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Converging Cylindrical Shock Waves in a Nonideal Gas With an Axial Magnetic Field

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ABSTRACT

This paper analyses the propagation of converging cylindrical shock waves in a nonideal gas, in the presence of an axial magnetic field. Chester-Chisnell-Whitham's method has been employed to determine the shock velocity and the other flow-variables just behind the shock in the cases, when (i) the gas is weakly ionised before and behind the shock front, (ii) the gas is strongly ionised before and behind the shock front, and (iii) nonionised gas undergoes intense ionisation as a result of the passage of the shock. The effects of the nonidealness of the gas, the conductivity of the gas, and the axial magnetic field have been investigated. It is found that in the case (i), an increase in the value of parameter (δ) characterising the nonidealness of the gas accelerates the convergence of the shock. In the case (ii), the shock speed and pressure behind the shock increase very fast as the axis is approached; and this increase occurs earlier if the strength of the initial magnetic field is increased. In the case (iii), for smaller values of the initial magnetic field, the shock speed, and pressure behind the shock decrease very fast after attaining a maximum; and for higher values of the initial magnetic field, the tendency of decrease appears from the beginning. This shows that the magnetic field has damping effect on the shock propagation. In the case (iii), it was also found that the growth of the shock in the initial phase and decay in the last phase were faster when it was converging in a nonideal gas in comparison with that in a perfect gas. Further, it has been shown that the gas-ionising nature of the shock has damping effect on its convergence.

Keywords: Converging shock waves, axial magnetic field, CCW method, nonideal gas

1. INTRODUCTION

The converging shock and detonation waves offer interesting possibilities of attaining extremely high temperature, pressure, and density. In fact, even the applications to thermonuclear fusion, synthesising materials, phenomenon of sonoluminescence, and treatment of stones in the human body (lithotripsy) were considered[1-6]. The problem of converging shock was first solved by Guderley[7]. The similarity solution was presented and the self-amplifying character of the wave was suggested. This gives a shock of infinite strength at the centre of convergence. Payne[8] has given a numerical solution for a converging cylindrical shock. Stanyukovich[9] has discussed the problem of a contracting spherical or cylindrical shock front propagating into a uniform gas at rest. Zeldovich and Raizer[10] have summarised two kinds of self-similarity, and in the second kind, have described the implosion of a spherical shock in a gas and the collapse of spherical bubbles in a liquid. The problem has been further extended wrt simple and accurate

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determination of similarity exponent or to include the effects of non-homogenity or nonidealness of the medium or the effects of thermal radiation and magnetic field[3,4,11-16]. The self-similar solution, describing imploding shocks, have asymptotic character because they describe the process of implosion in a region near the axis (or centre) in which it is no more influenced by the initial conditions. The global solution, that is, a solution that can describe the whole history of the fluid motion, from the initial stage to the focusing stage, was presented by Mastuo[17] by a non-similar approximated method.

Whitham[18] has given a very simple and effective rule for the analysis of imploding shocks. This method has found numerous applications[9], eg, the description of the propagation process of converging shock waves through a channel of variable section[19] in a dusty gas with variable density[20] and in a solid[21] and the analysis of imploding detonation waves[22]. Although the Whitham's rule is approximate, but it agrees well with exact solutions and with experimental results[22-24].

Nagayama[25] has given and experimentally verified (Nagayama and Mashimo[26]) a method for achieving very high pressure by compression of magnetic flux by means of a converging shock wave in a semiconducting material, which becomes highly conductor due to passage of shock wave. Tyl and Wlodarczyk[27] has given a relatively wide theoretical analysis of this compression process of magnetic flux. The analysis was made by Whitham's rule[18], the computation was performed for crystalline silicon and silicon powder, and the results are found in good agreement with the experimental results[26] Tyl and Wlodarczyk[28] have suggested to use the above method for a system for isentropic compression of materials. They indicated that the experimental realisation of the proposed system is simpler and the degree of compression of the material is higher than achieved with systems in which traditional methods are used. In continuation of these works, Tyl[29] has presented a detailed theoretical analysis of implosion process of a cylindrical shock wave in an ideal gas in the presence of a magnetic field.

