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Predicting the Dynamics and Trajectory of a Projectile Using a Six Degrees of Freedom Model

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ABSTRACT

A numerical code is developed based on a six-degree of freedom (6-DOF) model to examine the trajectory of flight and dynamic behavior of a mechanically guided projectile. The coupled differential equations are solved following the one-step, fourth order Runge-Kutta scheme. The novelty lies in its application and its comprehensive and detailed representation of projectile dynamics. A self-developed 120mm mortar round is used as a representative projectile for the work. The analysis is carried out for different muzzle velocities (V_m) [102, 165,192,250, and 318] m/s at different Quadrant Elevation (QE) angles 45^0, $65^{0,}$ and 85^0 to predict the path behavior such as maximum range and altitude, time, drift and pitch. The study is further extended to understand the effect of wind velocities (V_w) on projectile trajectory from different directions. The developed model can accurately predict the flight trajectory, range, time, and dynamic motion of the projectile during its time of flight.

Keywords: Guided projectile; 6-DOF; 120 mm mortar; Numerical code

1. INTRODUCTION

The development of an accurate flight dynamics model to track precise trajectory of a projectile has been an area of major concern for both weapon designers and researchers in the field of external ballistic. Earlier works for rapid trajectory prediction were mostly based on the approximate linear theory[1-3]. However, the linear theory based model does not accurately represent the system under nonlinear behavior. Several models have been developed so-far over the years for predicting the flight trajectory such as vacuum trajectory model, point mass model, modified point mass model. However, every model fails to cumulatively capture the effects of all the forces and moments affecting the projectile trajectory. This leads to the development of six-degree of freedom (6-DOF) projectile tracking model. The 6-DOF associated with this model are three translational motion and three rotational motion namely pitch, yaw and roll. It tracks the projectile in two inertial frames, one is the frame of reference of the projectile and the other is the frame of reference from the coordinate system moving with the projectile.

The 6-DOF model proposed by Fowler[4], *et al.* found to be the most accurate out of the several methods introduced so-far. Later, this model was made more rigorous. Rao[5] studied a 6-DOF trajectory to optimize the launching parameters of titan launch vehicle. The program NPSOL 4.0 was used to solve this model which uses a projected Lagrangian formulation with sequential quadratic programming. Gkritzapis[6-7] performed a study for the application of 6-DOF and modified point mass model in 7.62 mm sierra bullet. Later, he compared the results of simulated ranges and maximum height of point mass model, modified point mass model and 6-DOF model.Thuresson[8] performed a comparative analysis of 6-DOF and point mass model trajectory for 155 mm artillery projectile. Altufyuayl[9] developed 6-DOF trajectory simulation model for asymmetric projectile and simulated for 155 mm M107 projectile. The results were compared with PRODAS-V3 Program. It was found that the developed model has slightly higher range than predicted byPRODAS-V3 program. Zipfel[10] performed aerospace vehicle dynamics study, which later become the base to develop BALCO[11] flight dynamics model. BALCO is now a standard code for six degree of freedom model which is used within NATO. Though it is only limited to symmetric projectile but it gives an excellent benchmark for 6-DOF simulations. While performing the literature survey it was evident that a robust model to perform trajectory simulation is not available in any public domain. Prodas is the only program available that is mostly used for performing simulations, but not easy to procure or unavailable in many countries. Other developed 6-DOF models give good results but they lack the integration of many forces and moments in the equation of motion. Ange du [12] reported a comparative study between the 6-DOF model and point mass model. Gorecki[13] developed a generic 6-DOF model for missile. McCoy[14] performed a detailed study of the exterior ballistic process and performed experiments and numerical studies for different projectile simulation models. The work in this paper reports the development of a comprehensive 6-DOF model to predict the projectile flight dynamics of indigenously designed 120 mm mortar.

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The developed numerical code is based on the Fortran programming language. The code is developed to aid the indigenous ballistic research and development work that's being done in the institute. Here's an outline of the key aspects that contribute to its novelty:

1.1 Comprehensive Motion Representation

Unlike simpler models that consider only a subset of the projectile's motion, a 6-DOF model accounts for translation in three axes (x, y, z) and rotation about three axes (roll, pitch, yaw). This allows for a complete depiction of the projectile's trajectory and orientation throughout its flight.

1.2 Realistic Aerodynamic Forces and Moments

Aerodynamic Coefficients: The model typically incorporates detailed aerodynamic coefficients that vary with the projectile's velocity, angle of attack, and sideslip angle. These coefficients are often derived from empirical data, wind tunnel tests, or Computational Fluid Dynamics (CFD) simulations.

1.3 Nonlinear Effects

The 6-DOF model can include nonlinear aerodynamic effects, such as cross-coupling between translational and rotational motions, which simpler models often neglect

1.4 Gravity and Coriolis Effects

The model includes the influence of gravity and, if necessary, the Coriolis force due to the Earth's rotation, providing more accurate long-range predictions.

1.5 Wind and Atmospheric Conditions

The model can incorporate varying wind conditions and atmospheric properties (density, temperature, pressure) along the flight path, enhancing its realism.

1.6 Shape and Mass Distribution

The model can account for the specific geometry and mass distribution of the projectile, which significantly affects its flight dynamics. This includes the effects of fins, canards, or other control surfaces.

1.7 Spin and Gyroscopic Effects

For spinning projectiles, the 6-DOF model includes gyroscopic effects, which influence stability and accuracy.

1.8 Trajectory and Orientation Data

The model provides detailed outputs, including the projectile's position, velocity, orientation, and angular velocity at each time step, which are critical for analysing and optimising performance.

1.9 Versatility

The model can be applied to various projectiles, from small arms bullets to artillery shells and guided munitions, making it versatile.

