

## Gravity Modulation Effects on Transient-free Convection Flow Past an Infinite Vertical Isothermal Plate

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### ABSTRACT

An exact solution to the free convection flow influenced by gravity modulation has been derived by the Laplace-transform technique. Velocity profiles, penetration distance from the leading edge, and the skin-friction expressions are derived. Velocity profiles and penetration distance are shown graphically, and the numerical values of the skin friction are tabulated. It was observed that the transient velocity decreased with increasing the frequency  $\omega$  of gravity-modulation or the Prandtl number but increased with increasing time  $t$ . Also, transition from conduction to convection was delayed with increasing  $\omega t$ , but it occurred nearer the leading edge as the frequency  $\omega$  or the Prandtl number increased.

**Keywords:** Gravity modulation, free convection flow, penetration distance, transient-free convection

### NOMENCLATURE

		$\varepsilon$	Amplitude
$C_p$	Specific heat at constant pressure	$\beta$	Coefficient of volume expansion
$g_0$	Acceleration due to gravity	$\nu$	Kinematic viscosity
$k$	Thermal conductivity	$\mu$	Dynamic viscosity
$t'$	Time	Pr	Prandtl number
$T$	Dimensionless time	$\rho$	Fluid density
$T'$	Temperature of fluid	$\theta$	Dimensionless temperature
$T'_w$	Temperature of the plate	$\tau$	Dimensionless wall shear stress
$T'_\infty$	Temperature of the fluid away from the plate	$\omega'$	Frequency
$u'$	Velocity component	$\omega$	Non-dimensional frequency
$u$	Non-dimensional velocity component	$L, t_0$	Reference length, time
$u_0$	Reference velocity	$y'$	Coordinate normal to the vertical plate
$X_p$	Penetration distance	$y$	Non-dimensional coordinate normal to the plate

$p, q, X, Y$	Dummy variables
$f, f_1, f_2$ $f_3, H$	Functions of dummy variables
$\eta, y/(2\sqrt{t})$	
$a, b$	Constants

## 1. INTRODUCTION

From convection flows of an incompressible viscous fluid past a semi-infinite or infinite vertical plate were studied during the past many years. Both steady and unsteady cases were studied because of their important applications in atmospheric and oceanic circulation, power transformer, nuclear reactor, vortex chambers, etc.

Transient-free convection flows past an infinite vertical plate were studied extensively in the 1960's by Gebhart<sup>1</sup>, Chung and Anderson<sup>2</sup>, Schetz and Eichhorn<sup>3</sup>, Goldstein and Briggs<sup>4</sup>, etc, who presented exact solutions under different physical situations. Siegel<sup>5</sup> studied unsteady-free convection flow near a semi-infinite vertical plate due to step-change in wall temperature or surface heat flux employing the momentum integral method, and established that the initial behaviour of the temperature and velocity fields for the semi-infinite vertical plate is the same as for the doubly infinite vertical flat plate.

Under this condition, the temperature field is the solution of an unsteady one-dimensional heat conducting equation. The transition from conduction to convection begins only when some effect from the leading edge has propagated up the plate to the particular point in question. This phenomenon was first presented by Siegel<sup>5</sup> and before this, it was not known that the plate has a leading edge. These studies were devoted to the physical situation where all physical properties of the fluids and all the forces acting were constant.

However, oscillating buoyancy forces also exist in nature and in microgravity, these flows are induced by a modulating gravity field or g-jitter. Compared with earth-based systems, the flow intensity is found to be drastically reduced as a result of a significant reduction in gravitational forces. A large number

of papers have been published on the well-known topic of Benard convection under modulated gravity field in the case of both linear and nonlinear phenomena. Noted are those by Clever<sup>6,7</sup>, *et al.*, and Fu and Shieh<sup>8</sup>, etc.