When the flow takes place at high temperatures, the assumption that the gas is ideal is no more valid. Anisimov and Spiner[30] have taken an equation of state for low-density nonideal gases in a simplified form, and investigated the effect of parameter for nonidealness on the problem of a strong point explosion. Ranga Rao and Purohit[31] and Ojha[32] have also studied the propagation of shock waves in gases with the above equation of state. In the present work, the convergence of a cylindrical shock wave in a nonideal gas has been analysed with the equation of state given by Anisimov and Spiner[30], in the presence of an axial magnetic field. The effects of the nonidealness of the gas, the conductivity of the gas, and the axial magnetic field have been investigated.

During the experiments involving the implosion of a shock wave in a gas, the following states may occur[29]:

(a) The gas is weakly ionised before and behind the shock front, ie, $R_m << 1$, where R_m is the magnetic Reynolds number.

(b) The gas is strongly ionised before and behind the shock front, ie, $R_m >> 1$ or $\sigma \to \infty$, where σ is the electrical conductivity.

(c) Nonionised (or weakly ionised) gas undergoes intense ionisation as a result of the passage of the shock, ie, σ increases in a jump like manner from 0 to ∞.

In the present, all the three cases have been analysed by taking a constant axial magnetic field. Chester-Chisnell-Whitham's method[18,19,33] (Whitham's rule) was employed to determine the shock velocity and the other flow variables just behind the shock.

2. FUNDAMENTAL EQUATIONS AND BOUNDARY CONDITIONS

The equation of state for a nonideal gas is borrowed from the statistical physics[34] which has been simplied by Anisimov and Spiner[30] in the form:

$$p = R^* \rho T \left(1 + \bar{b}\rho\right) \quad (1)$$

where $\bar{b}$ (<< 1) is the internal volume of the molecules and R^* is the gas constant. p, ρ and T are the pressure, density, and temperature of the gas, respectively. Roberts and Wu[3, 4] have used an equivalent equation of state to study the shock wave theory of sonoluminescence.

The internal energy (E) per unit mass is given by Singh and Singh[35], Ojha[32], as

$$E = \frac{p}{\rho(\gamma - 1)(1 + \bar{b}\rho)} \qquad (2)$$

which implies that

$$C_p - C_v = R^* \left(1 + \frac{\bar{b}^2 \rho^2}{1 + 2\bar{b}\rho}\right) \cong R^* \qquad (3)$$

neglecting the term $\bar{b}^2\rho^2$. Here C_p and C_v are the specific heats of the gas at constant pressure and constant volume, respectively.

The basic equations governing the unsteady and cylindrically symmetric motion of a weakly conducting nonideal gas (*Case* I, $R_m << 1$) are given by Tyl[29] and Sakurai[36] as

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\frac{\partial u}{\partial r} + \frac{\partial u}{r} = 0 \qquad (4)$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r}\right) + \frac{\partial p}{\partial r} = -\sigma B_0^2 u \qquad (5)$$

$$\left(\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial r}\right) - a^2\left(\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r}\right) = (\gamma - 1)(1 + \bar{b}\rho)\sigma B_0^2 u^2 \qquad (6)$$

$$\frac{\partial B}{\partial r} = \mu\, \sigma\, B_0\, u \qquad (7)$$

where u, B are the velocity and axial magnetic induction at distance r from the axis of symmetry; B_0 is the value of B in the undisturbed state; γ is the ratio of specific heats; μ is the magnetic permeability; and a the speed of sound in the nonideal gas, is given by

$$a^2 = \frac{\gamma p}{\rho}\left(\frac{1 + 2\bar{b}\rho}{1 + \bar{b}\rho}\right)$$

Equations (4) to (6) can be combined to obtain the characteristic equation[18,29] as

$$dp - \rho a du + \frac{\rho a^2 u\, dr}{u - a\; r} = \left[\frac{(\gamma - 1)(1 + \bar{b}\rho)u^2 + ua}{u - a}\right]\sigma B_0^2 dr \qquad (8)$$

along the negative characteristic

$$\frac{dr}{dt} = u - a \qquad (9)$$

The fundamental equations governing the unsteady flow behind a cylindrical magnetogasdynamic (*Case* II, $R_m >> 1$) or gas-ionising (*Case* III, $\sigma : 0 \to \infty$) shock are given by Whitham[18], Vishwakarma and Yadav[37] as

