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Transfer Alignment for Space Vehicles Launched from a Moving Base

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ABSTRACT

Alignment of the inertial measurement unit (IMU) is a prerequisite for any space vehicle with self-contained navigation and guidance for any mission-critical application. Normally, inertial measurement unit is aligned through gyro-compassing using the stored data for heading. In case of launch from a moving base, it is essential to align the inertial measurement unit in the vehicle (slave unit) with that mounted on the moving platform (master unit). The master inertial navigation system is more accurate, stable, and calibrated wrt the slave unit. An error propagation system involving the misalignment between the master and the slave has been formulated involving the three misalignment angles, three velocity errors, and three positional errors. The manoeuvre of the moving base excites the sensors of both the master and the slave inertial navigation systems for the generation of data to be used in aligning the slave inertial measurement unit of the inertial navigation system (strapdown mode). The entire duration of manoeuvre has to be reduced to a minimum with minimum effort of manoeuvre. This involves the deployment of an adaptive estimator and a linear quadratic Gaussian regulator for alignment of the strapdown slave inertial navigation system.

Keywords: Space vehicles, navigation, guidance, launches, Kolman filter algorithm, strapped-down navigation algorithm, moving platform, inertial navigation system, launch platforms, transfer alignment scheme

1. INTRODUCTION

Simulation studies precede design and implementation, due to the real-time computational overhead at each phase of the alignment using strapdown navigation algorithm. Hence, two ways exist to remove the undetermined factors associated with this class of problem, viz. use of a preprogrammed trajectory with an initial stored heading, and dynamically adjustable trajectory for moving bases. Aircraft navigation uses the former one, as the flight course is well decided for any particular mission, where the preprogrammed manoeuvre generates data for the estimation of the errors between the master unit and the slave unit for every flight trial due to initial stored heading information. Other classes of space vehicles launched from the mobile bases suffer from the drawback of reduced accuracy due to the absence of estimation of the errors of initialisation

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of the vehicle inertial measurement unit wrt inertial measurement unit of the moving base. Normally, it is assumed that the master inertial navigation system is more accurate and less prone to errors. Dynamically adjustable trajectories are used to excite sensors for the minimum amount required for the purpose within the range of the moving base dynamics. Normally, slow movement with gradual change of velocity is ideally suited. For preliminary verification of the algorithm of transfer alignment, both the master and the slave inertial measurement units are put on the mobile van, which runs smoothly over a closed path. Thus, slowly varying velocity of the moving base is used for the estimation of errors between the master and the slave inertial navigation systems.

2. SYSTEM MODEL

Basic equation of motion in inertial frame is as follows:

$$\frac{dR}{dt} = V$$

$$\frac{dV}{dt} = A + g_m(R)$$

$$A = \frac{d^2R}{dt^2}|_I - g_m(R)$$

where R, V, A and g_m represents range, velocity, sensed acceleration, and modelled gravity component, respectively. Relationship between the inertial (I) and the earth-centered inertial (E) velocities (including Coriolis effect) may be expressed as

$$\frac{dR}{dt}|_I = \frac{dR}{dt}|_E + \Omega \times R = V + \Omega \times R$$

Here, Ω and V represent earth rate (inertial) and true velocity of the vehicle wrt the earth. On differentiation of the above equation along with substitution for earth-centred inertial velocities, one obtains:

$$\frac{d^2R}{dt^2}|_I = \frac{dV}{dt}|_I + \Omega \times (V + \Omega \times R)$$

Derivative of velocity V wrt inertial frame is related to the navigation frame by the following expression:

$$\frac{dV}{dt}|_I = \frac{dV}{dt}|_N + \omega \times V$$

$$\frac{d^2R}{dt^2}|_I = \frac{dV}{dt}|_N + (\Omega + \omega) \times V + \Omega \times (\Omega \times R)$$

where the acceleration components due to Coriolis effect and centripetal effect are obtained separately. So, the expression for A will be reduced to the respective components as

$$A = \frac{dV}{dt}|_N + (\Omega + \omega) \times V + (\Omega \times \Omega \times R) - g_m(R)$$

wherefrom an expression for effective gravity component involving earth rate is found as

$$g(R) = g_m(R) - \Omega \times (\Omega \times R)$$

The acceleration in navigation frame may be obtained from the accelerometer data in body frame as

$$\frac{dV}{dt}|_N = A - (\Omega + \omega) \times V + g(R)$$

$$= C_b^n A_b - (\Omega + \omega) \times V + g(R)$$

where $C_b^n A_b$ is the direction cosine matrix (body to navigation frame) transforming acceleration in body frame.