2. METHODOLOGY

The 6-DOF model considers the projectile as a rigid body, considering both translational and rotational movements. By incorporating three rotations (pitch, yaw, and roll) and three translations (surge, sway, and heave), the model provides a comprehensive representation of the projectile's motion. The most complex part of the code is to model the cumulative effect of all the forces and moments. Two equations of motion are derived that collectively take care of all the forces and moments, which are later expanded into six equations for every degree of freedom. Effect of Earth rotation is also one of the major problem that arises while developing a model to track projectiles. Earth rotation plays a major role since the long range projectile can hit off the target due to the Earth rotation. Direction and coordinate of projectile also play an important role in this aspect. The influence of the Earth rotation on the projectile is taken care by the Coriolis force. The details of the mathematical formulation along with the initial conditions are available in McCoy[14] and are not repeated here for the purpose of brevity. The 6-DOF code consists of several coupled differential Eqn.[14] which are solved simultaneously. One-step, fourth order Runge-Kutta method is used[15] to obtain the optimum solution.

3. VALIDATION OF RESULTS

The 6-DOF model is validated by comparing results of 105 mm spin stabilized projectile with McCoy's flight dynamics model[14].The simulation is done for muzzle velocity (V_m) of 205 m/s with twist rates 1/18 and 1/25. Twist rate indicates inches per turn i.e. 1/18 twist in the barrel will spin the projectile one revolution in 18 inches. Figure 1 illustrate the projectile trajectory of 105mm projectile at V_m of 205 m/s for Quadrant elevation (QE) angles of 45° and 70°. The maximum range achieved by the projectile at QE 45 degree is 3781 m and the maximum altitude achieved is1004 m, while for QE 70°, the range achieved is 2327 m and maximum altitude is 1756 m. The McCoy model shows similar trajectory for the same V_m and QE angle with maximum range of 3760 m and altitude of 1000 m for QE 45° and maximum range of 2320 m and altitude of 1750m for QE 70°. Figure 2 (a,b) shows the variation of drift

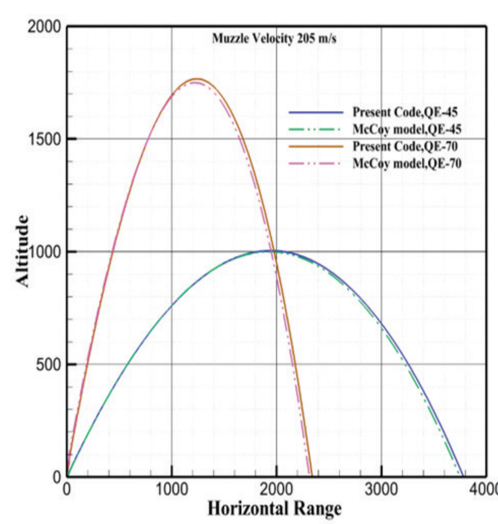
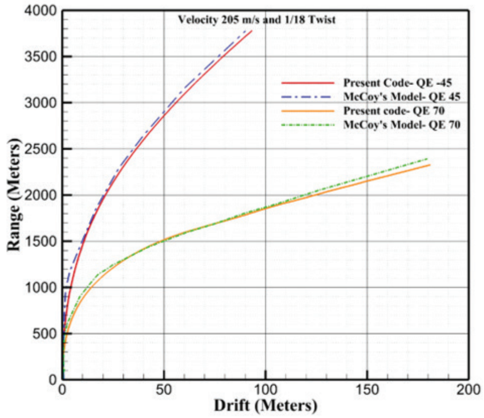
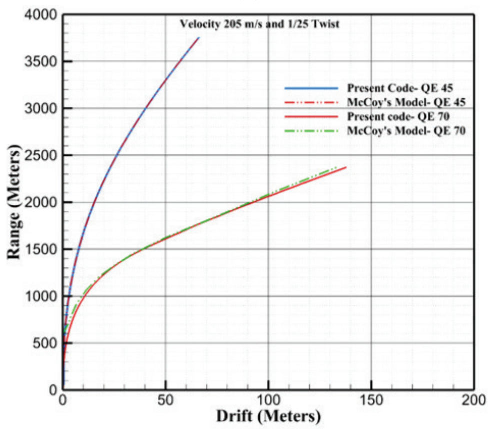


Figure 1. Altitude and range achieved by the projectile for muzzle velocity of 205 m/s and QE = 45°, 70°.



(a)



(b)

Figure 2. Drift and range for and V_m = 205 m/s for QE = 45°, 70°, (a) 1/18 twist, and (b) 1/25 twist.

with range for different twist (1/18 and 1/25) at V_m of 205 m/s. The derived plots are compared with McCoy's flight dynamics model and the data shows very similar results for 1/18 twist (Fig. 2(a)) and almost identical trajectory for 1/25 twist (Fig. 2(b)). The maximum deviation in the range is found as 21 m, which is 0.5 % different from the available McCoy's data.

Table 1. Physical data of 120 mm mortar

Reference diameter	119.6 mm
Projectile total length	538.72 mm
Projectile weight	14.262 kg
Axial moment of Inertia	0.025568 Kg-m^3
Transverse moment of Inertia	0.191042 Kg-m^3
Centre of gravity	321.23 cm from the base

4. RESULTS AND DISCUSSION

This study aims to numerically predict the flight trajectories and dynamic behavior of a projectile (mortar) through a comprehensive 6-DOF model. Physical data of the mortar are obtained by self-developed full-fledged model. Figure 3 portrays the model of the projectile and Table 1 illustrates the physical specification of the model. Table 2 depicts data of the aerodynamic coefficients for the projectile.