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\frac{\partial u}{\partial r} + \frac{\rho u}{r} = 0 \qquad (10)$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r}\right) + \frac{\partial p}{\partial r} + \frac{B}{\mu}\frac{\partial b}{\partial r} = 0 \qquad (11)$$

$$\left(\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial r}\right) - a^2\left(\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r}\right) = 0 \qquad (12)$$

$$\frac{\partial B}{\partial t} + \frac{1}{1}\frac{\partial}{\partial r}(rBu) = 0 \qquad (13)$$

Equations (10) to (13) can be combined to obtain the characteristic equation[18] as

$$dp + \frac{1}{\mu}BdB - \rho c du + \frac{\rho c^2 u}{u - c}\frac{dr}{r} = 0 \qquad (14)$$

along the negative characteristic

$$\frac{dr}{dt} = u - c \tag{15}$$

where c is the effective speed of sound given by $c^2 = a^2 + b^2$ and $b^2 = B^2/\mu\rho$.

Since σ is small in the *Case* I, and σ is zero ahead of the shock in the *Case* III, the magnetic induction may be taken continuous across the shock in these cases[36,38] The strong shock conditions (ie, the pressure ahead of the shock is assumed negligible) in the Cases I and III are, therefore,

$$u_1 = \frac{D(\beta-1)}{\beta}$$
$$\rho_1 = \rho_0 \beta$$
$$\rho_1 = \rho_0 D^2 \frac{(\beta-1)}{\beta} \tag{16}$$
$$a_1 = \sqrt{\frac{\gamma\,(\beta-1)\,(\gamma-1+2\delta\beta)}{(\gamma-1+\delta\beta)}}\,\frac{|D|}{\beta}$$

where D is the speed of the shock front, and the indices 1 and 0 refer to the states just behind and just ahead of the shock.

Also

$$\frac{1}{\beta} = \frac{(\gamma-1-\delta) + \sqrt{\{(\gamma-1-\delta)^2 + 4\delta\,(\gamma+1)\}}}{2\,(\gamma+1)}$$

and $\delta = (\gamma - 1)\,\overline{b}\,\rho_0$.

In the pure magnetogasdynamic case (*Case* II), the gas is strongly ionised, ie, highly conducting, before and behind of the shock front, upon which the magnetic induction may be discontinuous at the shock front resulting from a sheet current there[36] considering the constant ambient pressure $p_{o,}$ the shock conditions, in this case, may be written in the form (Whitham[18] and Tyl[29] :

$$\rho_1 = \xi\rho_0$$

$$B_1 = \xi B_0$$

$$|u_1| = \frac{2(\xi-1)^2}{[(\gamma+1)\xi - (\gamma-1)\xi^2 + \delta\xi^2\,(1-\xi)]}$$
$$\left(\frac{a_0^2}{\gamma\,(\gamma-1+2\delta)}\left\{\gamma(\gamma-1) + \xi\delta^2 + \gamma\delta\,(1+\xi)\right\} + b_0^2\left[\left(1-\frac{\gamma}{2}\right)\xi + \frac{\gamma}{2} + \frac{\delta\xi}{2}(1-\xi)\right]\right)$$

$$p_1 = p_0 + \frac{2\rho_0\,(\xi-1)}{[(\gamma+1) - (\gamma-1)\xi + \delta\xi\,(1-\xi)]}$$
$$\left(\frac{a_0^2}{\gamma\,(\gamma-1+2\delta)}\left[\gamma(\gamma-1) + \xi\delta^2 + \gamma\delta\,(1+\xi)\right] + \frac{b_0^2}{4}(\xi-1)^2\left\{(\gamma-1) + \delta\xi\right\}\right)$$

$$D = \frac{u_1\xi}{\xi-1} \tag{17}$$

In the *Case* III, where a non-conducting (or weakly conducting) gas becomes highly conducting due to passage of a strong shock, and there is no jump of the magnetic induction across the shock, the medium behind the shock acts as a piston compressing the magnetic flux and pushing it into the region ahead of the shock. In fact, the speed of the shock is higher than the speed of the conducting medium behind the shock, therefore the magnetic flux is transported by the convection from the compression region even if the medium behind the shock in an ideal conductor. The convection of the magnetic flux leads to the formation of a current-carrying layer of considerable thickness behind the shock front, and this fact increases the compression of the magnetic flux. In this way, the magnetic flux decreases in the compression region and increases infront of the shock during the implosion process, and therefore, there is an additional law of conservation of magnetic flux in the form[25,27,29] as

$$\frac{dB}{B} = -\frac{2u_1}{D}\frac{dR}{R} \tag{18}$$

where R is the shock radius.

