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Angular Stabilisation on an Unstable Platform

Deepak Kumar[1] and R.N. Mishra[2]

[1]*948 AD Msl Regt (SP) Wksp, C/o 56 APO*

[2]*Indian Institute of Technology Roorkee, Roorkee, Uttaranchal–247 667*

ABSTRACT

The paper studies the problem of angular stabilisation of direction-sensitive devices placed on a moving platform subjected to instability in pitch, roll, and yaw. The mathematical model and the quantitative correction required for stabilising the same in bearing and elevation have been evolved.

Keywords: Stabilisation system, pitch, roll, yaw, platform stabilisation, angular stabilisation, unstable platform, moving platform, mathematical model

NOMENCLATURE

p,r,y	Pitch, roll, and yaw subjected to unity direction vector, respectively
b	Bearing of unity direction vector
e	Elevation of unity direction vector
D	Unity direction vector
$\boldsymbol{D}_x,\boldsymbol{D}_y,\boldsymbol{D}_z$	Components of unity direction vector along X, Y, and Z-axes, respectively
$\boldsymbol{D}_p$	Unity direction vector subjected to pitch p
$\boldsymbol{D}_{xp},\boldsymbol{D}_{yp}, \boldsymbol{D}_{zp}$	Components of unity direction vector along X, Y, and Z-axes, subjected to pitch p
$\boldsymbol{D}_{px},\boldsymbol{D}_{py}, \boldsymbol{D}_{pz}$	Components of unity direction vector subjected to pitch p, along X, Y, and Z-axes, respectively
$\boldsymbol{D}_{pr}$	Unity direction vector subjected to pitch p and roll r
$\boldsymbol{D}_{pxr},\boldsymbol{D}_{pyr}, \boldsymbol{D}_{pzr}$	Components of unity direction vector subjectedn to pitch p, along X, Y, and Z- axes subjected to roll r
$\boldsymbol{D}_{prx},\boldsymbol{D}_{pry}$ $\boldsymbol{D}_{prz}$	Components of unity direction vector subjected to pitch p and roll r, along X, Y and Z-axes
b_{Dpr}	Bearing of unity direction vector subjected to pitch p and roll r
e_{Dpr}	Elevation of unity direction vector subjected to pitch p and roll r
b_{error}	Error in bearing
e_{error}	Error in elevation
$\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}$	Unity direction vector along X,Y, and Z-axes, respectively of the stabilised reference frame

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1. INTRODUCTION

Present-day trend in fighting a war involves continuous mobility of the weapon systems over undulating terrain. In such a scenario, consider a tracked armoured tank with its gun locked in the direction of the target to be destroyed while moving continuously. Obviously, its operator will need to continuously align the direction of the gun with the target to cater for the misalignment due to movement of the platform. Consider again a tracking radar mounted on a tracked armoured vehicle moving continuously on such a terrain with its radar antenna locked on a flying target. The movement of the vehicular platform will result in misalignment of the radar beam with the target and will need reacquisition of the target in the radar beam and locking it on to the target[1].

The misalignment of the gun or the radar beam will be due to the displacement of either the moving platform or the moving target, or both, while moving from one place to another. This misalignment due to the displacement aspect will be gradual and within the autolocking capability of the radar as well as the skill of the gun operator. The misalignment of the gun or the radar beam will also be by virtue of instability in pitch, roll, and yaw of the platform due to sudden jerks experienced while moving over an undulating terrain. This misalignment due to the instability aspect will be erratic and needs compensation.

To provide immunity against sudden jerks due to movement of a platform over undulating terrain, a stabilisation system is needed. Angular stabilisation is a common feature found in most of the modern-day weapon systems[2]. It enables the systems to continue performing the intended tasks even though the platforms on which these are mounted are unstable and give the weapon systems the capability of fire-on-move and track-while-move.

2. FRAME OF REFERENCE AND ANGULAR REFERENCES

Before evolving the mathematical model for angular stabilisation, the stabilised frame of reference and the angular references for measurement of the variables involved are defined.