In space technology, in space vehicle, there are transient or time-varying perturbations to the gravity field at a point. This g-jitter phenomenon arises from spacecraft manoeuvres and mechanical vibrations. An excellent account of this physical feature has been described by Ostrach<sup>9</sup>. Li<sup>10</sup>, has studied the effect of g-jitter and the magnetic field on the natural convection phenomenon in a channel bounded by long vertical plate. He has represented the g-jitter as a sum of Fourier harmonic components with distinct frequencies and amplitudes, and studied different types of natural convection in magneto-hydrodynamic (MHD) channel flow.

However, the effect of oscillating gravitational field on the transient-free convection has not been studied in the past as literature study reveals. Because of good applications of this phenomenon, both in space and on the earth, it is necessary to study this phenomenon, and hence, it is now proposed to study the effect of oscillating gravity field on the transient-free convection flow past an infinite vertical isothermal plate.

## 2. MATHEMATICAL ANALYSIS

Consider unsteady-free convection flow of an incompressible viscous fluid past an infinite vertical plate. The  $x'$ -axis is taken along the plate in the vertical direction and the  $y'$ -axis is taken normal to the plate. Initially, the plate and the fluid are assumed to be at a uniform temperature  $T_\infty'$  and stationary. At time  $t' > 0$ , the plate temperature is instantaneously raised to  $T_w'$  and stationary. At time  $t' > 0$ , the plate temperature is instantaneously raised to  $T_w'$  causing a temperature difference  $T_w' - T_\infty' > 0$  which then causes convective motion of the fluid near the plate, and at the same time, the gravitational acceleration also starts oscillating with frequency  $\omega'$  and amplitude  $\epsilon$ , which is assumed constant. Then under usual Boussinesq's approximation, the flow can be shown to be governed by the following equations:

$$\frac{\partial u'}{\partial t'} = \beta g_0 (1 + \varepsilon \sin \omega' t') (T' - T'_\infty) + \nu \frac{\partial^2 u'}{\partial y'^2} \quad (1)$$

$$\rho C_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial y'^2} \quad (2)$$

with the following initial and boundary conditions:

$$\left. \begin{array}{l} u' = 0, T' = T'_\infty \text{ for all } y', t' \leq 0 \\ u' = 0, T' = T'_w \text{ at } y' = 0 \\ u' = 0, T' = T'_\infty \text{ as } y' \rightarrow \infty \end{array} \right\} t' > 0 \quad (3) \text{ As}$$

the plate is assumed to be infinite in length, the physical variables are functions of  $y'$  and  $t'$  only. Also, all the physical variables are defined in the nomenclature. The following scaling factor may be chosen for the velocity, length, and time.

$$\begin{aligned} u_o &= \left\{ \nu g_o \beta (T'_w - T'_\infty) \right\}^{1/3} \\ L &= \left\{ g_o \beta (T'_w - T'_\infty) / \nu^2 \right\}^{-1/3} \\ t_o &= \left\{ g_o \beta (T'_w - T'_\infty) \right\}^{-2/3} / \nu^{-1/3} \end{aligned} \quad (4)$$

and then the non-dimensional quantities are defined as follows:

$$\begin{aligned} u &= u' / u_o, \quad y = y' / L, \quad t = t' / t_o, \quad \omega = \omega' t_o \\ \theta &= (T' - T'_\infty) / (T'_w - T'_\infty), \quad Pr = \mu C_p / k \end{aligned} \quad (5)$$

Introducing Eqn (5) into Eqns (1)-(3), one has

$$\frac{\partial u}{\partial t} = \theta (1 + \varepsilon \sin \omega t) + \frac{\partial^2 u}{\partial y^2} \quad (6)$$

$$Pr \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} \quad (7)$$

with the following initial and boundary conditions:

$$\left. \begin{array}{l} u = 0, \theta = 0 \text{ for all } y, t \leq 0 \\ u = 0, \theta = 1 \text{ at } y = 0 \\ u = 0, \theta = 0 \text{ as } y \rightarrow \infty \end{array} \right\} t > 0 \quad (8)$$