3. SOLUTION OF THE PROBLEM

The shock speed D and the flow variables just behind the shock are obtained using the Whitham's rule in all the three cases. For converging shocks, the rule is to apply the characteristic equation (valid along a negative characteristic) to the flow quantities just behind the shock front.

Case I

Shock Wave in a Weakly Conducting Nonideal Gas, $R_m << 1$

Using the values of the flow variables just behind the shock, given by the Eqn (16), into the characteristic [Eqn (8)] bearing in the mind that *D* and *u* are negative, one obtains:

$$K_1 \frac{d|D|}{dR} + K_2 \frac{|D|}{R} = K_3 \frac{\sigma B_0^2}{2\rho_0}$$

where

$$K_1 = 1 + \frac{1}{2} K_4 (\beta - 1)$$
$$K_2 = \frac{\gamma(\gamma - 1 + 2\delta\beta)}{2(\gamma - 1 + \delta\beta)\left[1 + K_4\right]} \tag{19}$$
$$K_3 = 1 - \frac{(\gamma + \delta\beta)}{1 + K_4}$$

and

$$K_4 = \sqrt{\frac{\gamma(\gamma - 1 + 2\delta\beta)}{(\gamma - 1 + \delta\beta)(\beta - 1)}}$$

Integrating the Eqn (19), with the initial conditions $D = D_i$ at $R = R_i$, one obtains the solution as

$$\frac{D}{D_i} = (1 - K_5)\, \lambda^{-K_6} + K_5 \lambda \tag{20}$$

where $\lambda = \dfrac{R}{R_i}$

$$K_5 = \frac{B_0^2 R_m \beta}{2\rho_0 D_1^2 (\beta - 1)\, \mu} \frac{K_3}{(K_1 + K_2)}$$

$$R_m = |u_{1i}| R_i \; \sigma\mu$$

$$K_6 = \frac{K_2}{K_1}$$

and u_{1i} is the value of u_1 at $R = R_i$. Also, from the shock conditions [Eqn(16)], one obtains:

$$\frac{u_1}{u_{1i}} = \frac{D}{D_i}$$

$$\frac{P_1}{P_{1i}} = \left(\frac{D}{D_i}\right)^2 \tag{21}$$

where p_{1i} is the values of p_1 at $R = R_i$.

Case II

Pure Magnetogasdynamic Shock Wave, $R_m >> 1$

Using the values of flow variables just behind the shock, given by Eqn (17), into the characteristic equation [Eqn (14)], one obtains after some simplifications:

$$\frac{d\xi}{d\lambda} = \frac{-c_1^2 |u_1| \Big/ \left(|u_1| + c_1\right)}{\lambda \left[\frac{d|u_1|}{d\xi} \left\{ \frac{2|u_1|}{\xi - 1} + c_1 \right\} - \frac{u_1^2}{\xi(\xi - 1)^2} \right]} \tag{22}$$

where

$$u_1(\xi) = \frac{2(\xi - 1)^2}{\left\{(\gamma + 1)\xi - (\gamma - 1)\xi^2 + \delta\xi^2 (1 - \xi)\right\}}$$
$$\left[\frac{a_0^2}{\gamma(\gamma - 1 + 2\delta)} \left\{ \gamma(\gamma - 1) + \delta^2 \xi + \gamma\delta(1 + \xi) \right\} + b_0^2 \left\{ \left(1 - \frac{\gamma}{2}\right) \xi + \frac{\gamma}{2} + \frac{\delta\xi}{2} (1 - \xi) \right\} \right]$$

$$c_1(\xi) = \sqrt{\frac{(\gamma - 1 + 2\delta\xi)\gamma P_1}{(\gamma - 1 + \delta\xi)\rho_0\xi} + \frac{B_0^2\xi^2}{\mu\rho_0\xi}} \tag{23}$$

Also, from Eqn (17), one has:

$$\frac{\rho_1}{\rho_{1i}} = \frac{\xi}{\xi_i};\ \frac{B_1}{B_{1i}} = \frac{\xi}{\xi_i} \text{ and } \frac{D}{D_i} = \frac{u_1\xi(\xi_i - 1)}{(\xi - 1)u_{1i}\xi_i} \tag{24}$$

where ρ_{1i}, B_{1i}, D_i and ξ_i are respectively the values of ρ_1, B_1, D and ξ at $\lambda = 1$.