3. ERROR PROPAGATION EQUATION

The system equation variables changeover a wide range during the complete cycle of navigation. To frame the sets of equations amenable to methods of solutions in successive steps decided by the navigation process, error propagation relationship between the master and the slave inertial navigation systems is chosen to be the set of differential equations, for replacement of the original equations by their nonlinear time-varying coefficients. Thus, state space model for error propagation is invoked for the purpose. The steps involve perturbation of the dependent variables about their initial/ updated values. Velocity error propagation is obtained by perturbation of navigation equation as

$$\Delta V = \Delta A - \Delta[(\Omega+\omega)\times V] + \Delta g$$

Again, misalignment error in sensed acceleration (navigation frame) may be represented as

$$\Delta A = A\times\Psi + W_a$$

where W_a is the accelerometer noise and $\Psi = [\alpha\beta\gamma]^T$ represents the misalignment angles between the master and the slave inertial measurement unit orientations in North West Vertical (N-W-V) frame. Velocity error propagation is expressed as

$$\Delta\dot{V} = A\times\Psi - \Delta[(\Omega+\omega)\times V] + \Delta g + W_a \quad (1)$$

Misalignment vector propagation is given by

$$\dot{\Psi} = -\omega\times\Psi + \Delta\omega + W_g \quad (2)$$

due to rotation of navigation frame wrt inertial frame, and the presence of gyro noise, W_g in sensing angular rates. In N-W-V frame (locally levelled frame) the earth rate, ω_e is resolved as $[\omega_e \cos L, 0, \omega_e \sin L]^T$. Again, the vehicle with linear velocity components, V_N, V_W and V_Z wrt the earth (navigation frame) has the corresponding angular velocities as $(-V_W / R + \omega_e \cos L,\ V_N / R,\ -(V_W / R)\tan L + \omega_e \sin L)$. Hence, for an initial misalignment of $\Psi \neq$ the state equations involving time derivative of components of Ψ are as follows:

$$\begin{bmatrix}\dot{\alpha}\\ \dot{\beta}\\ \dot{\gamma}\end{bmatrix} = \begin{bmatrix}0 & -\gamma & \beta\\ \gamma & 0 & -\alpha\\ \beta & \alpha & 0\end{bmatrix}\times\begin{bmatrix}-V_W/R+\omega_E\cos L\\ V_N/R\\ -V_w\tan L/R+\omega_E\sin L\end{bmatrix}$$

$$+\Delta\begin{bmatrix}-V_W/R+\omega_E\cos L\\ V_N/R\\ -V_W\tan L/R+\omega_E\sin L\end{bmatrix}+\begin{bmatrix}W_{g1}\\ W_{g2}\\ W_{g3}\end{bmatrix}$$

$$\begin{bmatrix}0 & -V_W\tan L/R+\omega_E\sin L & -V_N/R\\ -(-V_W\tan L/R+\omega_E\sin L) & 0 & -V_W/R+\omega_E\cos L\\ V_N/R & -(-V_W/R+\omega_E\cos L) & 0\end{bmatrix}$$

$$\times\begin{bmatrix}\alpha\\ \beta\\ \gamma\end{bmatrix} + \begin{bmatrix}0 & -1/R & 0\\ 1/R & 0 & 0\\ 0 & -\tan L/R & 0\end{bmatrix}\times\begin{bmatrix}\Delta V_N\\ \Delta V_W\\ \Delta V_Z\end{bmatrix}$$

$$+\begin{bmatrix}-\omega_E\sin L & 0 & V_W/R^2\\ 0 & 0 & -V_N/R^2\\ -V_w/R^2+\omega_E\cos L & 0 & V_W\tan L/R^2\end{bmatrix}\times\begin{bmatrix}\Delta L\\ \Delta l\\ \Delta h\end{bmatrix}+\begin{bmatrix}W_{g1}\\ W_{g2}\\ W_{g3}\end{bmatrix} \quad (3)$$