These are coupled linear system of equations, which can be solved by the usual Laplace-transform technique. Then the solutions are:

$$\theta = \text{erfc}(\eta \sqrt{Pr}) \quad (9)$$

$$\begin{aligned} u &= \frac{1}{Pr-1} [H(\eta) - H(\eta \sqrt{Pr})] \\ &+ \frac{\varepsilon}{\omega} Rl \left\{ \begin{array}{l} e^{iat} f(\eta, a) - e^{bt} f(\eta, b) \\ + e^{bt} f(\eta \sqrt{Pr}, b-a) \end{array} \right\} \\ &+ \frac{\varepsilon \cos \omega t}{\omega} \text{erfc}(\eta \sqrt{Pr}) \end{aligned} \quad (10)$$

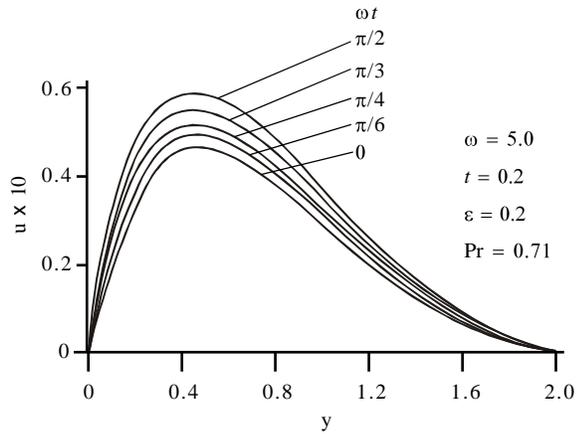
where

$$\begin{aligned} \eta &= y / 2\sqrt{t}, \quad a = i\omega, \quad b = aPr / (Pr - 1) \\ f(p, q) &= \frac{1}{2} \left\{ \begin{array}{l} e^{2p\sqrt{qt}} \text{erfc}(p + \sqrt{qt}) + \\ e^{-2p\sqrt{qt}} \text{erfc}(p - \sqrt{qt}) \end{array} \right\} \\ H(p) &= t \left\{ (1 + 2p^2) \text{erfc}(p) - 2pe^{-p^2} / \sqrt{\pi} \right\} \end{aligned}$$

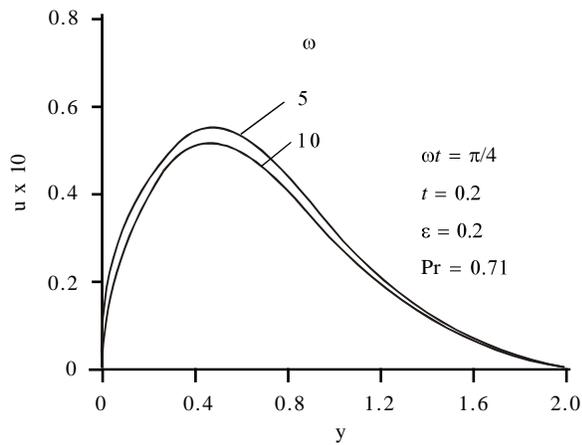
Here  $Rl\{ \}$  denotes the real part of the quantity enclosed and  $p, q$  are dummy variables. Moreover, the temperature profile given by Eqn (9) is well known.

To understand the effect of different parameters like  $Pr, \omega$ , etc. the numerical values of  $u$  have been computed by taking the real part of Eqn (10) and these are plotted in Figs 1 and 2. It has been observed from these figures that the transient velocity increases with increasing  $\omega t$  for both air and water, but the transient velocity decreases with increasing  $\omega$  for both air and water. The transient velocity also decreases with increasing the  $Pr$ .

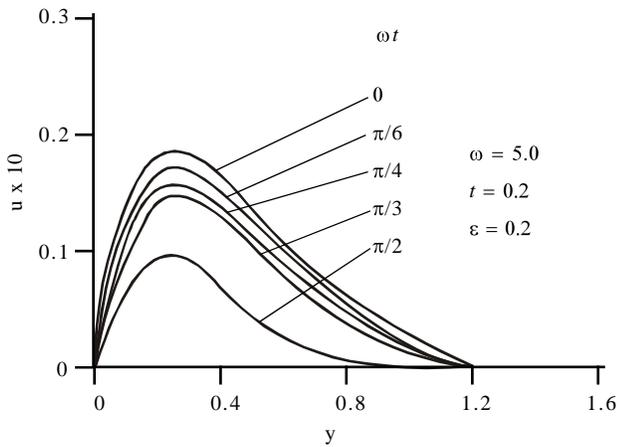
Now, it is interesting to understand the effect of gravity-modulation on the transition from conduction to convection, which is studied by deriving the



(a)



(b)



(c)

Figure 1. Velocity profiles.