In the 3-D space, a frame of reference is required as datum wrt which the angular measurements of the variables pitch, roll, yaw, bearing, and elevation are made[3]. The frame of reference will have three mutually perpendicular axes, named arbitrarily as X, Y, and Z-axes. Let the XY plane of this frame be always perpendicular to the direction of the gravitational force and parallel to perfectly horizontal surface of the earth. The XZ and the YZ planes are perpendicular to XY plane as well as to each other. Further, the centre of gravity of any platform placed in this reference frame is always coincident with the origin of this reference frame. However, the reference frame has no particular orientation wrt the earth's north and aligns itself with the platform as described subsequently. Such an imaginary reference frame, as described here, is the stabilsed reference frame.

The angular references are required for quantifying the angular measurements of the variables pitch, roll, yaw, bearing, and elevation in the stabilised frame of reference under consideration. Pitch is measured in XZ plane positively from + X-axis towards + Z-axis. Roll is measured in YZ plane positively from + Y-axis towards + Z-axis. Yaw is measured in XY plane positively from +X-axis towards + Y-axis. Bearing is measured similarly as yaw, and elevation is measured similarly as pitch except that it is measured from XY plane towards + Z-axis.

3. DEGREES OF INSTABILITY

In the stabilised reference frame, consider a static platform placed on the ground with its longitudinal, lateral, and vertical axes aligned along X, Y, and Z-axes and the centre of gravity coincident with the origin, as shown in Fig.1. While moving on an uneven surface, the antenna or the gun mounted on the platform will exhibit a misalignment with the target due to a change in pitch in the XZ plane, roll in YZ plane, and yaw in XY plane. The alignment of the antenna or the gun is on the other hand always done in bearing (azimuth) and elevation. Thus, the whole effort is concentrated in estimating

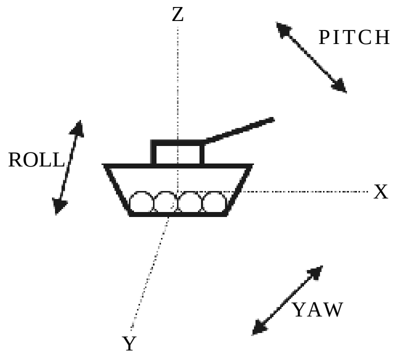

Figure 1. Instability in pitch, roll, and yaw of a platform in the stabilised reference frame.

the quantum of correction that needs to be effected in bearing and elevation for a certain pitch, roll, and yaw experienced by the moving platform to enable the system to maintain its alignment with the distant object.

4. SYSTEM MODEL

Consider a unity vector $\boldsymbol{D}$ in the stabilised reference frame, as shown in Fig. 2. The vector represents the angular orientation of any device such as the gun or the antenna mounted on the platform, which is initially without any pitch, roll, and yaw. The unity vector $\boldsymbol{D}$ is at an elevation e wrt the XY plane and bearing b wrt the XZ plane. The components of the unity vector along the X, Y, and Z-axes are:

$$\boldsymbol{D}_x = (\cos e \cos b)\ \boldsymbol{x} \qquad (1)$$

$$\boldsymbol{D}_y = (\cos e \sin b)\ \boldsymbol{y} \qquad (2)$$

$$\boldsymbol{D}_z = (\sin e)\ \boldsymbol{z} \qquad (3)$$

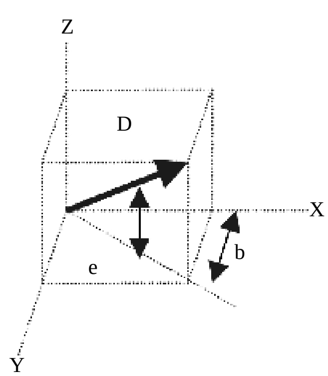

Figure 2. Unity direction vector in the stabilised frame of reference.