Simulation is performed for different muzzle velocities and quadrant elevation angles. Maximum range (total distance travelled) and maximum altitude are calculated along with the total time of flight and path of the projectile is traced by the 6-DOF code. Dynamic behavior of the projectile during its course of action is also studied. The representative projectile is a fin stabilised projectile with un-canted fins and there is no roll and spin about the axis for the entire duration of flight. Initially, the wind velocity is taken as zero in all directions and the effect on projectile trajectory due to wind is studied separately. The projectile is fired from equator i.e. latitude 0^o at an azimuth of 90^o i.e. along the equator line. The initial yaw of the projectile is 3° while the initial angle of attack is 0°.

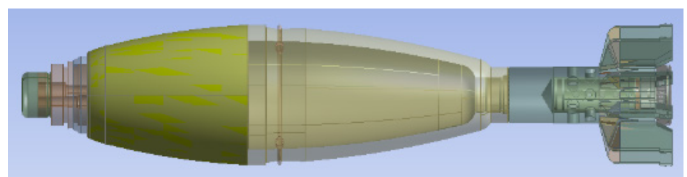


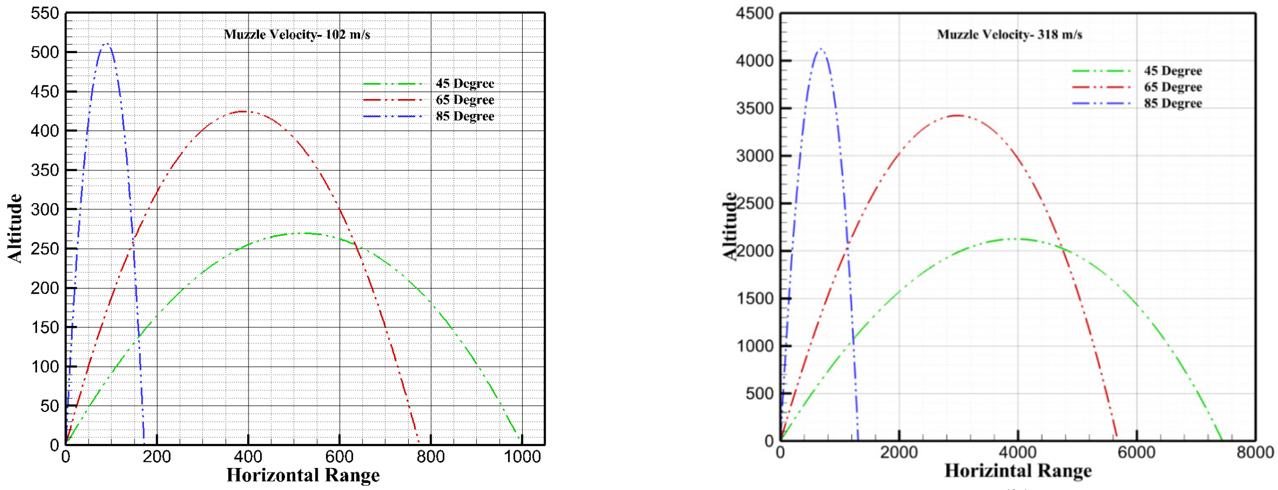
Figure 3. 120 mm projectile model.

4.1 Prediction of Projectile Motion

Figure 4(a,b) showcases the horizontal range and maximum altitude obtained at different muzzle velocities and quadrant elevation angles. It is evident that the range and the altitude achieved are function of quadrant elevation angle and muzzle velocity. With increase of QE angle at constant muzzle velocity, horizontal range of the projectile decreases while the altitude of the projectile increases. More the velocity, larger are the range and the altitude. The projectile shows parabolic path with peak at midst of flight (maximum elevation) and starts descending towards the ground. The maximum range is found to be 7447 m for V_m of 318 m/s at QE angle of 45^o. The maximum altitude obtained is 4123 m for QE 85^o at V_m of 318 m/s. Minimum range of projectile is found to be 172m at V_m of 102 m/s and QE angle of 85^o, that gives an impact range of 7275 m.

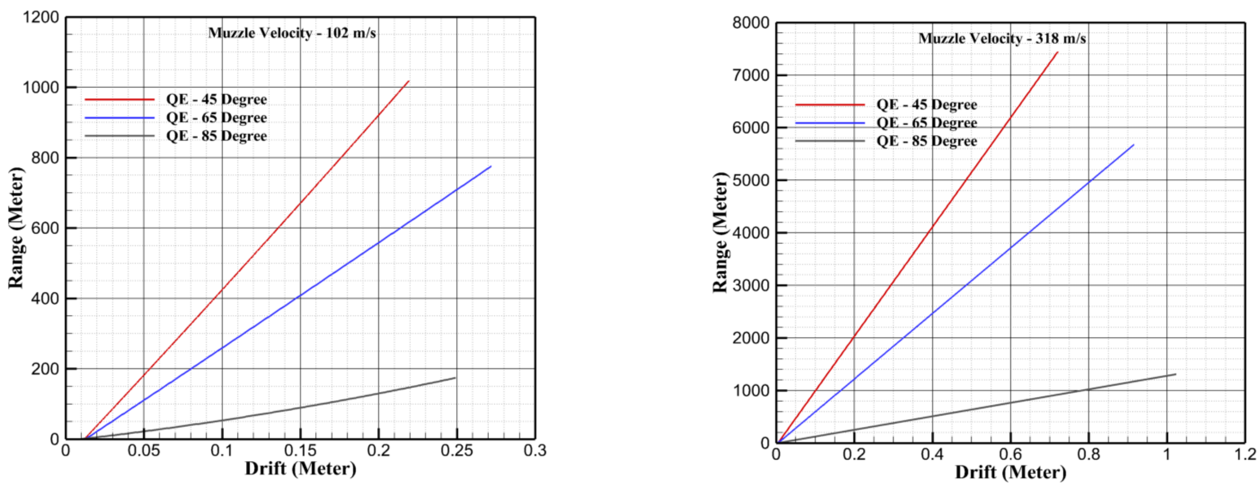
Table 2. Aerodynamic coefficients of 120 mm mortar at different Mach numbers [14]