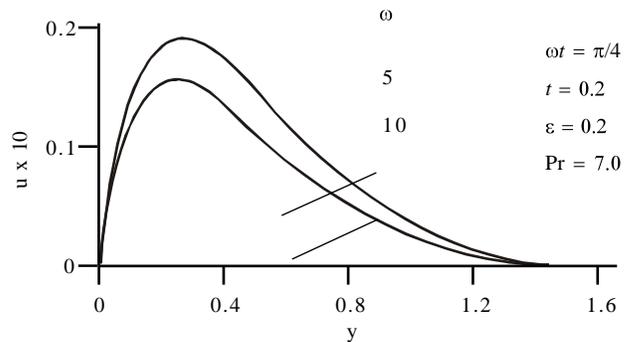
penetration distance of the point from the leading edge. Here, the leading edge is defined as the distance from any point  $x' = 0$  taken along the infinite plate in the upward direction.

It is given in non-dimensional form as and in terms of the Laplace transform and inverse transform, one has

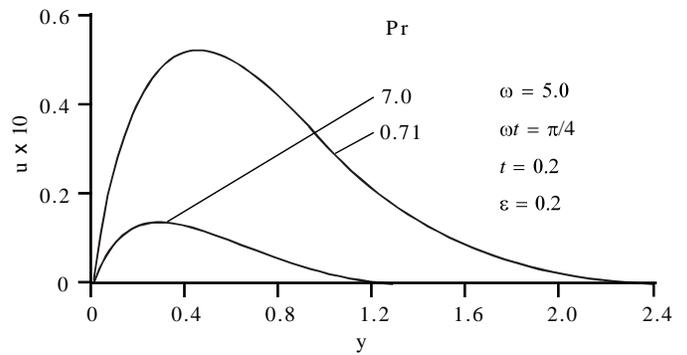
$$X_p = \int_0^t u(y,t) dt \tag{11}$$

Substituting for  $\bar{u}(y,s) = L\{u(y,t)\}$  from Eqn (10) in Eqn (11) and carrying out the algebra, one has

$$X_p = L^{-1} \left\{ \frac{1}{s} \bar{u}(y,s) \right\} \tag{12}$$



(a)



(b)

Figure 2. Velocity profiles.

$$X_p = \frac{1}{Pr-1} \left[ f_3(\eta) - f_3(\eta\sqrt{Pr}) \right] + \frac{\varepsilon}{\omega} Rl \left[ \frac{1}{a} \left\{ e^{ia\eta} f(\eta, a) - \operatorname{erfc}(\eta) + f(\eta\sqrt{Pr}, -a) \right\} + \frac{1}{b} \left\{ \operatorname{erfc}(\eta) - e^{b\eta} f(\eta, b) + e^{b\eta} f(\eta\sqrt{Pr}, b-a) - f(\eta\sqrt{Pr}, -a) \right\} \right] - \frac{\varepsilon \cos \omega t}{a\omega} \operatorname{erfc}(\eta\sqrt{Pr}) \quad (13)$$

where

$$f(X, Y) = \frac{1}{2} \left\{ \begin{array}{l} e^{2X\sqrt{Y}} \operatorname{erfc}(X + \sqrt{Yt}) \\ -e^{2X\sqrt{Y}} \operatorname{erfc}(X - \sqrt{Yt}) \end{array} \right\}$$

$$f_1(X) = 2\sqrt{t} \left\{ \frac{e^{-X^2}}{\sqrt{\pi}} - X \operatorname{erfc}(X) \right\}$$