Numerical integration of the differential Eqn (22) with initial conditions $\xi = \xi_i$, $\lambda = 1$ gives the values of ξ as λ decreases from 1 to zero. Then the values of $\left(\frac{P_1}{P_{1i}}\right), \left(\frac{\rho_1}{\rho_{1i}}\right), \left(\frac{u_1}{u_{1i}}\right)$ and $\left(\frac{D}{D_i}\right)$ can be obtained from the Eqns (23) and (24).

Case III

Gas-ionising Shock Wave ($\sigma : 0 \rightarrow \infty$)

Making use of the first of the equation [Eqn (16)], one can write the Eqn (18) in the form:

$$\frac{dB}{B} = -2\frac{(\beta - 1)}{\beta}\frac{dR}{R} \tag{25}$$

which on integration gives

$$B = B_0\lambda^{-2(\beta-1)/\beta}, \text{ where } B = B_0 \text{ at } R = R_i \tag{26}$$

Equation (26) shows that the magnetic induction just ahead of the shock (= magnetic induction just behind of the shock) does not remain constant but increases according as $R^{-2(\beta-1)/\beta}$ during the convergence process. This phenomenon is due to the fact that there is transport of the magnetic flux from the region behind the shock to the region ahead of it (as indicated in Section 2).

Using the flow variables just behind the shock given by the Eqns (16) and (25) for the flow variables in the characteristic equation [Eqn (14)], one obtains after some simplifications:

$$\frac{dD}{d\lambda} = \frac{D}{\lambda}\frac{(\beta - 1)}{\beta}\frac{[1 - 2F(D,\lambda)]}{F(D,\lambda)} \tag{27}$$

$$2F(D,\lambda) = \frac{(2 + q\beta)}{2 + q\beta - \frac{q^2\beta^2}{(\beta - 1) + q\beta} + \frac{B^2\beta}{\mu\rho_0 D^2(\beta - 1)}}$$

$$q = \left\{\frac{\frac{\gamma}{\beta^2}\frac{(\gamma - 1 + 2\delta\beta)}{(\gamma - 1 + \delta\beta)}(\beta - 1)}{\left[1 + \frac{1}{\gamma}\frac{(\gamma - 1 + \delta\beta)}{(\gamma - 1 + 2\delta\beta)}\frac{B^2\beta}{\mu\rho_0 D^2(\beta - 1)}\right]}\right\}^{\frac{1}{2}}$$

and B, in terms of λ is given by Eqn (26).

Further, from equations (16) and (26), one has:

$$\frac{u_1}{u_1 i} = \frac{D}{D_i};\ \frac{P_1}{P_{1i}} = \left(\frac{D}{D_i}\right)^2 \text{ and } \frac{B_1}{B_{1i}} = \lambda^{-2(\beta-1)/\beta} \tag{28}$$

where $B_{1i} = B_0$ and $D_i = \frac{u_{1i}\beta}{\beta - 1}$

By numerical integration of the differential equation [Eqn(27)] with initial conditions $D = D_i$ at $\lambda = 1$, one can calculate the values of D/D_i, u_1/u_{1i}, p_1/p_{1i} in terms of λ.

4. RESULTS AND DISCUSSION

For the purpose of numerical calculations, the values of R_m, δ, γ, B_0, μ, ρ_0, u_{1i}, p_0 given by Tyl[29] were used.

R_m = 0.001 (in the *Case* I only); δ = 0, 0.025, 0.05; γ = 1.4; B_0 = 0.5, 5.0 Tesla

$\mu = 4\pi \times 10^{-7}$ Henary/metre; $\rho_0 = 1$ kg/m^3;

$|u_{1i}|$ = 4 x 10^3 m/s; and p_0 = 10^5 pascal (in the *Case* II, only). The value δ = 0 corresponds to the case of a perfect gas.

In the *Case* I, the value of D/D_i are calculated from the Eqn (20) in terms of λ. It is found that an increase in the value of the parameter δ characterising the nonidealness of the gas, accelerated the convergence of the shock (Fig. 1). The effect of a change in the strength of the initial magnetic induction (B_0) is negligible (not shown in the figure). This is due to the fact that R_m << 1, in this case.