The velocity error propagation components are expressed as follows in accordance with previously defined components:

$$M_1 = A\times\Psi = \begin{bmatrix}0 & -A_z & A_w\\ A_z & 0 & -A_N\\ -A_W & A_N & 0\end{bmatrix}\times\begin{bmatrix}\alpha\\ \beta\\ \gamma\end{bmatrix}$$

$$M_2 = \Delta[V\times(\omega+\Omega)]$$

$$= \Delta\begin{bmatrix}(-V_W^2/R)\tan L + 2V_W\omega_E\sin L - V_N V_z/R\\ -V_W V_Z/R + 2V_Z\omega_E\cos L + V_N V_W/R - 2V_N\omega_E\sin L\\ (V_N^2+V_w^2)/R - 2V_w\omega_E\cos L\end{bmatrix}$$

$$M_3 = \Delta g = \begin{bmatrix}0\\ 0\\ \Delta g_Z\end{bmatrix}$$

where

$$\Delta g_Z = \Delta\left[\frac{g_0}{(1+h/R)^2}\right]$$

$$\cong \Delta\left[\frac{g_0}{1+2h/R}\right] = -\frac{2g_0}{R}\Delta h$$

$$[\Delta\dot{V}_N, \Delta\dot{V}_W, \Delta\dot{V}_Z]^T = M_1 + M_2 + M_3 + M_{4_N} \quad (4)$$

Combining Eqns (3) and (4) after exclusion of $\Delta\dot{V}_Z$ and $\Delta\dot{h}$, the following state equation is obtained:

$$\dot{X} = FX + W$$

where

$$W = [W_{g1} \quad W_{g2} \quad W_{g3} \quad W_{a1} \quad W_{a2} \quad 0 \quad 0]^T$$

and

$$X = [\alpha \quad \beta \quad \gamma \quad \Delta V_N \quad \Delta V_W \quad \Delta L \quad \Delta l]^T$$

Accordingly, the measurement model is formulated as

$$Z = HX + v$$

where

$$Z = [\Delta V_N \quad \Delta V_W \quad \Delta L \quad \Delta l]^T$$

and the relationship between the state vectors and the elements of measurement matrix are given by H, with measurement noise components of the reference unit as v.

The block diagram (Fig. 1) shows implementation of the transfer alignment scheme citing the application of the Kalman filter for the estimation of states and filtering the noise-corrupted measurements. It is mentioned that the correctness of the estimation depends mostly on the manoeuvre profile (simulated trajectory) indicating the acceleration components.

The conventional Kalman filter algorithm often fails in such cases due to loss of the observability property of the state propagation matrix. The robustness of the system may be enhanced using the actual parameters of the kinematics (truth model using nonlinear time-dependent simultaneous equations) instead of