Let the vector $\boldsymbol{D}$ be subjected to a pitch p and be represented by a new vector $\boldsymbol{D}_p$. Subjecting a pitch p to the vector $\boldsymbol{D}$ is equivalent to subjecting a pitch to the three components $\boldsymbol{D}_x$, $\boldsymbol{D}_y$, and $\boldsymbol{D}_z$. The components $\boldsymbol{D}_x$ and $\boldsymbol{D}_z$ get an angular shift of pitch p in the XZ plane, whereas the component $\boldsymbol{D}_y$ is unaffected (Fig. 3). Resolving the angularly shifted components $\boldsymbol{D}_x$ and $\boldsymbol{D}_z$ along the X and Z axes, one gets:

$$\boldsymbol{D}_{xp} = (\cos e \cos b \cos p)\ \boldsymbol{x} + (\cos e \cos b \sin p)\ \boldsymbol{z} \qquad (4)$$

$$\boldsymbol{D}_{yp} = (\cos e \sin b)\ \boldsymbol{y} \qquad (5)$$

$$\boldsymbol{D}_{zp} = -(\sin e \sin p)\ \boldsymbol{x} + (\sin e \cos p)\ \boldsymbol{z} \qquad (6)$$

Thus, all the components of $\boldsymbol{D}_{xp}$, $\boldsymbol{D}_{yp}$, and $\boldsymbol{D}_{zp}$ put together represent the vector $\boldsymbol{D}_p$. Segregating the $\boldsymbol{x}$, $\boldsymbol{y}$, and $\boldsymbol{z}$ components and renaming these, one gets:

$$\boldsymbol{D}_{px} = (\cos e \cos b \cos p - \sin e \sin p)\ \boldsymbol{x} \qquad (7)$$

$$\boldsymbol{D}_{py} = (\cos e \sin b)\ \boldsymbol{y} \qquad (8)$$

$$\boldsymbol{D}_{pz} = (\cos e \cos b \sin p + \sin e \cos p)\ \boldsymbol{z} \qquad (9)$$

The vectors $\boldsymbol{D}_{px}$, $\boldsymbol{D}_{py}$, and $\boldsymbol{D}_{pz}$ which are aligned along X, Y, and Z-axes when put together, represent $\boldsymbol{D}_p$ vector.

Let the vector $\boldsymbol{D}_p$ be subjected to roll r and be represented by a new vector $\boldsymbol{D}_{pr}$. Thus the vector $\boldsymbol{D}_{px}$, $\boldsymbol{D}_{py}$, and $\boldsymbol{D}_{pz}$ are subjected to roll r. The

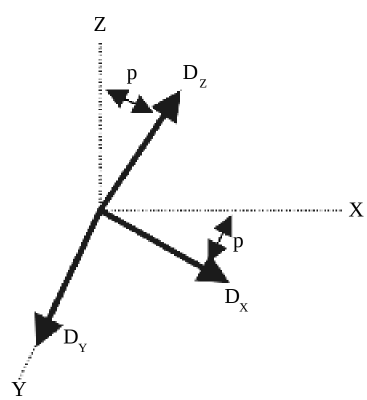

Figure 3. Components of unity direction vector subjected to pitch *p*.

vector $\boldsymbol{D}_{py}$ and $\boldsymbol{D}_{pz}$ get an angular shift of roll r in the YZ plane, whereas $\boldsymbol{D}_{px}$ remains the same (Fig. 4). Resolving these along X, Y and Z-axes, one gets:

$$\boldsymbol{D}_{pxr} = (\cos e \cos b \cos p - \sin e \sin p)\ \boldsymbol{x} \quad (10)$$

$$\boldsymbol{D}_{pyr} = (\cos e \sin b \cos r)\ \boldsymbol{y} + (\cos e \sin b \sin r)\ \boldsymbol{z} \quad (11)$$

$$\boldsymbol{D}_{pzr} = -(\cos e \cos b \sin p \sin r + \sin e \cos p \sin r)\ \boldsymbol{y} + (\cos e \cos b \sin p \cos r + \sin e \cos p \cos r)\ \boldsymbol{z} \quad (12)$$

Thus, all the components $\boldsymbol{D}_{pxr}$, $\boldsymbol{D}_{pyr}$, and $\boldsymbol{D}_{pzr}$ put together represent the vector $\boldsymbol{D}_{pr}$. Segregating the x, y, z components and renaming these, one gets:

$$\boldsymbol{D}_{prx} = (\cos e \cos b \cos p - \sin e \sin p)\ \boldsymbol{x} \quad (13)$$

$$\boldsymbol{D}_{pry} = (\cos e \sin b \cos r - \cos e \cos b \sin p \sin r - \sin e \cos p \sin r)\ \boldsymbol{y} \quad (14)$$

$$\boldsymbol{D}_{prz} = (\cos e \sin b \sin r + \cos e \cos b \sin p \cos r + \sin e \cos p \cos r)\ \boldsymbol{z} \quad (15)$$

