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Perturbation of Initial Stability of an FSAPDS Projectile

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ABSTRACT

For a spinning projectile, the initial stability condition is $\sigma^2 = 1 + (4\,K_3 / K_2^2) > 0$. In the present study, this condition has been modified for the malalignments arising due to pressure gradient and damping moment for an FSAPDS projectile. The equations of motion are established for the first phase of motion. A mathematical model for the first phase of motion has been developed. The effect of perturbation on the trajectory and stability of motion are discussed. It is proved that if K_3'' (a parameter appearing due to perturbation) $\leq (-K_2^2\,\sigma^2/4)$, the initial stability of motion will breakdown.

Keywords: FSAPDS projectile, spinning projectile, equation of motion, mathematical model, projectile trajectory simulation

NOMENCLATURE

O-XYZ	Inertial coordinate system
I_{xx}	Moment of inertia about the X-axis
I_{yy}	Moment of inertia about the Y-axis
m	Mass of the overall projectile
C	Mass centre of projectile
x, y, z	Range, altitude, and drift, respectively
F_1/M_1	Propellant gas force/Moment
F_2/M_2	Aerodynamic force/Moment
F_3/M_3	Gravity force/Moment
$C_{Mp\alpha}$	Magnus moment efficient
$(C_{Mqt}+C_{Mq\alpha})$	Pitch damping moment coefficient
$(C_{Nq}+C_{N\alpha})$	Pitch damping force coefficient.
u, v, w	Velocity components in projectile frame
$\omega_1, \omega_2, \omega_3$	Angular velocity components of projectile
r_p	Position vector of the effective pressure point and centre of mass of the projectile
Ω	Angular velocity wrt inertial frame
δ_1, δ_2	Angles made by projectile axis in velocity frame
η_1, η_2	Angles made by the propellant gas direction in projectile frame
$\varepsilon_1, \varepsilon_2$	Angles made by propellant gas direction in inertial frame

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P	Pressure in gas flow
ρ	Air density
l	Projectile characteristics length
p	Axial spin
S	Projectile reference area
V	Velocity of projectile
$C_{L\alpha}$ / $C_{N\alpha}$	Lift /Normal coefficient
C_D / C_X	Drag /Axial coefficient
$C_{Np\alpha}$	Magnus force coefficient
C_{tp}	Spin damping moment
$C_M\alpha$	Overturning moment coefficient
θ	Angle of attack
Φ	Angle of side slip

Convention

X	Vector cross product

Suffix

1	Denotes the projectile coordinate systems
2	Denotes the velocity coordinate systems

1. INTRODUCTION

Dynamical study of a fin-stabilised armour piercing discarding sabot (FSAPDS) round is very important till the sabots are separated and the thin penetrator moves ahead. The sabots are separated from the projectile under the action of aerodynamic forces. Yang[1] has discussed the complete dynamic modelling of a sabot discard process of an FSAPDS round. With three turning points, he has divided the sabot discard process in four phases and two transition periods (Fig. 1). The motions points are defined as

0 At the muzzle end.

1 The first turning point is at the instant when the fixed circle of sabots reaches the limit stress state and its groove teeth break due to the aerodynamic force.

2 The second turning point is at the instant when circle groove teeth of a sabot component separate from those of projectile body and their mechanical interaction vanishes.

3 The third turning point is at the instant when the intersect point between the projectile and the shock wave at the head of a sabot moves to the projectile base and the projectile gets free from the influence of sabot.

The turning points are defined for each sabot, separately. The first turning point for the three

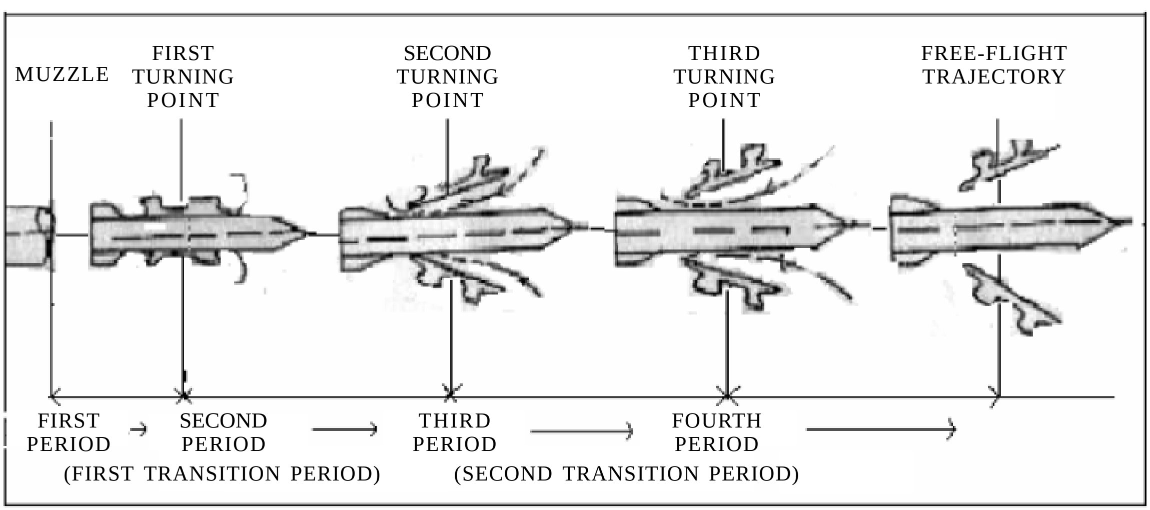

Figure 1. Turning points, four phases and transition periods of sabot discarding process.

sabots is at the same time and the same position relative to the projectile body. The second turning point is at the same position relative to the projectile body but is at different time, whereas the third turning point is at different time and different relative positions, which can be studied with the help of transition periods separately. The different phases of the motion are given below:

Phase I : Motion between points of 0 to 1

Phase II : Motion between points of 1 to 2

Phase III : Motion between points of 2 to 3

Phase IV : Motion after point 3 onwards.

First transition period is motion during the second turning point of the first sabot component to second turning point of the third sabot component. Second transition period is motion during the third turning point of the first sabot component to third turning point of the third sabot component motion condition at the end of the previous phase is the initial condition for the next phase, but the initial conditions for different sabots are distinct.

In this paper, the trajectory and stability of motion during phase I of the motion has been discussed. Propellant gases released from the muzzle-end develop a flow field around the projectile. At the muzzle, it commences with the formation of a strong blast and a jet which continues till the gun tube becomes empty. The flow field around the projectile continues till ambient conditions are reached. The projectile moves forward in this flow field. The propellant gases expand supersonically, due to which the pressure drops rapidly. Due to asymmetric flow of gases, the pressure is misaligned. The pressure acts at a point on the projectile having position vector ($\bar{r}_p$), from the centre of gravity of the body. Though damping force is small and can be ignored, damping moment may affect the stability of the projectile. These two aspects of the FSAPDS projectile have been considered. In the first part, the trajectory of the projectile is modelled and simulated. In the second part, the stability criterion of motion has been discussed.