Mach No.	$C_{D_{\partial 0}}$	Mach No.	$C_{D_{\partial^2}}$	Mach No.	$C_{L_{\alpha 0}}$	Mach No.	$C_{L_{\alpha 2}}$	Mach No.	$C_{M_{\alpha 0}}$	Mach No.	$C_{M_{\alpha 2}}$
0	0.119	0	2.32	0	1.75	0	14.8	0	-0.2	0	-15.1
0.7	0.119	0.4	2.44	0.6	1.95	0.5	14.8	0.4	-1.02	0.45	-15.1
0.85	0.120	0.6	2.66	0.8	2.02	0.6	4.5	0.6	-1.62	0.6	-12.7
0.87	0.122	0.7	2.87	0.9	2.06	0.63	1.4	0.8	-2.41	0.7	-8.5
0.90	0.126	0.75	3.01	0.95	2.08	0.7	0.4	0.9	-2.72	0.75	-4.5
0.93	0.148	0.85	3.55	-	-	0.8	8.8	0.92	-2.75	0.8	1.5
0.95	0.182	0.90	4.03	-	-	0.9	28.3	0.95	-2.71	0.85	13.9
-	-	0.95	5.20	-	-	0.95	40	-	-	0.90	30.2
				-	-	-	-	-	-	0.95	59.9



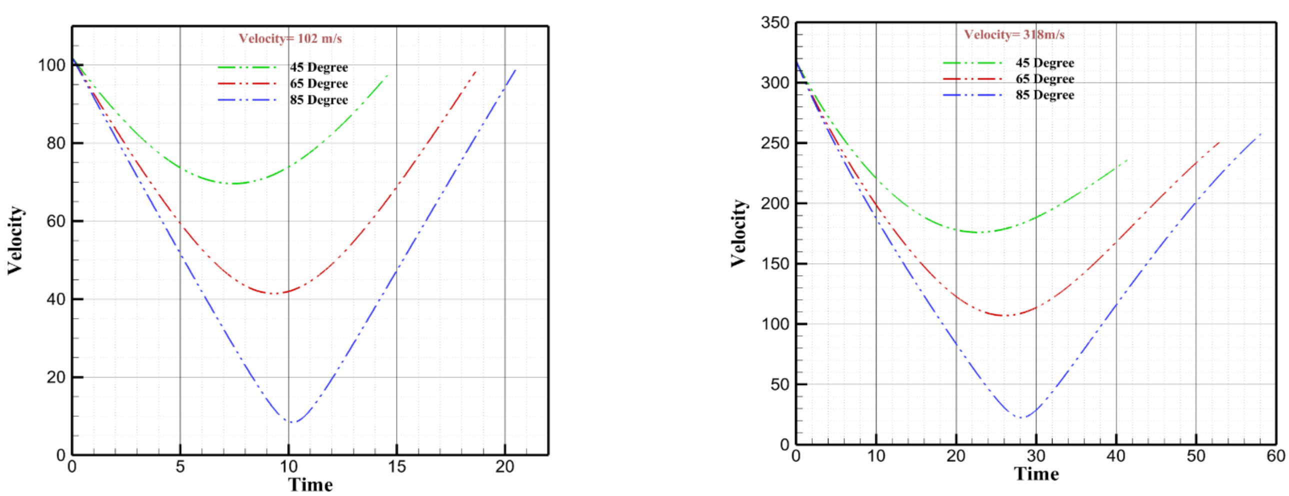
(a) (b)

Figure 4. Altitude and range at different Vm and QE angles, (a) Vm =102 m/s; and (b) Vm =318 m/s.



(a) (b)

Figure 5. Drift and range at different Vm and QE angles, (a) V_m =102 m/s, and (b) V_m =318 m/s.



(a) (b)

Figure 6. Velocity profile at different V_m and QE angles, (a) V_m =102 m/s; and (b) V_m =318 m/s.

Figure 5(a,b) shows the lateral motion (drift) of the projectile i.e. the motion of the projectile in the z-direction. Fin stabilized projectile doesn't have spin due to which spin induced forces and moments don't act on it. Therefore, the drift in fin stabilized projectile is very less compared to spin stabilized projectiles and is mostly due to yawing motion of the projectile. Spin stabilized projectile have curve trajectory (from top view perspective), while the fin stabilized projectile is linear as can be seen from Fig. (6). For a constant muzzle velocity, the drift is directly proportional to the QE angle. The maximum drift (1.02 m) can be seen for the maximum muzzle velocity and the QE angle is 85°. It is also noticeable that with increase of the muzzle velocity, drift also increases.

Figure (6) shows the velocity profile of the projectile. Velocity profile depicts the velocity variation with respect to total time of flight.

Velocity profiles at two extreme muzzle velocities i.e. 102 m/s and 318 m/s are plotted with their respective time of flight at different QE angles. The velocity starts decreasing as it gathers altitude and is least at the maximum altitude of the flight. It pivots the trend and gradually starts increasing till it reaches the impact point i.e. the target. Figure (6) shows a valley type plots for velocity-time variation, and the valley gets deeper with increase of QE angle. The projectile loses kinetic energy with the gain in elevation. At the peak elevation, the KE becomes zero, and the projectile starts descending due to the gravitational pull. The velocity of projectile increases till the point of impact. With increase of the QE angle, the velocity of projectile steeply decreases till it reaches the maximum altitude and then steeply increases till the point of impact.