In the *Case* II, ξ the density ratio across the shock varies as the shock propagates, which is in contrast with the cases using strong shock assumptions (*Cases* I and III), where it is equal to β, a constant given in the Eqn (16). Values of ξ are obtained by numerical integration of the differential equation [Eqn (22)] and then P_1/P_{1i} and D/D_1 are calculated from Eqn (23) and (24) in terms of λ (Fig. 2 and 3). It is found that, in general, the shock speed and the pressure behind the shock increase very fast as the axis is approached. This increase occurs earlier if the initial magnetic induction, B_0 is increased. Earlier increase of shock speed means that the fall of the total pressure (gas pressure + magnetic pressure) ahead of the shock is more rapid in comparison with that behind of the shock for higher values of B_0. This fact is verified from Table 1 which shows that the density ratio across the shock (a measure of shock strength) increases rapidly during the convergence of the shock for higher values of B_0. The reduction of shock frontal areas also causes the increase in shock speed near the axis, in general. A change in the value of the parameter of nonidealness of the gas, δ has small effect on the shock velocity and the pressure behind the shock (Figs 2 and 3).

In the *Case* III, the values of D/D_i and P_1/P_{1i} are obtained in terms of λ, by numerical integration of the Eqn (27) and using Eqn (28). It was found that for smaller values of B_0, the shock speed and the pressure behind the shock decrease very fast after attaining a maximum; and for higher values of B_0 the tendency of decrease appears from the beginning (Figs 4 and 5). This shows that the magnetic field has damping effect

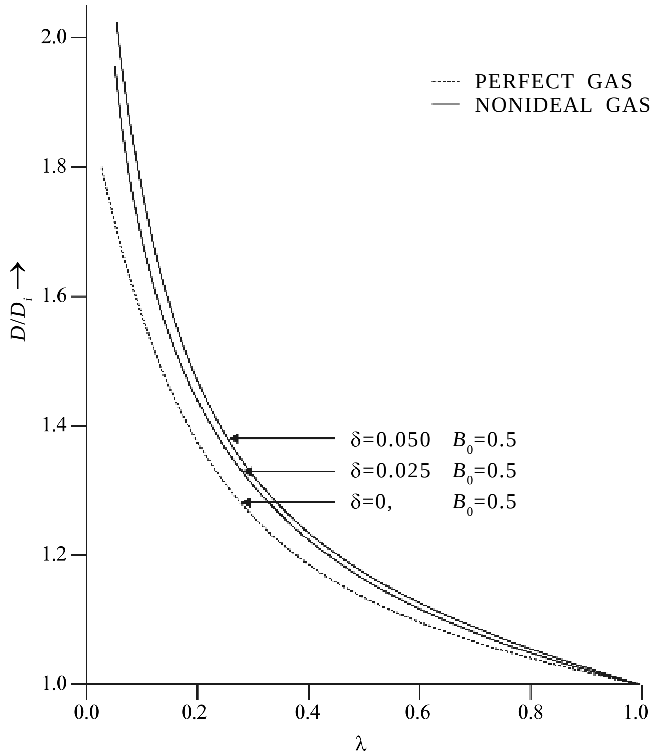

Figure 1. Variation of shock velocity with distance in *Case* I (R_m<<1).

on the convergence of gas-ionising shock. The phenomenon of decrease of the shock speed and post-shock pressure near the axis, which is in contrast with the *Cases* I and II, may be physically interpreted in the following way.

- During the convergence of a gas ionising shock, the magnetic flux in the conducting region behind the shock is compressed and pushed into the region ahead of the shock (Sections 2 and 3). This results in rapid increase of the ambient magnetic induction near the axis [according to Eqn (26)], which increases the magnetic pressure in the region ahead of the shock very fast in comparison with that behind of it, causing the strong decay of the shock.