2. COORDINATE SYSTEMS

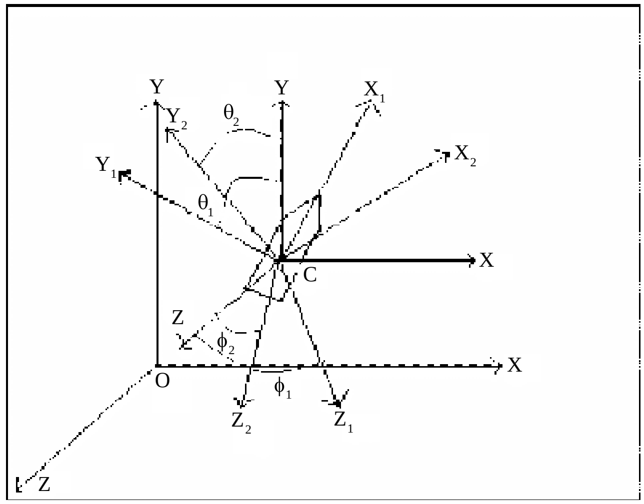

Figure 2. Coordinate system.

The motion has been studied with the help of three coordinate systems defined in Fig. 2.

(a) *Inertial Coordinate System* (O-XYZ)

- O Origin at the muzzle-end along the barrel axis
- OX Along the barrel direction
- OY Vertical axis normal to the OX
- OZ Completes the right-handed system.

(b) *Projectile Coordinate System* (C-$X_1Y_1Z_1$)

- C Origin at the centre of mass of the projectile
- CX_1 Along the axis of rotation of the projectile
- CY_1 Normal to the CX_1 in the plane of reflection symmetry of the projectile
- CZ_1 Completes the right-handed system.

(c) *Velocity Coordinate System* (C-$X_2Y_2Z_2$)

- C Origin at the centre of mass of the projectile
- CX_2 Along the instantaneous velocity direction
- CY_2 Normal to the CX_2 in the plane of motion
- CZ_2 Completes the right-handed system.

2.1 Transformation Matrices

- *Transformation from the Inertial Coordinate System to Velocity System*

$$(\hat{i}_2, \hat{j}_2, \hat{k}_2) = A^X(\pi/2)A^Z(\theta_2)A^Y(\phi_2)(\hat{i}, \hat{j}, \hat{k})$$

$$\therefore \begin{bmatrix} \hat{i}_2 \\ \hat{j}_2 \\ \hat{k}_2 \end{bmatrix} = \begin{bmatrix} \cos\theta_2 \cos\phi_2 & \sin\theta_2 & -\cos\theta_2 \sin\phi_2 \\ \sin\phi_2 & 0 & \cos\phi_2 \\ \sin\theta_2 \cos\phi_2 & -\cos\theta_2 & -\sin\theta_2 \sin\phi_2 \end{bmatrix} \begin{bmatrix} \hat{i} \\ \hat{j} \\ \hat{k} \end{bmatrix}$$

- *Transformation from the Projectile Coordinate System to Velocity System*

$$(\hat{i}_2, \hat{j}_2, \hat{k}_2) = A^Z(\delta_1)A^Y(\delta_2)(\hat{i}_1, \hat{j}_1, \hat{k}_1)$$

$$= \begin{bmatrix} \cos\delta_1 \cos\delta_2 & \sin\delta_1 & -\cos\delta_1 \sin\delta_2 \\ -\sin\delta_1 \cos\delta_2 & \cos\delta_1 & \sin\delta_1 \sin\delta_2 \\ \sin\delta_2 & 0 & \cos\delta_2 \end{bmatrix} \begin{bmatrix} \hat{i}_1 \\ \hat{j}_1 \\ \hat{k}_1 \end{bmatrix}$$

- *Transformation from the Inertial Coordinate System to Projectile System*

$$\begin{bmatrix} \hat{i}_1 \\ \hat{j}_1 \\ \hat{k}_1 \end{bmatrix} = A^X(\gamma_1)A^Z(\theta_1)A^Y(\phi_1) \begin{bmatrix} \hat{i} \\ \hat{j} \\ \hat{k} \end{bmatrix}$$

$$= \begin{bmatrix} \cos\phi_1 \cos\theta_1 & \sin\theta_1 & -\sin\phi_1 \cos\theta_1 \\ -\cos\gamma_1 \cos\phi_1 \sin\theta_1 + \sin\gamma_1 \sin\phi_1 & \cos\gamma_1 \cos\theta_1 & \cos\gamma_1 \sin\phi_1 \sin\theta_1 + \sin\gamma_1 \cos\phi_1 \\ \sin\gamma_1 \cos\phi_1 \sin\theta_1 + \cos\gamma_1 \sin\phi_1 & -\sin\gamma_1 \cos\theta_1 & -\sin\gamma_1 \sin\phi_1 \sin\theta_1 + \cos\gamma_1 \cos\phi_1 \end{bmatrix} \begin{bmatrix} \hat{i} \\ \hat{j} \\ \hat{k} \end{bmatrix}$$

2.2 Angular Velocity

- *Angular Velocity of the Projectile* $\left(\overline{\Omega}_1\right)$

$$\overline{\Omega}_1 = \dot{\phi}_1 \hat{j} + \dot{\theta}_1 \hat{k} + \dot{\gamma}_1 \hat{i}$$

which can be obtained from

$$\begin{bmatrix} \cos\phi_1 & 0 & -\sin\phi_1 \\ 0 & 1 & 0 \\ \sin\phi_1 & 0 & \cos\phi_1 \end{bmatrix} \begin{bmatrix} 0 \\ \dot{\phi}_1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \dot{\phi}_1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \cos\theta_1 & \sin\theta_1 & 0 \\ -\sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ \dot{\phi}_1 \\ \dot{\theta}_1 \end{bmatrix} = \begin{bmatrix} \dot{\phi}_1 \sin\theta_1 \\ \dot{\phi}_1 \cos\theta_1 \\ \dot{\theta}_1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma_1 & \sin\gamma_1 \\ 0 & -\sin\gamma_1 & \cos\gamma_1 \end{bmatrix} \begin{bmatrix} \dot{\phi}_1 \sin\theta_1 + \dot{\gamma}_1 \\ \dot{\phi}_1 \cos\theta_1 \\ \dot{\theta}_1 \end{bmatrix}$$

$$= \begin{bmatrix} \dot{\phi}_1 \sin\theta_1 + \dot{\gamma}_1 \\ \dot{\phi}_1 \cos\theta_1 \cos\gamma_1 + \dot{\theta} \sin\gamma_1 \\ -\dot{\phi}_1 \cos\theta_1 \sin\gamma_1 + \dot{\theta}_1 \cos\gamma_1 \end{bmatrix}$$

$$\begin{aligned} \therefore \overline{\Omega}_1 &= (\dot{\phi}_1 \sin\theta_1 + \dot{\gamma}_1)\hat{i}_1 + (\dot{\phi}_1 \cos\theta_1 \cos\gamma_1 \\ &\quad + \dot{\theta}\sin\gamma_1)\hat{j}_1 + (-\dot{\phi}_1 \cos\theta_1 \sin\gamma_1 + \dot{\theta}_1 \cos\gamma_1)\hat{k}_1 \\ &= \omega_1 \hat{i}_1 + \omega_2 \hat{j}_1 + \omega_3 \hat{k}_1 \end{aligned} \qquad (1)$$

