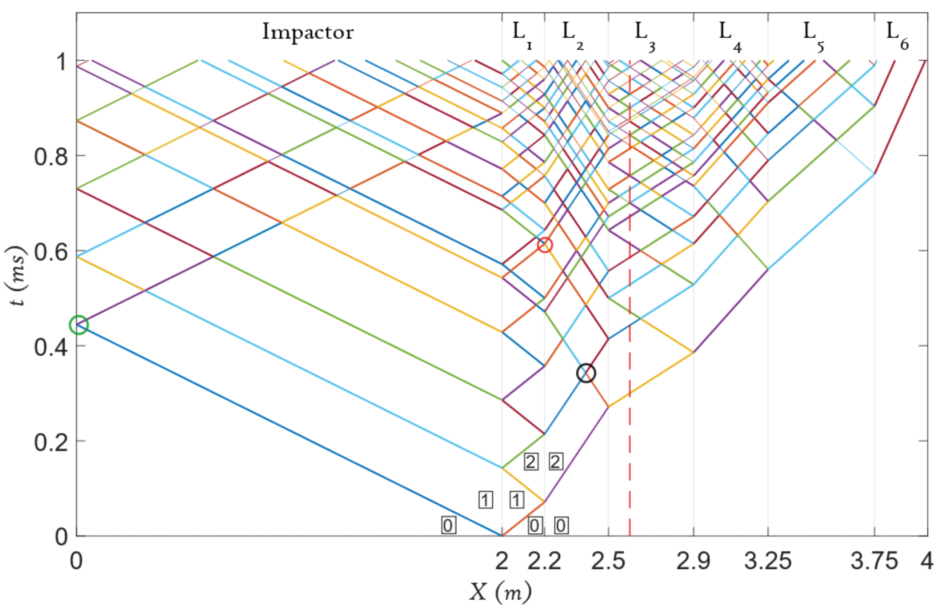


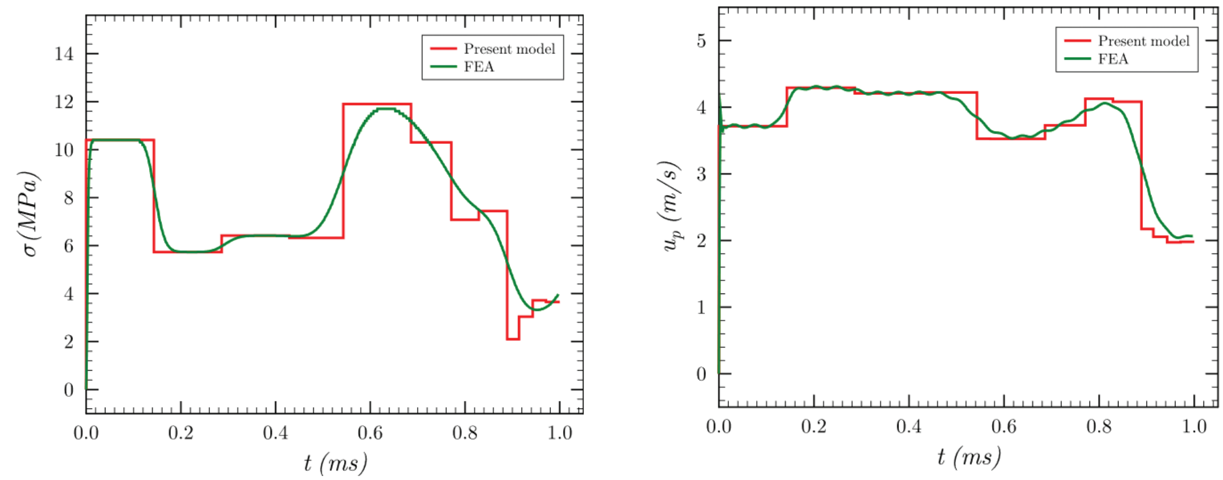
Figure 6. *X-t* graph for the general case studied showing the wave propagation in the impactor and the layered medium with layers L_1, L_2 … L_n.

X=2 *m* and *t*=0 *ms*. σ_1 and u_{p1} are calculated using Eqns (3) and (4). The particle velocity and stress values across the interface between the two layers are continuous. Thus, the state of both the impactor and the layer (L_1) goes to state 1 from state 0. The next wave interaction in the layered medium is the interaction of an elastic wave propagating in layer L_1 with the interface at location *X*=2.2*m*. The state of layer L_1 changes to state 2, and layer L_2 observes the shift from state 0 to state 2 because of the interaction of the wave with the material interface located at *X*=2.2*m*. σ_2 and u_{p2} are calculated using stress and particle velocity expressions given in Eqns (5) and (6). The interaction results in a reflected and a transmitted wave in layers L_1 and L_2, respectively, which further propagate and interact in the impactor and the layered medium. In the same way, wave interactions are solved repeatedly, and the impact response is obtained in terms of stress and particle velocity. The first instances of the interaction of two elastic waves inside a layer and the interaction of an elastic wave with a free surface are highlighted in the *X-t* diagram using a black and green circle, respectively. A red circle in Fig. 6 marks the interaction of two elastic waves at a material interface. The variation of stress and particle velocity with time is obtained at the impact interface (*X*=2*m*) and an arbitrary location (*X*=2.6 *m*, denoted by a red dashed line) in the layered medium.