The velocity at the point of impact is termed as the impact velocity of the projectile and Fig. 7 shows the variation of the impact velocity with variation of QE angle. It can be seen

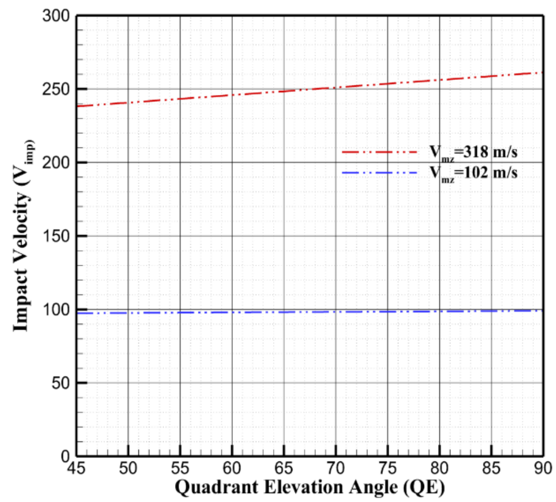
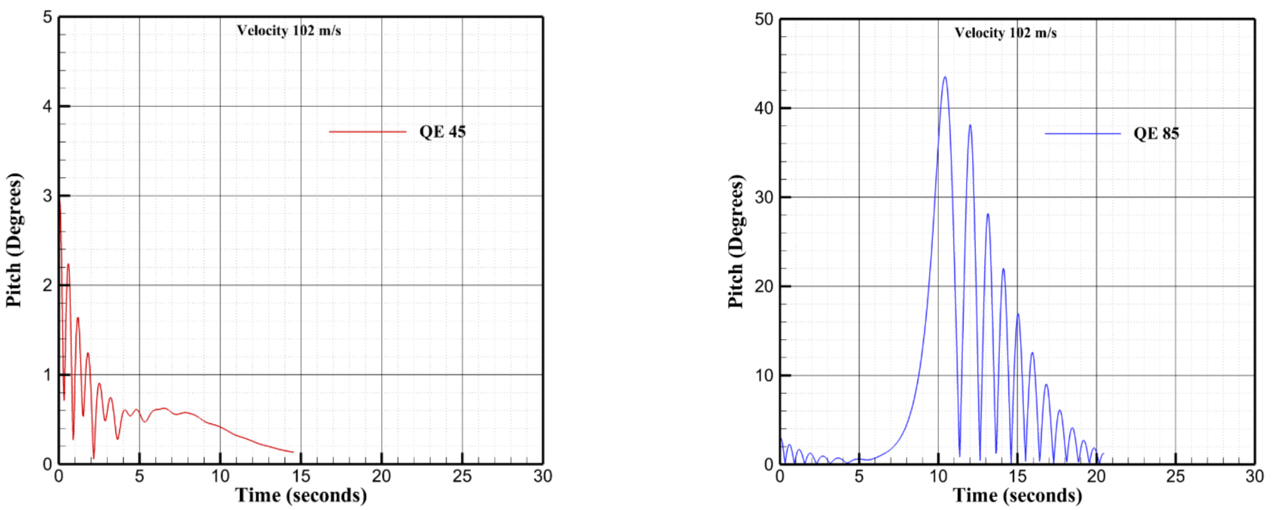
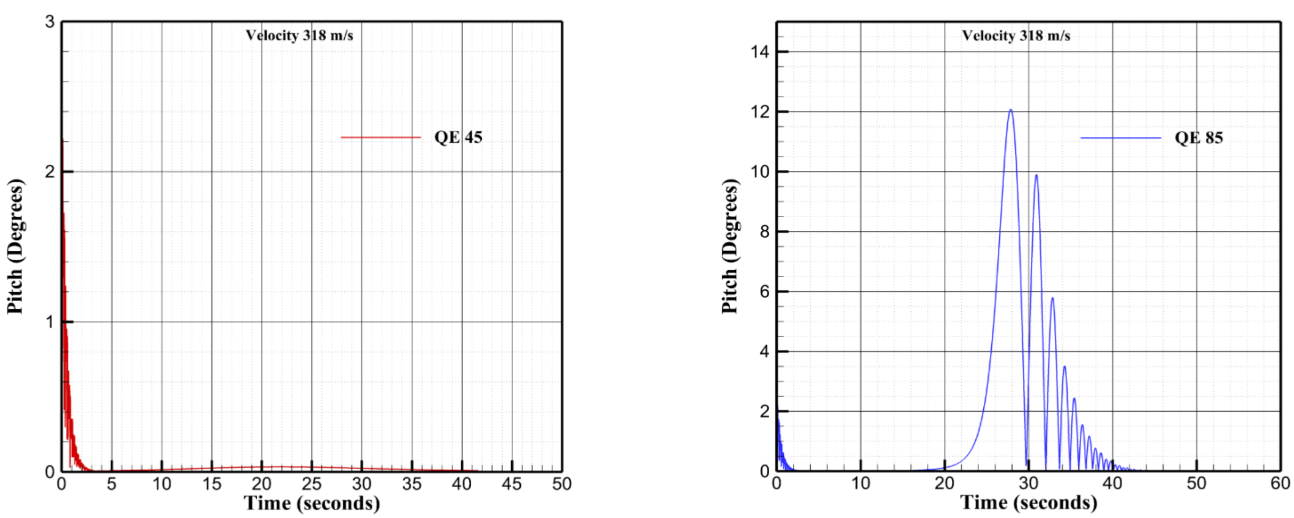


Figure 7. Impact velocity for different V_m and QE angles.



(a) (b)

Figure 8. Pitch during the time of flight for V_m=102 m/s at different QE angles. (a) QE = 45°; and (b) QE = 85°.