- *Angular Velocity of the Velocity Coordinate System* $\left(\overline{\Omega}_2\right)$

$$\overline{\Omega}_2 = \dot{\theta}_2 \hat{k} + \dot{\phi}_2 \hat{j}$$

$$\begin{bmatrix} \cos\phi_2 & 0 & -\sin\phi_2 \\ 0 & 1 & 0 \\ \sin\phi_2 & 0 & \cos\phi_2 \end{bmatrix} \begin{bmatrix} 0 \\ \dot{\phi}_2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \dot{\phi}_2 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \cos\theta_2 & \sin\theta_2 & 0 \\ -\sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ \dot{\phi}_2 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} \dot{\phi}_2 \sin\theta_2 \\ \dot{\phi}_2 \cos\theta_2 \\ \dot{\theta}_2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} \dot{\phi}_2 \sin\theta_2 \\ \dot{\phi}_2 \cos\theta_2 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} \dot{\phi}_2 \sin\theta_2 \\ \dot{\theta}_2 \\ -\dot{\phi}_2 \cos\theta_2 \end{bmatrix}$$

$$\therefore \overline{\Omega}_2 = \dot{\phi}_2 \sin\theta_2 \hat{i}_2 + \dot{\theta}_2 \hat{j}_2 - \dot{\phi}_2 \cos\theta_2 \hat{k}_2 \qquad (2)$$

3. FORCES AND MOMENTS FOR THE MOTION

3.1 Forces

Propellant Gas Force: The forces developed by the propellant gases will be acting on the projectile for some time till the effective force becomes zero. This force acts along the gas flow direction which may be different from the barrel direction as well as from the projectile direction.

$$\overline{F}_1 = PA\hat{i}_g \qquad (3)$$

where $\hat{i}_g$ is a unit vector along the gas direction.

3.1.1 Modelling of Pressure

The pressure affecting the projectile motion varies continuously. The pressure at the muzzle-end is muzzle pressure, which further decreases as the projectile moves in the propellant gas flow field till it reduces to atmospheric pressure. Generally, the pressure reduces to atmospheric pressure at a distance of 10-15 calibers.

It is assumed that the decrease in the pressure is continuous and linear. The pressure at any point at a distance X away from the muzzle is given by

$P = P_0$ (1- X/l), where P_0 is the muzzle pressure.

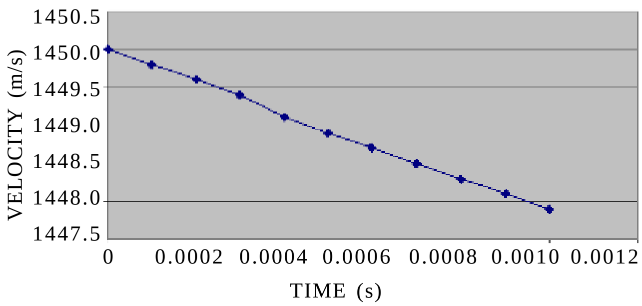

Figure 3. Trajectory of FSAPDS projectile in the first phase.

This change in the pressure P is calculated and plotted in Fig. 3.

- *Aerodynamic Force*

Here the aerodynamic forces due to drag, lift, magnus and pitch damping forces are considered and are given below[2]:

$$\overline{F}_2 = \overline{F}_D + \overline{F}_L + \overline{F}_M + \overline{F}_P$$

$$\overline{F}_D = -(1/2)\rho S V^2 C_D \hat{i}_2 \quad \text{(Drag force)} \tag{4}$$

$$\overline{F}_L = (1/2)\rho\, S\, C_{L\alpha} [\overline{V}\ X\ (\hat{i}_1\ X\ \overline{V})]$$
$$= (1/2)\rho\, S C_{L\alpha} V^2\ [\hat{i}_2\ X\ (\hat{i}_1\ X\ \overline{V})] \quad \text{(Lift force)} \tag{5}$$

$$\overline{F}_M = (1/2)\rho S V^2 (pl/V) C_{N_{p\alpha}} (\hat{i}_2\ X\ \hat{i}_1) \quad \text{(Magnus force)} \tag{6}$$

$$\overline{F}_P = (1/2)\rho S V l\ (C_{N_q} + C_{N\alpha})(d\,\hat{i}_1/dt) \quad \text{(Pitch damping force)} \tag{7}$$

By the transformation

$$C_D = C_X \cos\delta_1 \cos\delta_2 + C_N \sin\delta_1$$

and

$$C_{L\alpha} = -C_X\, s \sin\delta_1 \cos\delta_2 + C_N \cos\delta_1$$

The magnus force coefficient $C_{Np\alpha}$ is usually a small negative quantity. It always acts in a direction perpendicular to the plane of yaw. Pitch damping force acts in the plane of transverse angular velocity, which is not necessarily the same as the yaw plane. The pitch damping force contains two parts, first part proportional to transverse angular velocity (pitching velocity) and the second part proportional to the rate of change of total angle of attack.

$$\overline{F}_2 = (1/2)\rho S V^2 \{-C_D \hat{i}_2 + C_{L\alpha}\ [\hat{i}_2\ X\ (\hat{i}_1 X\ \hat{i}_2)]$$
$$+ (pl/V) C_{N_{p\alpha}} (\hat{i}_2\ X\ \hat{i}_1)$$
$$+ (l/V)\ (C_{N_q} + C_{N\alpha})(d\,\hat{i}_1/dt)\} \tag{8}$$

- *Gravity Force*

$$\overline{F}_3 = -mg\,\hat{j} \tag{9}$$

In the separation process, the mechanical force and shock wave force also act on the sabots. These forces are present in 2nd and 3rd phases. Hence in the first phase of motion, these forces and moments are ignored.