- Due to cumulation of magnetic flux near the axis during the convergence of a gas-ionising cylindrical shock wave, a very high pressure may be achieved, there. This pressure may be used for isentropic compression of materials, in a simpler way, as suggested by Tyl and

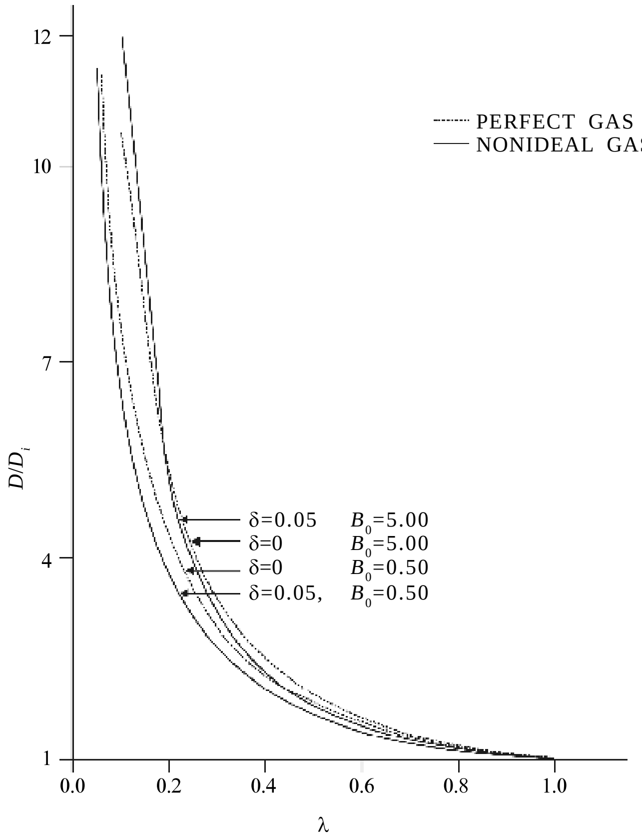

Figure 2. Variation of shock velocity with distance in *Case* II (R_m>>1).

Table 1. Variation of ξ, the density ratio across the shock with λ at different values of B_0 and δ, in the *Case* II (R_m >> 1)

λ	ξ			
	B_0 = 0.5		**B_0 = 5.0**	
	δ = 0	**δ = 0.05**	**δ = 0**	**δ =0.05**
1.0	5.3500	4.0700	1.9900	1.9620
0.9	5.4455	4.0988	2.1341	2.0918
0.8	5.5374	4.1261	2.3217	2.2550
0.7	5.6247	4.1518	2.5712	2.4610
0.6	5.7064	4.1757	2.9086	2.7196
0.5	5.7814	4.1975	3.3643	3.0343
0.4	5.8483	4.2169	3.9557	3.3918
03	5.9058	4.2337	4.6475	3.7507
0.2	5.9522	4.2474	5.3220	4.0456
0.1	5.9851	4.2573	5.8165	4.2184

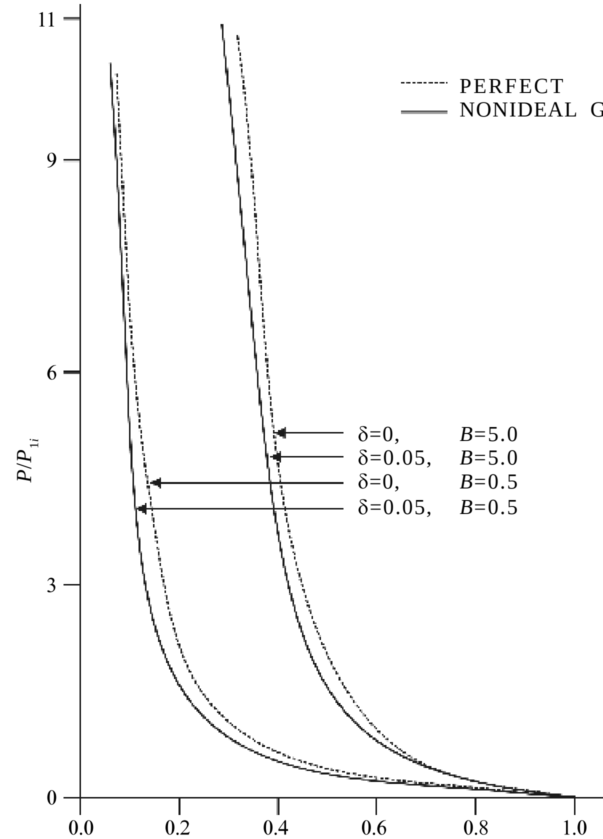

Figure 3. Variation of pressure with distance in *Case* II (R_m>>1).

Wlodarczyk[28]. In the case of a nonideal gas, the cumulation rate of magnetic flux, near the axis, is higher in comparison with that of a perfect gas, as the index 2(β – 1)/β in the Eqn (26) is smaller for a nonideal gas (Table 2). Thus, one may achieve comparatively higher pressure near the axis, when a shock is converging in a nonideal gas rather than in a perfect gas.