$$f_2(X) = \frac{2t}{3} f_1(X) - \frac{2X\sqrt{t}}{3} H(X)$$

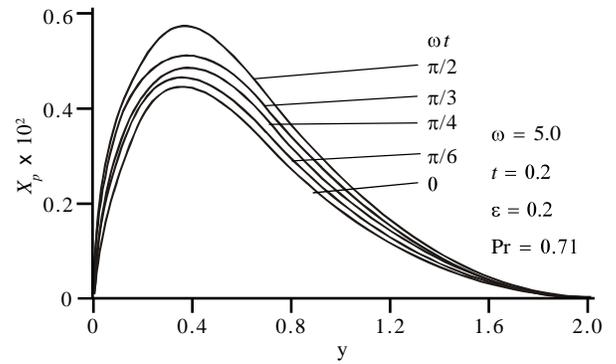
$$f_3(X) = \frac{t}{2} H(X) - \frac{X\sqrt{t}}{2} f_2(X)$$

where  $X, Y$  are dummy variables.  $X_p$  has been evaluated numerically and the variations in penetration distance are shown in Figs 3 and 4. It has been observed from these figures that the transition from conduction to convection is delayed as  $\omega t$  increases for both air and water, whereas transition from conduction to convection occurs nearer the leading edge as  $\omega$  increases or with increasing  $Pr$ .

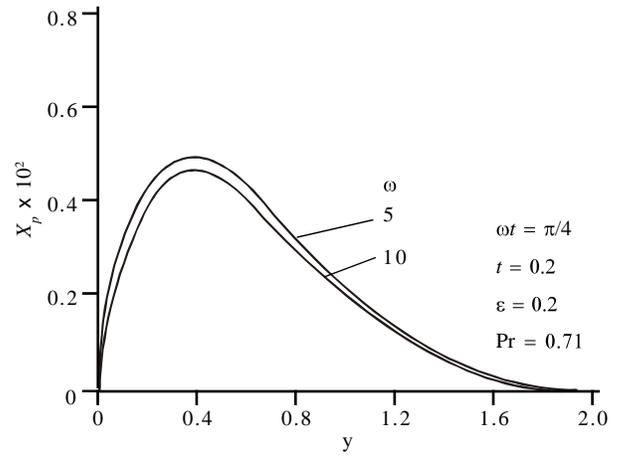
From the velocity field, it is now proposed to study the skin friction. It is given in non-dimensional form by

$$\tau = - \left. \frac{du}{dy} \right|_{y=0} \quad (14)$$

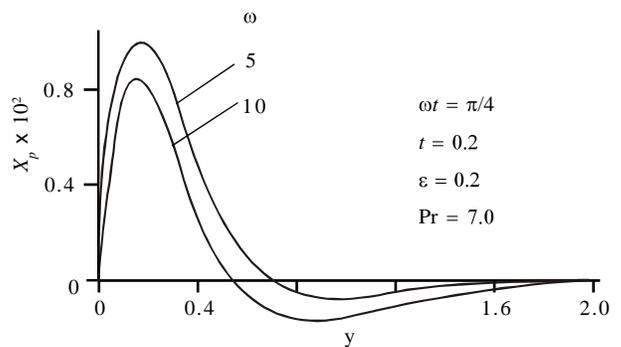
Then from Eqns (10) and (14), one has



(a)