The vectors $\boldsymbol{D}_{prx}$, $\boldsymbol{D}_{pry}$, and $\boldsymbol{D}_{prz}$ which are aligned along X, Y and Z-axes when put together represent the $\boldsymbol{D}_{pr}$ vector. From the above components, one can gets the new bearing and elevation of the vector $\boldsymbol{D}_{pr}$ as under:

$$b_{Dpr} = \tan^{-1} (\boldsymbol{D}_{pry} / \boldsymbol{D}_{prx}) \quad (16)$$

$$e_{Dpr} = \tan^{-1} [\boldsymbol{D}_{prz} / \sqrt{(\boldsymbol{D}_{prx}^2 + \boldsymbol{D}_{pry}^2)}] \quad (17)$$

Thus, the change in bearing and elevation that the direction vector $\boldsymbol{D}$ undergoes when subjected to pitch and roll, which is represented by vector $\boldsymbol{D}_{pr}$ is as under:

$$b_{error} = b - b_{Dpr} \quad (18)$$

$$e_{error} = e - e_{Dpr} \quad (19)$$

Finally, let the vector $\boldsymbol{D}_{pr}$ be subjected to yaw y. To compensate for yaw subjected to vector $\boldsymbol{D}_{pr}$, simply subtract yaw from b_{error} and the final equations emerge as

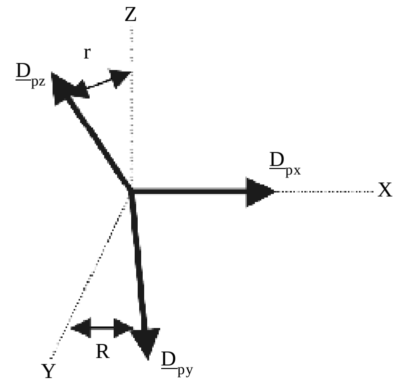

Figure 4. Components of unity direction vector with pitch *p*, subjected to roll *r*.

$$b_{error} = b - b_{Dpr} - y \quad (20)$$

$$e_{error} = e - e_{Dpr} \quad (21)$$

The Eqns (20)-(21) above are *angular stabilisation equations* where b_{Dpr}, e_{Dpr}, $\boldsymbol{D}_{prx}$, $\boldsymbol{D}_{pry}$, and $\boldsymbol{D}_{prz}$ are as given in Eqns (13)-(17) above.

Here pitch has been considered first and thereafter roll. It would make no difference had one considered these vice versa and the final result would have been the same. Also, pitch and roll have been considered compositely. Another way to approach this problem would have been to subject the vector $\boldsymbol{D}$ to only pitch or only roll, one at a time, calculate the new b_{error} and e_{error} for both and add them together by the principle of superposition to get the final result. However, this would be a lengthier method and would render the same result.

5. CONCLUSION

In this paper, the problem of platform's instability in pitch, roll, and yaw has has been reviewed. The quantum of correction required in elevation and bearing have been evolved for stabilising the same. The platform stabilisation equation can be implemented on a digital computer in real-time with input of pitch, roll, yaw, bearing, and elevation from the suitable points in the closed-loop servo system for bearing and elevation channels, and feeding back the output of error in bearing and elevation for compensating the instability in pitch, roll, and yaw[4].

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Contributors

Lt Col Deepak Kumar received his BTech (Elec Engg) from the G.B. Pant University in 1984, and ME (Elec Engg) from the Delhi College of Engineering in 1988. He is presently pursuing his PhD (Elec Engg) at the Indian Institute of Technology Roorkee. He has been closely associated with providing engineering support to a broad range of air-defence equipment in service with the Govt of India.

Prof (Dr) R.N. Mishra received his BE (Elec Engg) from the University of Jabalpur, MTech (Control Engg and Instrumentation) from the Indian Institute of Technology Delhi, and PhD from the University of Leeds, UK. He is Professor in the Department of Electrical Engineering at the IIT Roorkee. In the field of process instrumentation and image processing, he has guided researchers for their PhD, MPhil, and MTech Theses, and published several research papers in international journals of repute.