- Figure 4 also shows that the growth of the shock in the initial phase and the decay in the last phase are faster when the shock is converging in a nonideal gas (δ = 0.025, 0.05) in comparison with that in a perfect gas (δ = 0). This also confirms the generation of higher pressure near the axis in the case of a nonideal gas.

- On comparison of the curves in Figs 1, 2, and 4, it has been found that the conductivity of the gas has significant effect on the shock propagation. When conductivity (σ) is very high before and behind the shock (*Case* II), the velocity of the

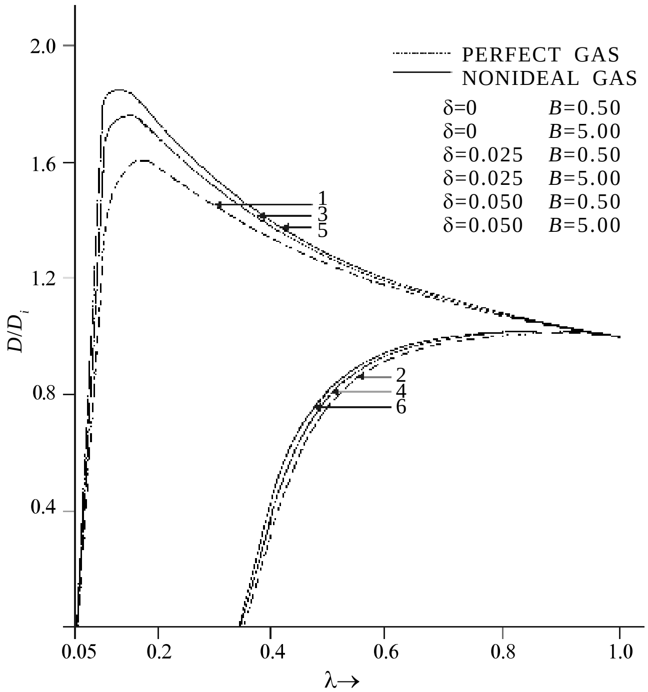

Figure 4. Variation of shock velocity with distance in *Case* III (σ : 0→ ∞ at the shock front).

shock increases very fast as it converges towards the axis (Fig. 2). In the *Case* I, where σ is very small (or zero) before and behind the shock, the shock velocity increases towards the axis, but slowly in comparison to the shock velocity increase in the *Case* II (Fig. 1). On the other hand, in *Case* III where σ is very small before the shock front and very high behind it, the shock decays very fast near the axis (Fig. 4). Thus, the gas-ionising nature of the shock has damping effect on its propagation.

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Table 2. Variation of the index 2(β – 1)/β in the Eqn (26) with the parameter of nonidealness of the gas δ for γ = 1.4

δ	0	0.025	0.050
2(β – 1)/β	1.66667	1.58669	1.53075

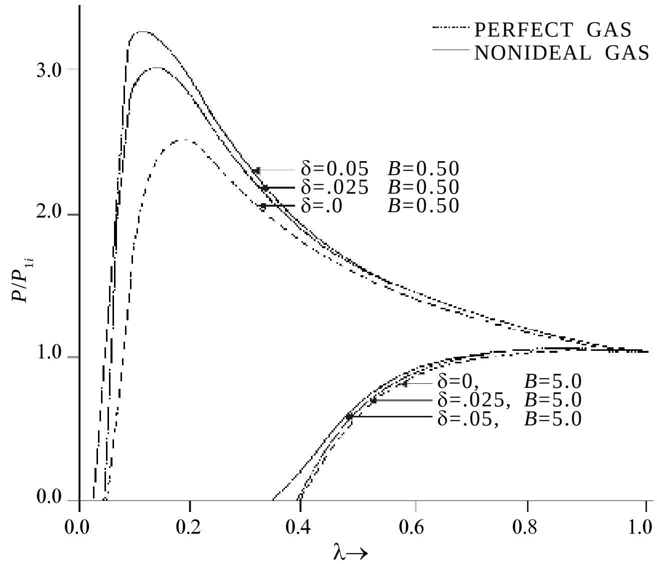

Figure 5. Variation of pressure with distance in *Case* III (σ : 0→ ∞ at the shock front).

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