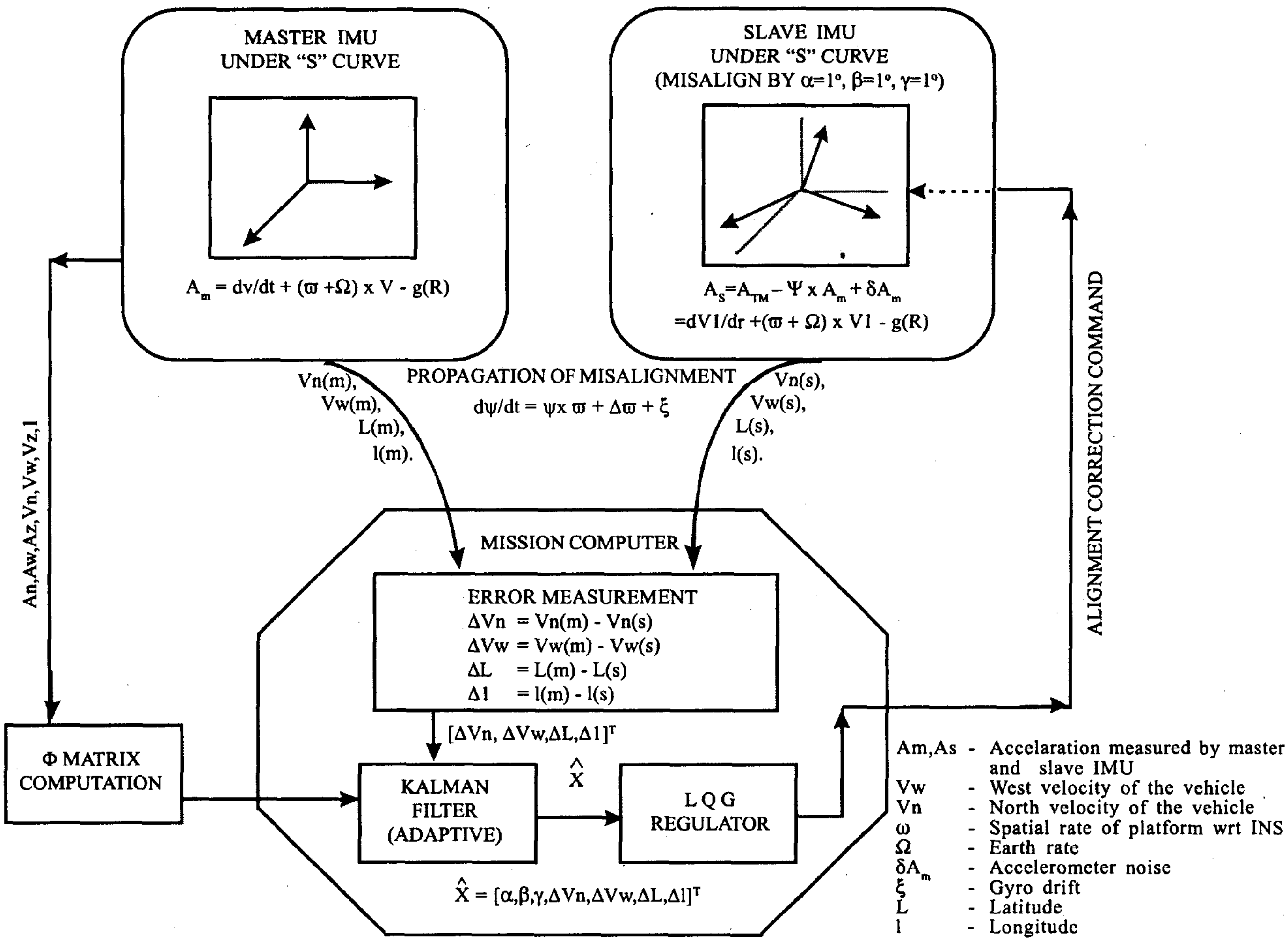

Figure 1. Kalman filter application in estimation filtering–transfer alignment

watching the propagation of the misalignment errors between the master and the slave navigation units[1].

4. FORMULATION OF KALMAN FILTER ALGORITHM

The Kalman filter, which has been implemented consists of non-real time initialisation of the measurement matrix, process noise covariance, measurement noise covariance matrices, and error covariance matrix, etc, followed by real-time Kalman cycles. The present scheme of transfer alignment is based on matching of position and velocities from the master and the slave inertial measurement units as shown in Fig. 1. The state equations and the measurement model in the discrete form may be rewritten as

$$X_{k+1} = \Phi X_k + (G.\Delta t).W + c$$

$$Z_k = H.X_k + v$$

where suffix K, K+1 represent consequent Kalman cycles, and

$$X = [\alpha, \beta, \gamma, \Delta V_0, \Delta V_N, \Delta L, \Delta l]$$

is the error state vector. $G, \Delta t, \Phi$ represent the disturbance matrix, time of one update cycle, state transition matrix between K and K+1 instants. W and v correspond to the plant noise and measurement noise vectors (white noise), respectively which are uncorrelated with each other and represent the misalignment error correction obtained by multiplying the estimated state vector X with the control matrix. The estimation error covariance matrix, P_K is updated as