The identical problem is also studied by doing a finite element analysis using ABAQUS to validate the results. A

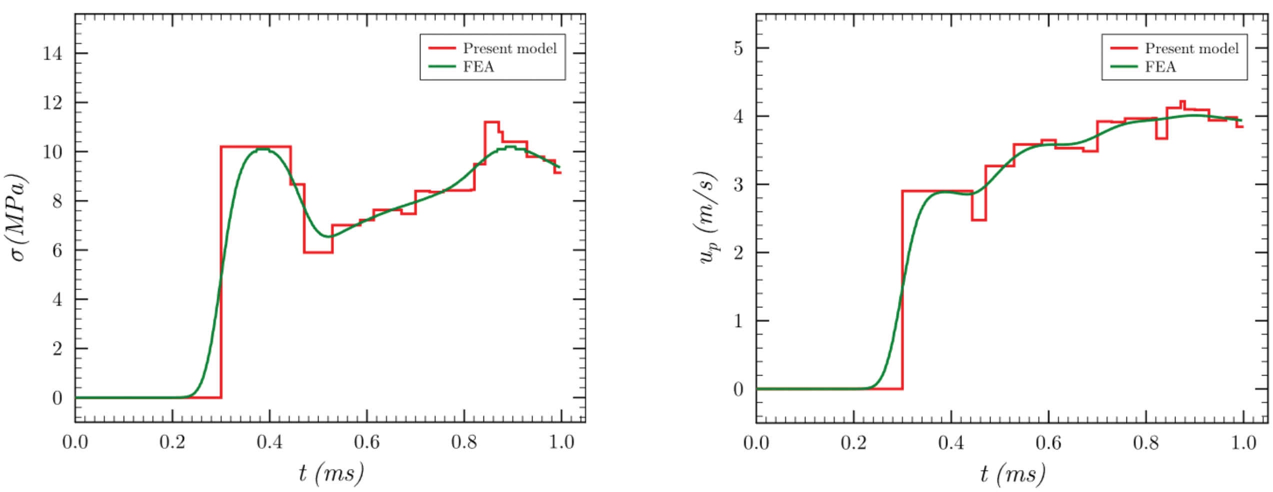
Table 1. Material properties of the impactor and different layers of the medium

Property (Units)	Impactor	L_1	L_2	L_3	L_4	L_5	L_6
Density (*kg/m³*)	1800	1000	1000	1000	1000	1000	1000
Wave speed (*m/s*)	4500	2800	1500	3500	2000	2500	1000
Thickness (*m*)	1	0.2	0.3	0.4	0.35	0.5	0.25



(a) (b)

Figure 7. (a) σ variation, (b) U_p variation with time obtained at the interface formed by the impactor and the layered medium (*X*=2*m*) using FE analysis and the present model.



(a) (b)

Figure 8. (a) σ variation, (b) u_p variation with time obtained at the *X*=2.6*m*, an arbitrary position inside the layered medium, using FE analysis and the present model.

three-dimensional element of type C3D8R with hourglass control and reduced integration is applied to represent the layered medium. Uniaxial strain behaviour is simulated by permitting the translation only along the *X* axis. The layered material's end face (*X*=4*m*) is kept free. The impactor's initial velocity is set to 5*m/s*. Table 1 lists the properties of the impactor and the layers of the medium used for the analyzed problem.

The variation of stress and velocity with time at *X*=2*m* and *X*=2.6*m* is procured. Figures 7(a) and 7(b) show the comparison of the stress and velocity value calculated from the FE analysis and the MATLAB code at *X*=2*m*, and Fig. 8(a) and 8(b) show the comparison of the stress and velocity value calculated from the FE analysis and the MATLAB code at *X*=2.6*m*, respectively. Both stress and particle velocity outcomes calculated from FE and the developed code match with reasonable accuracy at the interface between the impactor and layered medium and any position inside the layered medium. The magnitude and the pattern of variation in the stresses and the particle velocities are a good match; some changes that occur in short duration are not followed in the FE results as the finite element method can not take jumps in the material state due to wave interactions. The primary purpose for comparing the results obtained using the code with FE analysis is to verify the algorithm for the generalized case of an impact problem on an elastic layered medium.

5. CONCLUSIONS

In this research, the authors have examined the interaction between elastic waves and the interfaces formed by the layers of the medium to investigate the behaviour of an elastic layered media to a one-dimensional impact. The impact behaviour of the layered medium is determined by analyzing the interactions occurring in the impactor and the layered medium. A layered medium with layers having varying elastic impedances has shown the interaction of the elastic wave with the interface between the layers, producing reflected and transmitted waves. Exact analytical expressions for particle velocity and stress are obtained using the mass, momentum balance and constitutive relationship for each possible wave interaction happening in the impactor and the layered medium. The expressions are utilized in a computer program to study the impact behaviour by tracking each interaction in the impactor and the layered medium system. The results found using the present method are validated by investigating the identical problem using the finite element method. The present model uses expressions of stress and particle velocity to provide the exact solutions for the impact problem and also offers good computational efficiency over the finite element method as this algorithm does not require the mesh requirement over the whole domain, unlike FEM where the meshes following the CFL (Courant–Friedrichs–Lewy) criteria are required to get accurate results.

The proposed model can also be used for studying the mitigation of stress waves by using a layered medium with decreasing elastic impedance in the layers. The exact location in the layered medium where tensile stress is generated and whether that leads to a separation between the layers can also be studied using the present model. The work can be extended for the high-velocity impact cases that generate the stress above HEL, and only a shock wave is generated in the layered medium. There are some limitations to the current model. It finds the impact response of the layered medium with isotropic layers. The current version of the code is limited to one-dimensional analysis as tracking the waves and getting the material response in two and three-dimensional becomes complex. However, future projects can extend the present work to two-dimensional or three-dimensional impact conditions.

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