3.2 Moments

- *Propellant Gas Moment*

Due to malalignment, the pressure is acting at a point on the projectile having position vector $(\overline{r}_p)$ from the centre of gravity of the body. This pressure force generates a moment.

$$\overline{M}_1 = \overline{r}_p\ x\ \overline{F}_1, \qquad \overline{r}_p = r_p\,\hat{i}_p \tag{10}$$

where $\begin{bmatrix} \hat{i}_p \\ \hat{j}_P \\ \hat{k}_P \end{bmatrix} = A^Z(\eta_1) A^Y(\eta_2) \begin{bmatrix} \hat{i}_1 \\ \hat{j}_1 \\ \hat{k}_1 \end{bmatrix}$ and

$$\hat{i}_p = \cos\eta_1 \cos\eta_2\, \hat{i}_1 - \sin\eta_1 \cos\eta_2\, \hat{j}_1 + \sin\eta_2\, \hat{k}_1$$

Due to the above aerodynamic forces, the corrosponding moments[2] are:

- *Aerodynamic Moment*

(a) Spin damping moment

$$= (1/2)\rho S u^2 l C_{t_p} (pl/u)\overline{i}_1$$

(b) Overturning moment

$= (1/2)\rho S u^2 l C_{M_\alpha} (\bar{i}_2 X \bar{i}_1)$

(c) Magnus moment

$= (1/2)\rho S u^2 l C_{Mp\alpha} (pl/u)[(\bar{i}_1 X (\bar{i}_2 X \bar{i}_1)]$

(d) Pitch damping moment

$$= (1/2)\rho S u l^2 C_{Mqt} (\bar{i}_1 X d\bar{i}_1 / dt) \tag{11}$$

- *Malalinment (Off Set) Gravity Moment*

$\bar{r}_g$ is the position vector of off set from the centre of gravity of the body.

$$\bar{M}_3 = \bar{r}_g X \bar{F}_3 . \tag{12}$$

In this phase of motion, $\bar{r}_g$ is along the axis of the body, hence $\bar{M}_3 = 0$.

4. EQUATIONS OF MOTION

The equations of motion by Newton's second law, because of moving frame, are given as

$$m\frac{d\bar{V}}{dt} = m\frac{\partial\bar{V}}{\partial t} + \bar{\Omega} \times \bar{V} = \sum \bar{F}_i$$

$$\frac{d\bar{H}}{dt} = \frac{\partial\bar{H}}{\partial t} + \bar{\Omega} \times \bar{H} = \sum \bar{M}_i \tag{13}$$

5. TRAJECTORY SIMULATION

In the first phase of motion, the trajectory of the projectile as a whole body can be obtained by resolving the equation in velocity coordinate system. The pressure applies force on the base of the projectile which affects its trajectory. The moment equations are resolved in projectile coordinate system. The scalar equations are:

$$\dot{V} = (PA/m)\left(\cos\varepsilon_1 \cos\varepsilon_2 \cos\theta_2 \cos\phi_2 - \sin\varepsilon_1 \cos\varepsilon_2 \sin\theta_2 - \sin\varepsilon_2 \cos\theta_2 \sin\phi_2\right)$$

$$+ (1/2m)\rho S V^2 [-C_D + (l/V)(C_{Nq} + C_{N\alpha})[\sin\delta_1 (-\dot{\phi}_1 \sin\gamma_1 \cos\theta_1 + \dot{\theta}_1 \cos\gamma_1)$$

$$+ \cos\delta_1 \sin\delta_2 (\dot{\phi}_1 \cos\gamma_1 \cos\theta_1 + \dot{\theta}_1 \sin\gamma_1)] - g \sin\theta_2 \tag{14}$$

$$\dot{\phi}_2 = \{PA(\cos\varepsilon_1 \cos\varepsilon_2 \sin\phi_2 + \sin\varepsilon_2 \cos\phi_2) + (1/2)\rho S V^2 [-C_{L\alpha} \sin\delta_1 \cos\delta_2$$

$$- (pl/V) C_{Np\alpha} \sin\delta_2 + (l/V)(C_{Nq} + C_{N\alpha})[\cos\delta_1 (-\dot{\phi}_1 \sin\gamma_1 \cos\theta_1 + \dot{\theta}_1 \cos\gamma_1)$$

$$- \sin\delta_1 \sin\delta_2 (\dot{\phi}_1 \cos\gamma_1 \cos\theta_1 + \dot{\theta}_1 \sin\gamma_1)]\}/[-mV \cos\theta_2] \tag{15}$$

$$\dot{\theta}_2 = \{PA(\cos\varepsilon_1 \cos\varepsilon_2 \sin\theta_2 \cos\phi_2 + \sin\varepsilon_1 \cos\varepsilon_2 \cos\theta_2 - \sin\varepsilon_2 \sin\theta_2 \sin\phi_2)$$

$$+ (1/2)\rho SV^2\,[C_{L\alpha}\sin\delta_2 - (pl/V)C_{Np\alpha}\sin\delta_1\cos\delta_2$$

$$- (l/V)(C_{Nq} + C_{N\alpha})\cos\delta_2\,(\dot{\phi}_1\cos\gamma_1\cos\theta_1 + \dot{\theta}_1\sin\gamma_1)] + mg\cos\theta_2\,\}/[-mV] \tag{16}$$

The velocity in inertial frame is:

$$\dot{x} = V\cos\theta_2\cos\phi_2 \tag{17}$$

$$\dot{y} = V\sin\theta_2\cos\phi_2 \tag{18}$$

$$\dot{z} = -V\sin\phi_2 \tag{19}$$

Moment equations are:

$$I_{YY}(\partial/\partial t)(\dot{\phi}_1\cos\gamma_1\cos\theta_1 + \dot{\theta}_1\sin\gamma_1)$$

$$= (I_{YY} - I_{xx})(\dot{\phi}_1\sin\theta_1 + \dot{\gamma}_1)(-\dot{\phi}_1\sin\gamma_1\cos\theta_1 + \dot{\theta}_1\cos\gamma_1)]$$

$$+ r_p PA(\sin\eta_2\cos\varepsilon_1\cos\varepsilon_2 - \cos\eta_1\cos\eta_2\sin\varepsilon_2) + (1/2)\rho SV^2 l[-C_{Ma}\cos\delta_1\sin\delta_2$$

$$+ C_{M\alpha p}(pl/V)\sin\delta_1 + (l/V)(C_{Mqt} + C_{Mqi})(\dot{\phi}_1\cos\gamma_1\cos\theta_1 + \dot{\theta}_1\sin\gamma_1)] \tag{20}$$

$$[\,I_{YY}(\partial/\partial t)(-\dot{\phi}_1\sin\gamma_1\cos\theta_1 + \dot{\theta}_1\cos\gamma_1)$$

$$= (I_{xx} - I_{YY})(\dot{\phi}_1\sin\theta_1 + \dot{\gamma}_1)(\dot{\phi}_1\cos\gamma_1\cos\theta_1 + \dot{\theta}_1\sin\gamma_1)]$$

$$+ r_p PA(-\cos\eta_1\cos\eta_2\sin\varepsilon_1\cos\varepsilon_2 + \sin\eta_1\cos\eta_2\cos\varepsilon_1\cos\varepsilon_2)$$

$$+ (1/2)\rho SV^2 l[-C_{M\alpha}\sin\delta_1 + C_{M\alpha p}(pl/V)\cos\delta_1\sin\delta_2$$

$$+ (l/V)(C_{Mqt} + C_{Mqi})(-\dot{\phi}_1\sin\gamma_1\cos\theta_1 + \dot{\theta}_1\cos\gamma_1)] \tag{21}$$

5.1 Simulation Results

Equations (14) – (21) give the mathematical model for trajectory of the projectile in phase I. The trajectory for phase I over the time period 0.00 s to 0.001 s has been simulated for the following data with fixed step size, h = 0.0001.