(a) (b)

Figure 9. Pitch during the time of flight for V_m = 318 m/s at different QE angles; (a) QE = 45°; and (b) QE = 85°.

that with increase of the QE angle at constant V_m the impact velocity also increases. It is observed that the impact velocity of the projectile always remains much lower than the muzzle velocity. This is due to the motion of the projectile under the influences of various aerodynamic forces. The maximum impact velocity is found to be 258 m/s for muzzle velocity 318 m/s at QE 85°.

Projectile moving freely in air under the influence of aerodynamic forces shows a parabolic trajectory with constant change in the angle of attack and velocity throughout the flight. Figures 8,9 show the variation of the pitch in degrees during the time of flight for V_m = 102 m/s and 318 m/s at QE angles of 45°and 85°. It can be observed that as the QE angle increases, the maximum amplitude of the wave motion also increases. It remains below 3° for QE 45° but for higher QE angle of 85° it is about 43.5° as can be seen from Fig. 8.

It is observed that for higher muzzle velocity, the variation of pitch is less and for lower QE angles it shows a damping wave motion and becomes more stable after a point in time. Figure 9 depicts that the projectile is very much stable and shows high frequency and low amplitude of pitch for lower QE angles. The sudden spike of amplitude is also seen but it is comparatively very less for lower QE angle and for higher QE angle the maximum amplitude is about 12° and thereafter damping of the wave starts till it is damped completely. It is also observed that at higher V_m the pitch variation of the projectile is completely damped for all QE angles. It is also observed that with increase of the muzzle velocity, the maximum amplitude of pitch decreases and this trend is true for all QE angles.

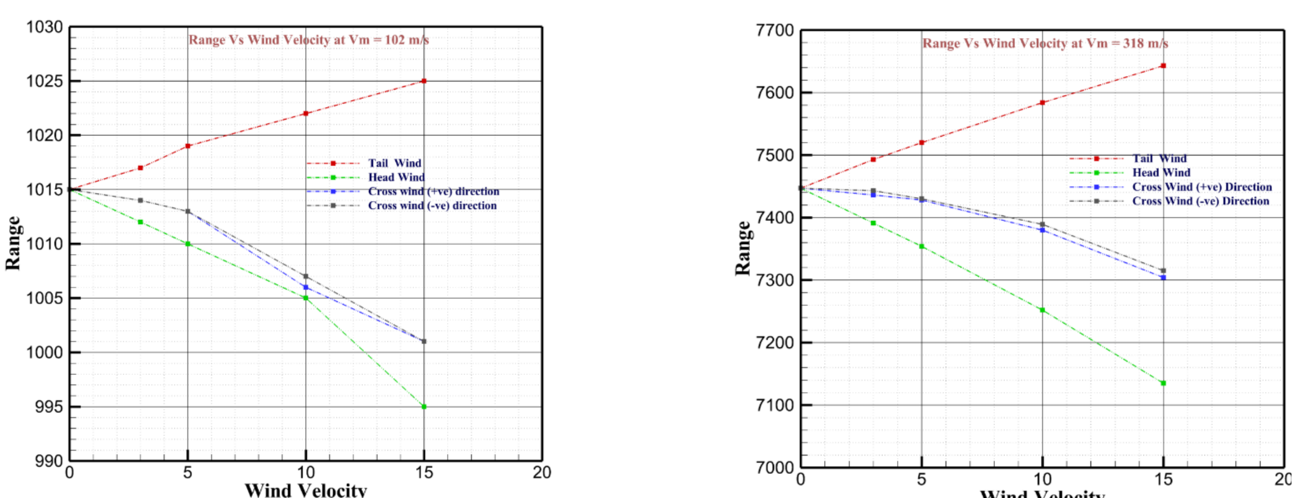
4.2 Effect of Wind

Wind is the motion of air mass through which projectile is travelling. Wind velocity can be categorised depending on the direction of wind. If the wind is flowing in the same direction that of the projectile it is termed as tail wind.

If the wind direction is in opposite to that of the projectile, the wind is termed as head wind. Lateral direction wind is called cross wind while wind in the direction perpendicular to the motion of projectile is termed as up and down wind. Tail wind tends to over range projectile while head wind resists the motion of projectile hence it under range. Cross wind adds an element of lateral force which drifts the projectile in the direction of wind. Figure 10 shows the effect of wind from different directions at various wind velocities on the range of the projectile for different muzzle velocities. It is observed that the range increases with the increase of wind velocity in the case of tail wind, but in all the other cases the range decreases.