$$P_{k+1}^{-} = GQG^T + \Phi_K P_K^{+} \Phi^T$$

where Q is the error covariance for the plant noise. Kalman filter gain, K is given as

$$K = P_K^{-} H^T (H P_K^{-} H^T + R)^{-1}$$

where R is the measurement noise covariance. Error states in the beginning of any Kalman filter cycle (after measurements) is updated using state transition matrix for the states at the end of the cycle (before the next measurement) as $X_{K+1}^{-} = \Phi_K X_K^{+}$. This being an *a priori* estimate, is updated to an *a posteriori* estimate as

$$X_{K+1}^{+} = X_{K+1}^{-} + K_K (Z_K - H_K X_{K+1}^{-})$$

It is worthwhile to note that the scheme of simulation as shown in Fig. 1 is not robust in the sense that the algorithm for estimation of the Euler angles may not converge at low values of acceleration in level plane within specified time due to the

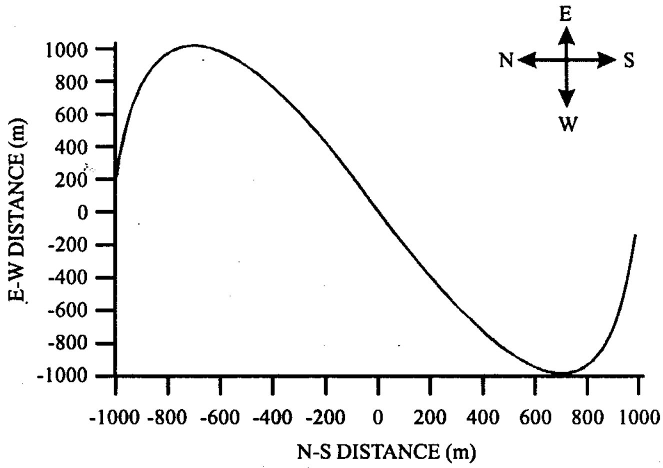

Figure 2. Manoeuvring profile

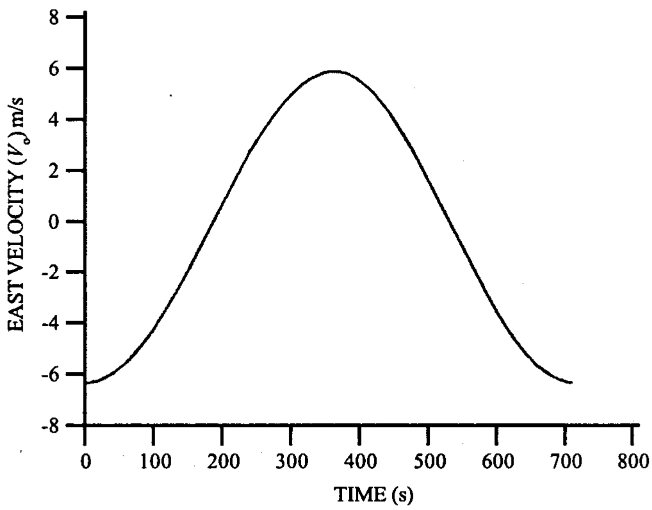

Figure 3. Manoeuvring profile

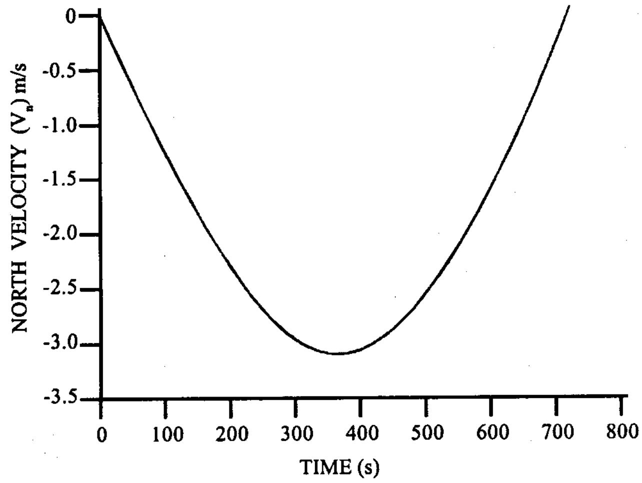

Figure 4. Manoeuvring profile

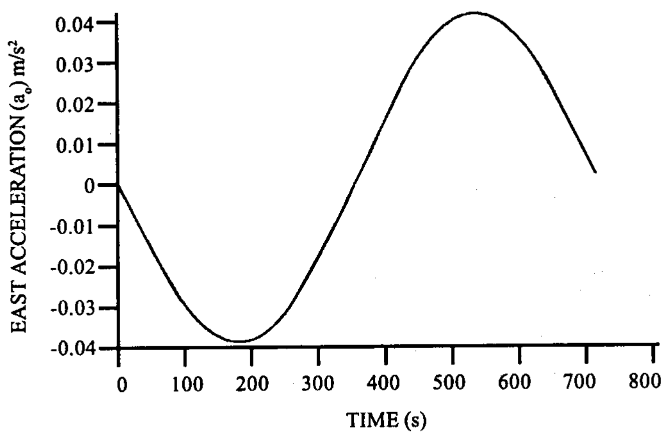

Figure 5. Manoeuvring profile

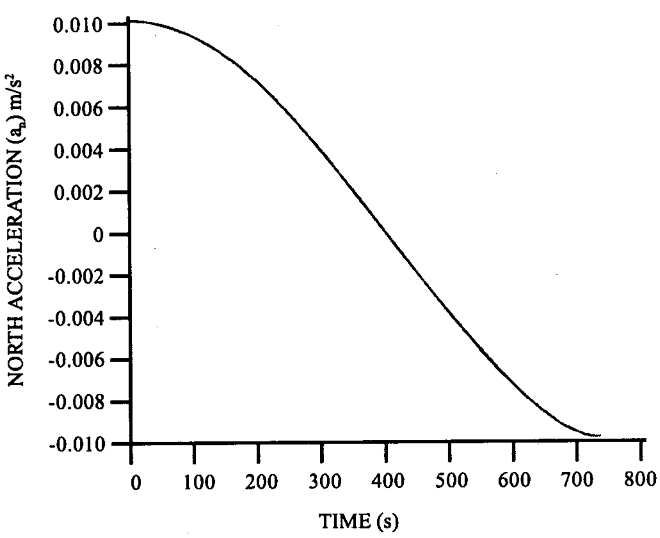

Figure 6. Manoeuvring profile

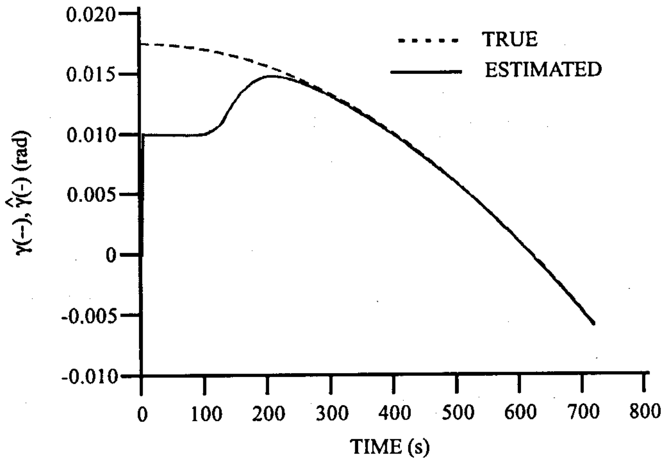

Figure 7. Propagation of γ (true) and $\hat{\gamma}$ (estimated) (open loop)