5.1.1 Data

P = 405 MPa	Ixx = 4.000 kg/m^3	ρ = 1.225 kg/m^3	Iyy = 560 kg/m^3
l = 0.486 m	m = 6.400 kg	p = 145 rpm	C_x = 1.250
C_N = 7.020	$C_{Np\alpha}$ = 0	$C_{Nq\alpha}$ = 0	C_{tp} = 0
$(C_{Mqt} + C_{Mq\alpha})$ = -570.600	C_{Nq} = 0	V = 1450 m/s	$C_{M\alpha}$ = 2.500
$\delta_1 = \Phi_1$ = 1.500°	$\delta_2 = \theta_1 - \theta_2$	θ_1 = 1.500°	
$\eta_1 = \eta_2 = \varepsilon_1 = \varepsilon_2 = \gamma_1 = 0$	$x = y = z = 0$		

5.1.2 Results

t (s)	V_P (m/s)	Φ_2 (deg)	Φ_2 (deg)	*x* (m)	*y* (m)	*z* (m)
.0000	14500	0	0.0999854	0.00000	0	0
.0001	1449.8	0.000992807	0.098788	0.14499	0.0002515	-1.256E-06
.0002	1449.6	0.00198550	0.0975907	0.28996	0.0005000	-5.025E-06
.0003	1449.4	0.002977963	0.0963934	0.43490	0.0007454	-1.13E-05
.0004	1449.1	0.003970255	0.0951903	0.57983	0.0009877	-2.009E-05
.0005	1448.9	0.004962374	0.093993	0.72473	0.0012270	-3.139E-05
.0006	1448.7	0.005954551	0.0927899	0.86961	0.0014632	-4.519E-05
.0007	1448.5	0.006946213	0.0915869	1.01450	0.0016963	-6.15E-05
.0008	1448.3	0.007937874	0.0903838	1.15930	0.0019263	-8.032E-05
.0009	1448.1	0.008928962	0.0891808	1.30410	0.0021533	-0.0001016
.0010	1447.9	0.009920051	0.0879720	1.44890	0.0023772	-0.0001255

Φ_1_dot (deg)	Φ_1_dot (deg)	Φ_1 (deg)	Φ_1 (deg)	P (MPa)
0.038395226	- 0.3874526	1.1998472	1.49980904	405.000
0.037623552	- 0.3881286	1.1997899	1.49980904	367.400
0.036851878	- 0.3888046	1.1997899	1.49980904	329.800
0.036079631	- 0.3894806	1.1997327	1.49980904	292.210
0.035306811	- 0.3901566	1.1996754	1.49980904	254.620
0.034533991	- 0.3908326	1.1996754	1.49980904	217.040
0.033760598	- 0.3915086	1.1996181	1.49980904	179.460
0.032986633	- 0.3921789	1.1995608	1.49980904	141.890
0.032212667	- 0.3928549	1.1995608	1.49980904	104.320
0.031438129	- 0.3935309	1.1995035	1.49980904	66.765
0.030663017	- 0.3942069	1.1994462	1.49986633	46.112

In this short interval of time, it is observed that

- The projectile travels a distance of 1.4489 m
- Velocity decreases approx. by 2.1 m/s
- Pressure reduces from 405 Mpa to 46.112 Mpa (Fig. 4).
- The altitude remains the same
- The projectile drifts slightly towards left (3.1 cm) (Fig. 5)
- Angle of attack, θ_2 decreases but θ_1 remains constant
- Angles of side slip, Φ_1 and Φ_2 remain constant

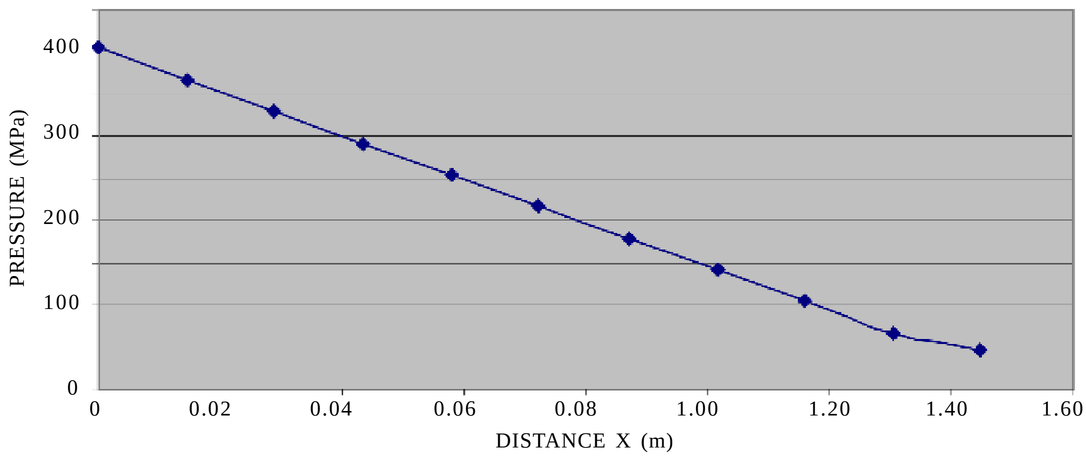

Figure 4. Pressure against distance.

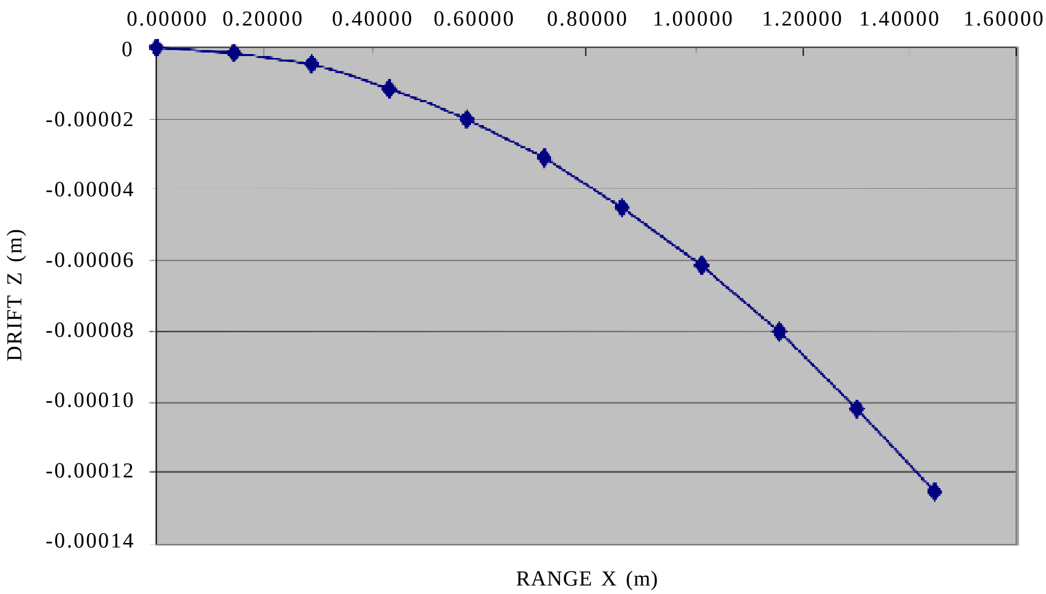

Figure 5. Range X against drift Z.