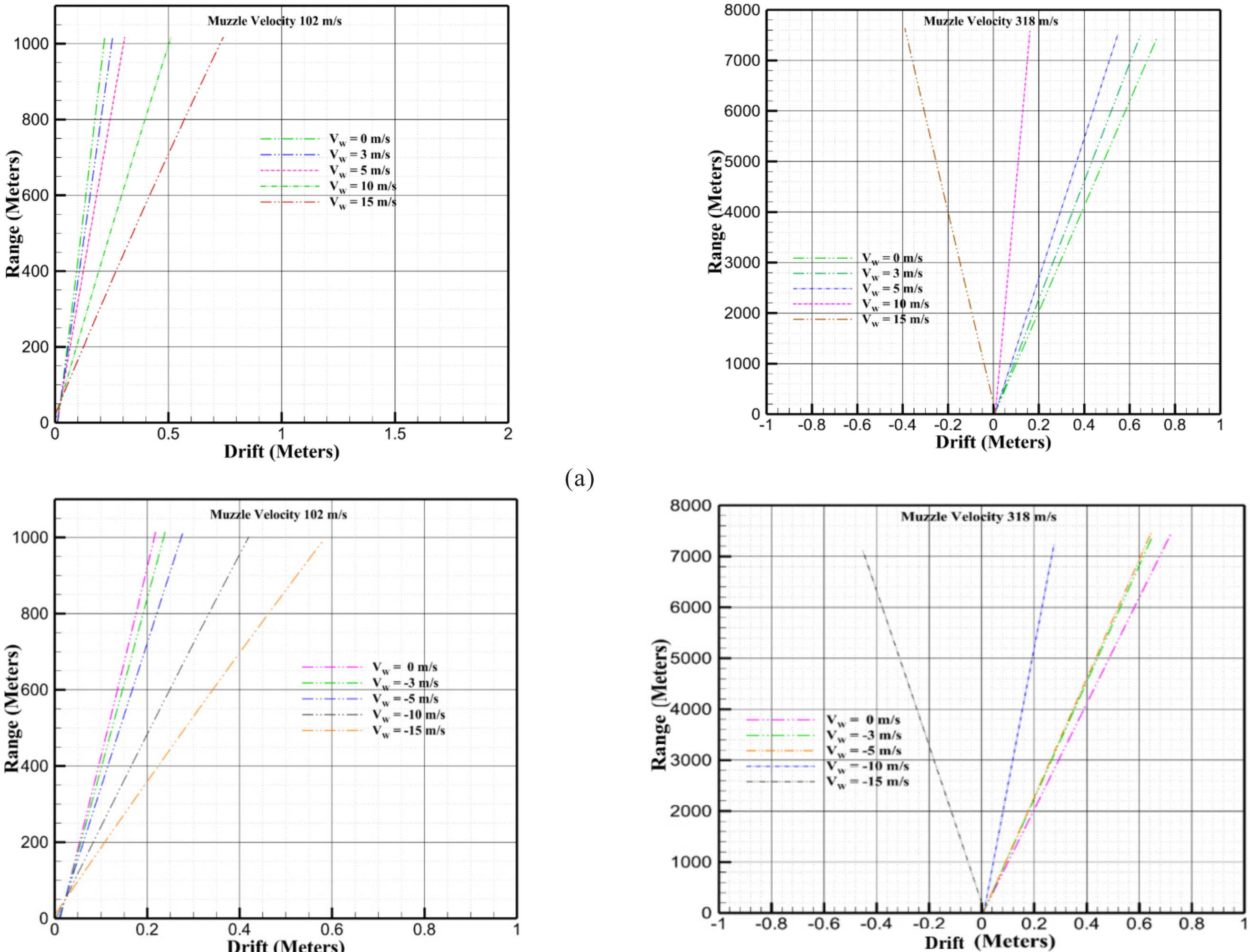
Figure 11(a) shows the induced drift due to tail wind. It is observed that tailwind over ranges the projectile and as the wind velocity increase so does the over ranging. In case of lower muzzle velocity, the difference in range is not that high but with higher muzzle velocity the difference in range is quite significant. It is due to decrease of drag on the projectile. When the projectile is fin stabilised the effect of wind is more on the fin than that of the body. Drag is function of projectile velocity with respect to air in which it is moving, so the movement of air causes the variation in drag. Tail wind creates a zone of high pressure on the base of the projectile that coincides with the low pressure zone of base drag, due to which there is a reduction in overall drag in case of tail wind. As a consequence, the projectile moves faster under the influence of the aiding tail wind. Hence, over ranging of the projectile is observed in case of tail wind.

Figure 11(b) shows that the head wind under ranges the projectile. For higher V_m the difference in ranges is even higher. Head winds impacts the projectile range significantly. At higher V_m, fins generate a side force that seems to dampen the drift, but at high wind velocity the side force becomes more dominating and the projectile starts to drift in the direction of side force. At lower V_m, the drift seems to be increasing with the wind velocity due to weak side force, which is dominated by wind effect on the projectile. It is observed that with the inclusion of wind velocity, the maximum altitude increases initially, but with further increase of wind velocity, maximum altitude decreases. Drag acts as a stabilizing agent in fin stabilised projectile and head wind aids the base drag which leads the projectile nose to resist the change of trajectory along the motion.



(a) (b)

Figure 10. Variation of range with wind velocity; (a) V_m = 102 m/s; and (b) V_m = 318 m/s.



(b)

Figure 11. Range vs drift at various (a) tailwind and (b) headwind velocities for V_m = 102 m/s and 318 m/s; (a) Tailwind velocities; and (b) Headwind velocities.

This makes the projectile travel lesser distance vertically and increased drag lead to under ranging of the projectile. Head wind overall acts as a destabilising agent to the projectile motion throughout.

Figure 12(a) illustrates the induced drift trajectory under the influence of positive cross wind in the projectile at V_m= 102m/s and 318 m/s respectively. It is observed from the figures that the increase of cross wind velocity at constant muzzle velocity leads to increase of drift and a slight decrease ofthe range. This variation is very evident in trajectory with higher V_m. Figure 12(b) depicts the drift induced due to negative varying cross wind for Vm= 102 m/s and 318 m/s respectively. It has to be noted that wind is not blowing away the projectile, instead in order to stabilize, the projectile turns into the cross wind direction to follow crosswind. This is evident from the trajectory of drift in both the respective cases and cross winds. The drift in fin stabilised projectile shows linear behavior in absence of cross wind but with cross wind the drift profile is curved in nature. The positive and negative cross winds show similar trends of reducing range and increasing drift with increase of cross wind velocity in their respective direction. Fin stabilised projectile lacks lateral forces that’s why they have lesser drift and almost a planner trajectory but under the influence of crosswind there is an induced lateral force in the direction of wind, hence, they drift. Proper understanding of projectile trajectory can help identifying the impact zone of the projectile better.