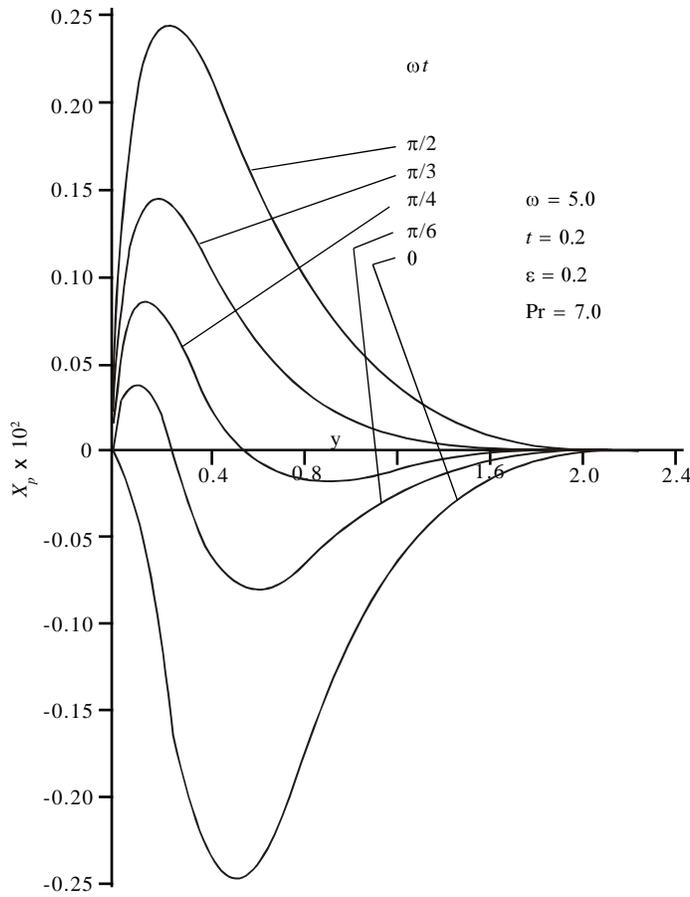
(b)



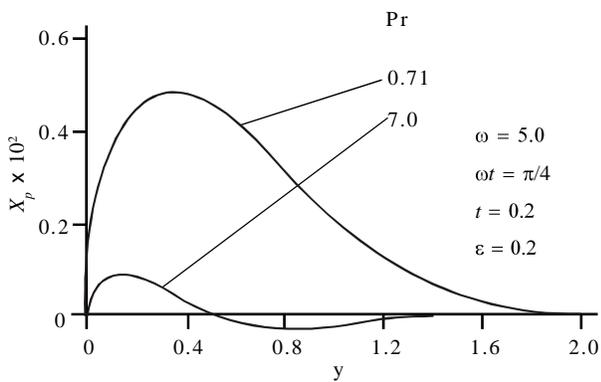
(c)

Figure 3. Penetration distance.

$$\tau = \frac{1}{Pr-1} \left[ \frac{2\sqrt{t}(1-\sqrt{Pr})}{\sqrt{\pi}} \right] - \frac{\varepsilon}{\omega} Rl \left\{ \begin{array}{l} \sqrt{a} e^{at} \operatorname{erf}(\sqrt{at}) - e^{bt} \sqrt{b} \operatorname{erf}(\sqrt{bt}) + \\ \sqrt{b} e^{bt} \operatorname{erf}(\sqrt{(b-a)t}) \end{array} \right\} + \frac{2\varepsilon\sqrt{Pr}}{\omega\sqrt{\pi t}} \cos \omega t \quad (15)$$



(a)



(b)

Figure 4. Penetration distance.

The numerical values of ( $\tau$ ) for  $\epsilon = 0.2$  and  $t = 2$  have been computed and are listed in Table 1.

It has been concluded from Table 1 that with increasing  $\omega$  or  $Pr$ , the skin friction decreases.

Table 1. Values of  $\tau$  for  $\epsilon = 0.2, t = 2$

$\omega$	$\omega t$	$Pr = 0.71$	$Pr = 7.0$
5	0	0.5605	0.4211
5	$\pi/6$	1.9936	1.8359
5	$\pi/4$	1.4204	1.2692
5	$\pi/3$	1.1337	0.9865
5	$\pi/2$	0.8471	0.7038
10	$\pi/4$	0.8539	0.7027
15	$\pi/4$	0.6492	0.4979

### 3. CONCLUSIONS

The following conclusions have been drawn:

- (a) The transient velocity increases with increasing the frequency  $\omega$  or  $Pr$  but increases with increasing  $\omega t$ .
- (b) The transition from conduction to convection is delayed due to increasing  $\omega$  but occurs nearer the leading edge as  $\omega$  or  $Pr$  increases.

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