- The trajectory remains unchanged due to the pressure
- The drift slightly decreases due to damping moment but range is unaltered
- There is no change in θ_1, Φ_1 and Φ_2 due to pressure and damping moment but θ_2 cntinuously decreases due to damping moment (Fig. 6).

6. STABILITY OF MOTION

The FSAPDS projectile is generally stabilised by spin in the first phase, as fins are not active. The stability can be discussed with the six DOF equations expressed in the projectile frame. The design of the projectile is stable. In the first phase, the pressure gradient and its malalignment may disturb the stability of the projectile. The modified stability criterion due to perturbations caused by pressure gradient and damping moment has been analysed.

The vector equations in projectile frame with these perturbations are:

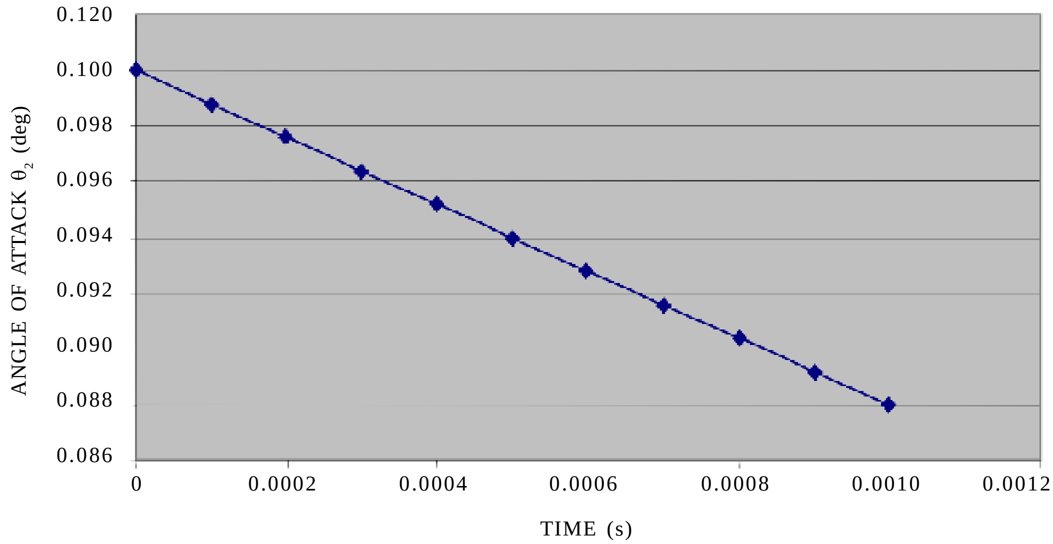

Figure 6. Angle of attack θ_2 against time.

$$m(\partial u/\partial t + \omega_2 w - \omega_3 v)\hat{i}_1 + m(\partial v/\partial t + \omega_3 u - \omega_1 w)\hat{j}_1 + m(\partial w/\partial t + \omega_1 v - \omega_3 u)\hat{k}_1$$

$$= \{PA(\cos\varepsilon_1 \cos\varepsilon_2) - (1/2)\rho S u^2 C_D - mg\sin\theta_1\}\hat{i}_1 + \{-PA\sin\varepsilon_1\cos\varepsilon_2$$

$$+(1/2)\rho S u^2[-C_{L\alpha}(v/u) + (pl/u)C_{M_{pa}}(w/u) + l(\omega_3/u)C_{N_q} + C_{Na})] - mg\cos\theta_1\}\hat{j}_1$$

$$+\{PA\sin\varepsilon_2 - (1/2)\rho S u^2[-C_{L\alpha}(w/u) + (pl/u)C_{M_{pa}}(v/u) + l(\omega_2/u)(C_{N_q} + C_{Na})]\}\hat{k}_1 \tag{22}$$

$$I_{xx}\frac{d\omega_1}{dt}\hat{i}_1 + [I_{yy}\frac{d\omega_2}{dt} + \omega_3 I_{xx}\omega_1 - \omega_1 I_{yy}\omega_3]\hat{j}_1 + [I_{yy}\frac{d\omega_3}{dt} + \omega_1 I_{yy}\omega_2 - \omega_2 I_{xx}\omega_1]\hat{k}_1$$

$$= [r_p PA(-\sin\eta_1\cos\eta_2\sin\varepsilon_2 + \sin\eta_2\sin\varepsilon_1\cos\varepsilon_2) + (1/2)\rho S u^2 C_{l_p}(pl/u)]\hat{i}_1$$

$$+\{r_p PA(\cos\eta_1\cos\eta_2\sin\varepsilon_2 - \sin\eta_2\sin\varepsilon_1\cos\varepsilon_2) + (1/2)\rho S u^2[C_{M_\alpha}(w/u)$$

$$+(pl/u)C_{M_{pa}}(v/u) + C_{M_{qt}} l(\omega_2/u)]\}\hat{j}_1 + \{r_p PA(-\cos\eta_1\cos\eta_2\sin\varepsilon_1\cos\varepsilon_2$$

$$+\sin\eta_1\sin\eta_2\cos\varepsilon_1\cos\varepsilon_2) + [-C_{M_a}(v/u) + (pl/u)C_{M_{pa}}(w/u) + C_{M_{qt}} l(\omega_3/u)]\}\hat{k}_1 \tag{23}$$

7. NONDIMENSIONALISATION

The motion can be studied in the cross plane of the body[3] for its stability. Let the nondimensional velocities in complex form can be defined as

$$\xi = \frac{v + iw}{u} \quad \text{and} \quad \eta = \left(\frac{\omega_2 + i\omega_3}{u}\right)l$$

where

$$B' = I_{xx} / I_{yy} \quad ; \ \varepsilon = (1/2m)Sl\rho \quad ; \ K_t^2 = I_{yy} / ml^2$$

$$\sigma^2 = 1 + (4K_3 / K_2^2) \quad \text{[Cranz stability parameter]}$$

The aerodynamic forces $\overline{F}\,(F_x, F_y, F_z)$ and moments $\overline{G}\,(G_x, G_y, G_z)$ in body frame can now be expressed in the complex plane as

$$\overline{F} = F_y + iF_Z \qquad\qquad \overline{G} = G_y + iG_Z$$

Changing the independent variable from time to nondimensional axial distance, one has:

$$\overline{x} = x / l, \qquad \frac{d}{dt} = \frac{u}{l}\frac{d}{d\overline{x}}$$

and ignoring the term $(w\omega_2 - v\omega_1)$ [as it is very small compared to $\dot{u}$] the equations in the complex plane reduce to

$$\xi' + [(g_x\, l/u^2) + (F_x^s\, l/u^2 m) + i\overline{\omega}_1]\xi - i\eta = (g_y l/u^2) + (Fl/u^2 m) \tag{24}$$

$$\eta' + [(g_x\, l/u^2) + (F_x^s\, l/u^2 m) - i(B'\zeta - \overline{\omega}_1)]\eta = (Gl^2)/(u^2 I_{yy}) \tag{25}$$

where

$\overline{\omega}_1 = \omega_1 l/u$ and $'$ denotes differentiation wrt $\overline{x}$.