5. CONCLUSIONS

We have developed a comprehensive 6-degree-of-freedom (6-DOF) model for predicting accurate flight trajectories of projectiles subjected to wind velocities from different directions. This model incorporates various aerodynamic considerations such as Mach number, total angle of attack, and both variable and constant aerodynamic coefficients. The evaluated flight trajectories cover a range of velocities from 102 m/s to 318 m/s and quadrant elevation angles from 45° to 85°. The model is effective in predicting both the range and altitude of the projectile. The deep insights into the dynamics involved in the projectile trajectory due to external factors suggest that the model provides a thorough understanding of how various parameters affect the motion of the projectile during flight. Overall, the model can be viewed as a robust and versatile tool for studying projectile trajectories, and its application to other guided projectiles opens up possibilities for further research

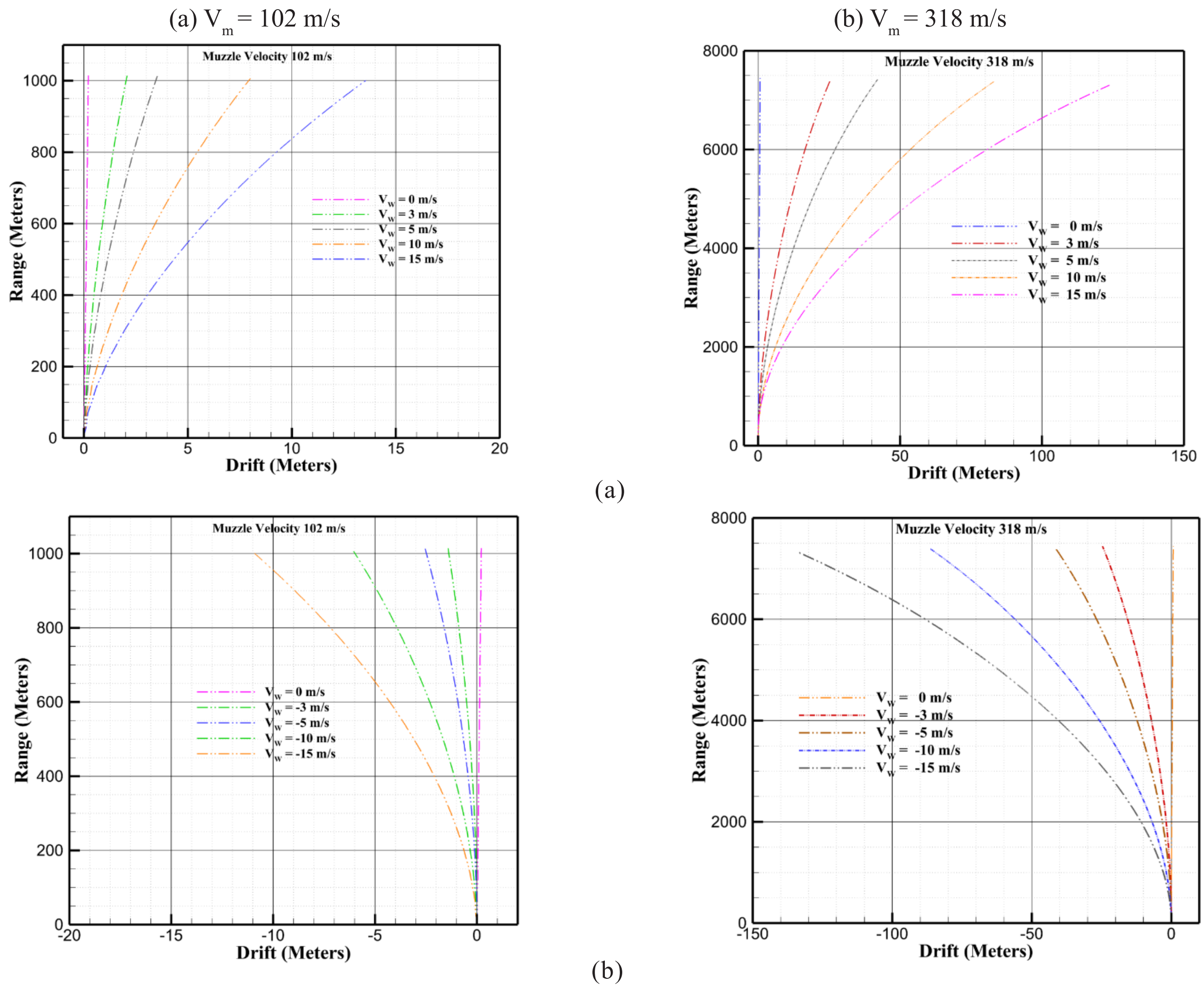


Figure 12. Drift vs range at various (a) positive and (b) negative cross wind velocities for V_m = 102 m/s and 318 m/s; (a) Positive cross wind velocity; and (b) Negative cross wind velocity.

and analysis in the field. Some of the specific observations from the study are itemised as follows:

- With increase of the elevation angle at a fixed muzzle velocity, horizontal range of the projectile decreases while the altitude increases. As the muzzle velocity increases, both the range and the altitude increase
- With increase of the muzzle velocity, drift increases. For a fixed muzzle velocity, the drift is directly proportional to the elevation angle
- With increase of the elevation angle, the velocity of the projectile steeply decreases till it reaches the maximum altitude and then steeply increases till the point of impact
- With increase of the elevation angle at fixed muzzle velocity, the impact velocity increases. However, the impact velocity always remains much lower than the muzzle velocity
- With increase of the muzzle velocity, the maximum amplitude of pitch decreases, thereby damping out the pitch variation at all elevation angles
- The range increases with the increase of wind velocity in the case of tail wind, but in all the other cases the range decreases. The projectile moves faster under the influence of the aiding tail wind. Head wind destabilizes the projectile motion throughout. Cross wind velocity causes increase of the drift and a slight decrease of the range of the projectile.

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