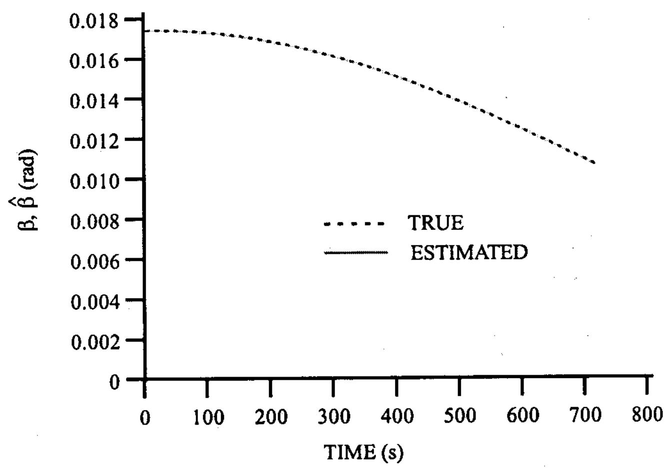

Figure 8. Propagation of β (true) and $\hat{\beta}$ (estimated) (open loop)

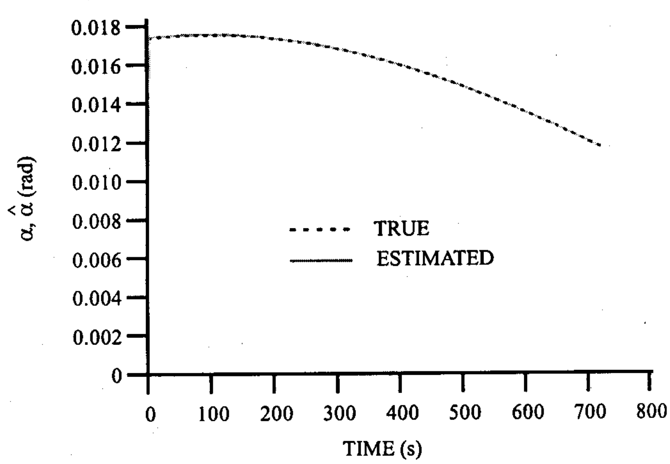

Figure 9. Propagation of α (true) and $\hat{\alpha}$ (estimated) (open loop)

presence of noise in the sensors. To alleviate such problems, there are two kinds of solutions, viz, extended Kalman filter approach[1] or the adaptive Kalman filter method employing the adaptation using the first-order moment of the measurement noise.

5. ADAPTIVE KALMAN FILTER METHOD

Some techniques such as fictitious noise injection, exponential data weighting, etc have been found to be effective for controlling the divergence of the Kalman filter by preventing the Kalman gain from approaching towards zero with suitable modification of the predicted error covariance P_K^- with increment of K. If residuals, ie, deviations of predicted measurement from the actual measurement are sufficiently small and consistent with their predicted statistics, then the Kalman filter is deemed to be operating satisfactorily. Rather than analysing residuals, on execution of the Kalman filter performance, this adaptive Kalman filter model provides feedback from residuals in real-life situations, in terms of system noise input levels. This degrades the actual estimation of error covariance matrix, increases the Kalman filter gain, making the Kalman filter sensitive to incoming data[2].

An empirical noise estimator for the noise statistics is derived in a batch form under assumption of constant values for q_i, r_i, Q_i, R_i, where $E[W_i]$, $E[v_i]$ represent q_i, r_i as the true means, respectively, whereas

$$E[(W_i - q_i)(W_j - q_j)^T] = Q_i \delta_{ij}$$

and

$$E[(v_i - r_i)(v_j - r_j)^T] = R_i \delta_{ij}$$

are the expressions for the true moments about the mean of the state and the observation noise sequences, respectively. The recursive equation for the Kalman filter gain algorithm is

$$K_K = P_K^- H_K^T (H_K P_K^- H_K^T + \hat{R}_K)^{-1}$$

$$\hat{X}_K^+ = \hat{X}_K^- + K_K[\hat{R}_K + r_k]$$

$$P_K^+ = (I - K_K H_K) P_K^-$$

6. RESULTS

The following results were obtained on simulation with van trial following a circular path. It has been found to have convergence with a very low acceleration (Figs 2 to 9). It is interesting to note that the azimuth misalignment (γ) could also be estimated within a reasonable error of a minute within an estimation time period of 400 s with peak-to-peak acceleration as low as ± 0.1 m/s^2 in the level channel.

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