From the above equations of ballistic equation, a second-order differential equation in ξ can be derived by eliminating η to study the motion in the cross plane. The equation is of the form

$$\xi'' + (-K_1 I + iK_2 J)\xi' + (K_3 I + iK_4)\xi = K_5 + iK_6 \tag{26}$$

Here, $K_i\,(i = 1,2,3,4)$ refers to damping, spin, positional and magnus parameters, respectively.

These parameters are functions of the density of air, aerodynamic coefficients, spin, frame velocity and nondimensional gravity groups for linear force systems.

$$K_1 = 2\varepsilon C_D - (2g_x\, l/u^2) - (2l/u^2)PA\cos\varepsilon_1 \cos\varepsilon_2 + \varepsilon K^{-2}{}_t\,(C_{Mqt} + C_{Mq\alpha})$$

$$K_2 = -B'\zeta + 2\overline{\omega}_1 - \zeta\varepsilon C_{Np\alpha}$$

$$K_3 = -\varepsilon K^{-2}{}_t C_{M\alpha} + \varepsilon^2 C_D (C_D - C_{L\alpha}) + (g_x l/u^2) - \overline{\omega}_1^2 - (g_x l/u^2)\varepsilon(2C_D - C_{L\alpha})$$

$$+ B'\zeta\overline{\Omega}_x - \overline{\omega}_1^2\,\varepsilon\, C_{Np\alpha} - B'\zeta C_x C_{Np\alpha} + (2d/u^2)PA\cos\varepsilon_1 \cos\varepsilon_2 [(2g_x l/u^2)$$

$$-\varepsilon(2C_D - C_{L\alpha}) - \varepsilon K^{-2}{}_t\,(C_{Mqt} + C_{Mq\alpha})] - \varepsilon K^{-2}{}_t\,(C_{Mqt} + C_{Mq\alpha})\,(g_x l/u^2) - \varepsilon(C_D - C_{L\alpha})]$$

$$K_4 = B'\zeta\varepsilon(C_D - C_{L\alpha}) - B'\zeta(g_x l/u^2) - \overline{\omega}_1[\varepsilon(2C_D - C_{L\alpha}) - (2g_x l/u^2) - \varepsilon^2 C_D C_{Np\alpha}$$

$$-(\varepsilon^2 C_{Np\alpha} - \varepsilon)K_t^{-2}(C_{Mqt} + C_{Mq\alpha})] - (2l/u^2)PA\cos\varepsilon_1\cos\varepsilon_2[B'\zeta - 2\overline{\omega}_1 + \overline{\omega}_1\varepsilon C_{Np\alpha}]$$

$$K_5 = -[\varepsilon C_D - g_x l/u^2 - (2l/u^2)PA\cos\varepsilon_1\cos\varepsilon_2 + \varepsilon K_t^{-2}(C_{Mqt} + C_{Mq\alpha})]$$

$$+ [g_y l/u^2 + (2l/u^2)PA\sin\varepsilon_1] + (B'\zeta - \overline{\omega}_1)(2l/u^2)PA\sin\varepsilon_2 - \varepsilon K_t^{-2} r_p PAB_2/(u^2 m)$$

$$+ [g_y l/u^2 + (2l/u^2)PA\sin\varepsilon_1] + (B'\zeta - \overline{\omega}_1)(2l/u^2)PA\sin\varepsilon_2 - \varepsilon K_t^{-2} r_p PAB_2/(u^2 m)$$

$$K_6 = (\overline{\omega}_1 - B'\zeta) - \left[g_y l/u^2 + \left(2l/u^2\right)PA\sin\varepsilon_1\cos\varepsilon_2\right] - \varepsilon K_t^{-2} r_p PAB_1/\left(u^2 m\right)$$

$$-(2l/u^2)PA\sin\varepsilon_2[g_x l/u^2 - \varepsilon C_D + (2l/u^2)PA\cos\varepsilon_1\cos\varepsilon_2 - \varepsilon K_t^{-2}(C_{Mqt} + C_{Mq\alpha})] \quad (27)$$

where

$$B_1 = \sin\eta_2\cos\varepsilon_1\cos\varepsilon_2 - \cos\eta_1\cos\eta_2\sin\varepsilon_2)$$

$$B_2 = (-\cos\eta_1\cos\eta_2\sin\varepsilon_1\cos\varepsilon_2 + \sin\eta_1\cos\eta_2\cos\varepsilon_1\cos\varepsilon_2)$$

To study the effect of pressure, its malalignment and damping moment on the stability of the projectile, the parameters are expressed as

$$K_i'\; s\ as\ K_i = K_i' + K_i'' \quad (28)$$

where K_i' s represent the terms due to aerodynamic, drag, lift, and gravity and K_i'' consists of the terms arising due to perturbation (due to pressure effect and damping moment)

$$K_1'' = -\left(2l/u^2\right)PA\cos\varepsilon_1\cos\varepsilon_2 + \varepsilon K_t^{-2}\left(C_{Mqt} + C_{Mq\alpha}\right)$$

$$K_2'' = -\zeta\varepsilon C_{Np\alpha}$$

$$K_3'' = -\overline{\omega}_1^2\varepsilon C_{Np\alpha} - B'\zeta C_x C_{Np\alpha} + (2l/u^2)PA\cos\varepsilon_1\cos\varepsilon_2[(2g_x l/u^2) - \varepsilon(2C_D - C_{L\alpha})$$

$$-\varepsilon K_t^{-2}(C_{Mqt} + C_{Mq\alpha})] - \varepsilon K_t^{-2}(C_{Mqt} + C_{Mq\alpha})[g_x l/u^2) - \varepsilon(C_D - C_{L\alpha})]$$

$$K_4'' = -\overline{\omega}_1[-\varepsilon^2 C_D C_{Np\alpha} - (\varepsilon^2 C_{Np\alpha} - \varepsilon)K_t^{-2}(C_{Mqt}C_{Mq\alpha})]$$

$$-(2l/u^2)PA\cos\varepsilon_1\cos\varepsilon_2[B'\zeta - 2\overline{\omega}_1 + \overline{\omega}_1\varepsilon C_{Np\alpha})$$

$$K_5'' = (2l/u^2)PA\cos\varepsilon_1 \cos\varepsilon_2 - \varepsilon K^{-2}{}_t\,(C_{Mqt} + C_{Mq\alpha}) + (2l/u^2)PA\sin\varepsilon_1$$

$$+ (B'\zeta - \overline{\omega}_1)(2l/u^2)PA\sin\varepsilon_2 - \varepsilon K^{-2}{}_t\, r_p PAB_2/(u^2 m)$$

$$K_6'' = (2l/u^2)(-PA\sin\varepsilon_1 \cos\varepsilon_2)(\overline{\omega}_1 - B'\zeta) - \varepsilon K^{-2}{}_t\, r_p PAB_1/(u^2 m)$$

$$-\ (2l/u^2)PA\sin\varepsilon_2\ [g_x\, l/u^2 - \varepsilon C_D + (2l/u^2)PA\cos\varepsilon_1 \cos\varepsilon_2 - \varepsilon K^{-2}{}_t\,(C_{Mqt} + C_{Mq\alpha})] \qquad (29)$$

8. STABILITY CRITERIA

The projectile is made stable by applying spin or by attaching fins to the projectile. For fin-stabilised projectile, $K_3' > 0$ and for fin-stabilised bodies, $K_3' < 0$. For FSAPDS round, K_3' can be positive or negative.

A spinning projectile is made stable by static stability parameter

$$S_S = \frac{I_{XX}^2 \omega^2}{4 I_{YY} C_{M\alpha}} \quad ;\ 1 < S_S < 3.5 \text{ for the stable bodies.} \qquad (30)$$

During motion due to perturbation and aerodynamic forces/moments, the projectile can become unstable. This stability is called as dynamic stability. The dynamic stability can be studied with the stability condition for the motion of projectile as given by Naik[4] is:

$$K_1'^2 K_3' + K_1' K_2' K_4' - K_3'^2 - K_4'^2 > 0 \qquad (31)$$

$$-\left(\frac{K_3'}{K_1'} - \frac{K_1'}{2}\right)^2 + \frac{K_1'^2}{4} + \frac{K_2'}{4} - \left(\frac{K_4'}{K_1'}\right)^2 > 0 \qquad (32)$$

so Eqn (31) reduces to

$$\frac{K_1'^2}{4} + \frac{K_2' K_4'}{4} - \left(\frac{K_4'}{K_1'}\right)^2 > 0\ . \qquad (33)$$

The modified stability parameter

$$S = 1 + \frac{(2K_4'/K_1') - K_2'}{\left[K_1'^2 + K_2'^2\right]^{1/2}} \text{ reduces this condition to } S(S-2) > 0 \qquad (34)$$

which means S lies between 0 to 2.

Equation (31) can be also be expressed as

$$-4\left(K_4' - \frac{K_1' K_2'}{2}\right)^2 + K_1'^2 (K_2'^2 + 4K_3') - 4K_3'^2 > 0 \qquad (35)$$

It is satisfied provided $K_2'^2 + 4K_3' > 0$, which is the same as gyroscopic stability given by Cranz stability parameter σ as $0 < \sigma < 1$. (36)

To study the effect due to perturbation, let Liapunov's approach be followed.

P-method [5] has been applied to develop the Liapunov function.

The motion is given by Eqn (26)

$$L \equiv X'' + (-K_1 I + K_2 J)X' + (K_3 I + K_4 J)X' = 0 \tag{37}$$

Define a generating function as

$$N \equiv 2\dot{X} + P \tag{38}$$

the inner product $< L, N > + < N, L > \equiv 0$ (39)

The Eqn (36) leads to a suitable Liapunov function V and its derivative $\dot{V}$

$$V = 2(\dot{X}^t \frac{X^t P^t}{2})(\dot{X} + \frac{PX}{2}) + X^t(2K_3 + 2Q - \frac{P^t P}{2})X \tag{40}$$

and

$$\begin{aligned}\dot{V} = \{&\dot{X}^t[-(P + P^t + 4K_1)]^{-\frac{1}{2}} - X^t(K_1 I + K_2 J)P + 2K_4 J + 2Q^t[-(P + P^y + 4K_1)\}^{-\frac{1}{2}} \\ &[(-(P + P' + 4K_1))^{-\frac{1}{2}}\dot{X} - (-(P + P^t + 4K_1)^{-1})^{\frac{1}{2}}\{(K_1 I + K_2 J)P + 2K_4 J + 2Q\}X] \\ &-X^t[\{(K_1 I + K_2 J)P + 2K_4 J + 2Q\}^t\{-(P + P^t + 4K_1)\}^{-1}\{(K_1 I + K_2 J)P \\ &+2K_4 J + 2Q\} + \{(K_3 I + K_4 J)P + P^t((K_3 I + K_4 J)^t\}]X\end{aligned} \tag{41}$$

Q can be selected as $Q = -K_3 + \dfrac{P^t P}{2}$ (42)

since P^t–P>0, V is always positive.

A choice of $P = -K_1 I + K_2 J$ gives the condition for $\dot{V} < 0$ as

$$K_1^2 K_3 + K_1 K_2 K_4 - K_3^2 - K_4^2 > 0 \tag{43}$$

which is similar to Eqn (31).

Following the same steps, the modify stability parameter is defined as

$$S_M = 1 + \frac{(2K_4 / K_1) - K_2}{\sqrt{K_1^2 + K_2^2}} \tag{44}$$

where the condition for stability is

$$S_M(S_M-2)>0$$

that is $0<S_M<2$ (45)

Separating the terms of K_i' *and* K_i'' the condition becomes:

$$S_M=S+\frac{2K_1'(K_1'^2+K_2'^2)K_4''-2(K_1'^2+K_2'^2)K_4'K_1''}{K_1'^2(K_1'^2+K_2'^2)^{3/2}}$$

$$-\frac{K_1''K_1'^2(2K_4'-K_1'K_2')+K_1'^4K_2'+2K_1'K_2'K_4'K_2''}{K_1'^2(K_1'^2+K_2'^2)^{3/2}}<2 \qquad (46)$$

The other condition from Cranz stability parameters becomes:

$$K_2^{\,2}+4K_3>0.$$

Substituting for K_2 and K_3, one gets:

$$K_3''>(-K_2^{\,2}\sigma^2/4). \qquad (47)$$

Equations (46) and (47) give the conditions for stability of FSAPDS round in the first phase due to malalignment.

For the given data, the value of stability parameter gets modified from S=1.859147 to S_M=1.891418 due to perturbation. The value of this parameter still lies between 0 to 2 and therefore, does not vary stability.

9. CONCLUSIONS

- For an FSAPDS projectile, a mathematical model has been developed in 6-DOFS. The trajectory has been simulated. It is observed that the addition of pressure and damping moment in the simulation does not affect the trajectory, in the first phase of motion.

- Stability of an FSAPDS projectile in the first phase is affected by the gas pressure as well as damping moment. A modified stability parameter has been developed using the Liapunov function with the help of generating matrix (P) and parametric matrix (Q). The stability in the cross plane can be achieved for $K_3''>(-K_2^2\sigma^2/4)$.

  For a particular data, it was verified that the stability will not be disturbed due to pressure and damping moment, as their contributions are very small.

- The two conditions

$$S_S=\frac{I_{XX}^{\,2}\omega^2}{4I_{YY}C_{M\alpha}} \quad ; \; 1<S_S<3.5 \;;$$

$$S=1+\frac{(2K_4'/K_1')-K_2'}{\left[K_1'^2+K_2'^2\right]^{1/2}} \quad ; \quad 0<S<2$$

  are sufficient for the designer to decide minimum spin.

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