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Effect of Gyroscopic Couple on Aircraft Landing Gear Shimmy

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ABSTRACT

This article presents the effect of gyroscopic couples on the shimmy response of landing gear. The gyroscopic effect between the rotational motions of the aircraft's longitudinal and vertical axes may be one of the causes of shimmy. The vertical load acting on the wheel axle plays a significant role in a tire's dynamic characteristics, which may influence shimmy oscillation. So, vertical dynamics also need to be considered for shimmy analysis along with lateral and torsional modes. Suitable mathematical models are required to study the system's shimmy oscillation and stability nature. The mathematical model was developed by introducing a gyroscopic couple with 5 degrees of freedom (DOF). A numeric analysis was made to investigate the influence of gyroscopic couples on the landing gear model. The simulation results of the model with and without gyroscopic couple are compared and the effect of each parameter on shimmy is studied.

Keywords: Gyroscopic couple; Critical velocity; Shimmy; Tire model; Wheel inertia

1. INTRODUCTION

Shimmy is a complex self-excited oscillation experienced by nose wheel landing gears of aircraft. Shimmy excitation may result in a severe reduction of the landing gear life, failure of fittings or loss of aircraft stability. The prediction and evaluation of shimmy are more difficult due to nonlinear parameters influencing the behavior of landing gears of aircraft. Many researchers studied shimmy oscillation by numerical techniques using various mathematical models. The models of shimmy excitation mainly differ by several DOF considered, analysis methods and tire models chosen. The stability of the shimmy system is evaluated based on bifurcation theory and the system's dynamic response exposed to road roughness excitation is investigated[1]. Harmonic Balance based method used in[2] to determine critical velocity region where shimmy occurs. Previously developed theories are confirmed by series of experimental investigation such as full scale test with automobile on road , full scale tire on drum test stand and experiment with mechanical model capable of representing wheel shimmy phenomenon is presented[3]. The coupling effects of the vehicle body are incorporated in shimmy model as illustrated[4]. Shimmy analysis with different tire models is presented[5]. By varying design parameters, landing gear shimmy vibration was investigated[6]. The gyroscopic effect on the dual-wheel nose gear model is investigated[7]. The influence of the gyroscope couple on the MLG system[8]. Various stages of Damping of the shimmy vibrations are presented[9]. The vibration of the landing gear about the fuselage was investigated with two degrees of freedom mathematical model[10]. Previously published theories on shimmy, oscillation is listed, discussed, and compared[11]. The gyroscopic couple and the distance between the two wheels influence dual-wheel nose landing gear shimmy oscillations are described in[12] using bifurcation theory. The effect of mass and runway irregularities on shimmy amplitude was investigated[13]. From a simple trailing wheel model to a complex model is considered for shimmy analysis are illustrated[14]. In[15] it is presented that landing gear stability depends on 15 parameters as predicted by theory and substantiated by experiment, the quantitative effect of each parameter.

As a summary, in most of previous studies only lateral and torsional mode considered for shimmy analysis. The novelty of this work is to develop 5DOF non-linear mathematical model of generic nose wheel landing gear to analyze effect of gyroscopic couple on the shimmy oscillation with consideration of axial displacement of sprung mass x_s, axial displacement of unsprung mass x_t along with strut yaw angle ψ, strut lateral bending angle δ, and tire slip angle α. By using developed model effect of gyroscopic couples on shimmy oscillation of landing gear and the effect of other parameters, such as the vehicle's mass, wheel inertia, aircraft's forward velocity, landing gear length are studied. The Dynamics equations of the model are derived using second Lagrange's equation.

2. DYNAMIC MODEL

In Fig. 1 mathematical model of single-wheel landing gear is shown. 20 per cent of the maximum take-off weight of aircraft is assumed as vertical force acts on this model can be represented as sprung mass m_s, a mass of wheel assembly mentioned as m_u, and aircraft velocity specified as v. The model has 5 dof such as the rotation of landing gear about strut axis described as shimmy angle ψ, lateral bending of gear

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about longitudinal axis denoted as bending angle δ, coupling of torsion and bending through tire ground interaction denoted by tire lateral deflection λ, the vertical displacement of the strut denoted as z_t and the vertical displacement of tire represented as z_s. The torsional, bending, axial stiffness and damping coefficients of the strut is represented as K_ψ, K_δ, K_s, C_ψ, C_δ, and C_s. The damping and stiffness coefficient of the tire is denoted as C_t and K_t repectively.

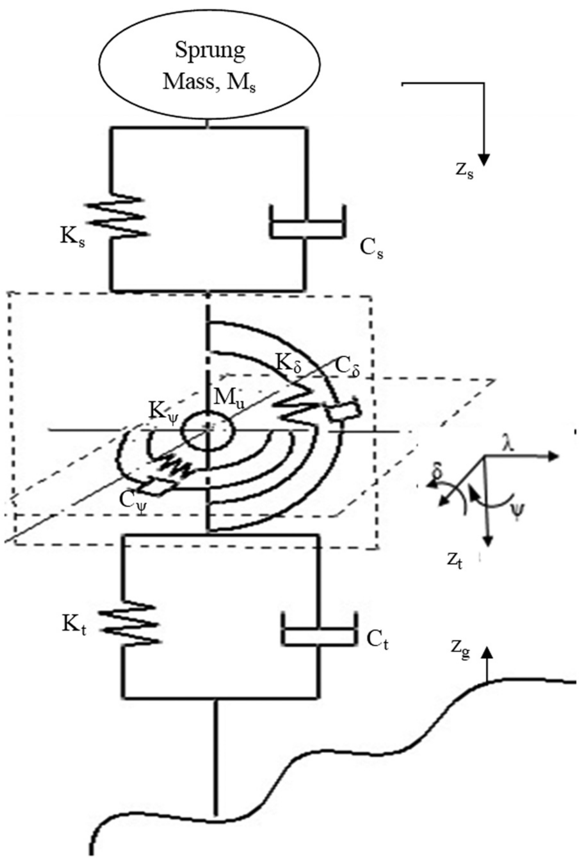


Figure 1. Dynamic model of landing gear.

3. EQUATION OF MOTION

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_i}\right)-\left(\frac{\partial T}{\partial x_i}\right)+\frac{\partial U}{\partial x_i}+\left(\frac{\partial D}{\partial \dot{x}_i}\right)=Q_i; i=1,2,3,\ldots, \quad (1)$$

Dynamic equations derived using the second Lagrange equation as expressed in Eqn (1). Where *T* represents kinetic energy, *U* denotes potential energy, *D* is the dissipative potential function, Q_i is the generalized force, and z_i represents the generalized coordinate of the system, which is expressed $[z_1, z_2, z_3, z_4, z_5]$ as $[\psi, \delta, z_t, z_s, \lambda]$

Expressions for *T, U,* and *D* are given in Eqns. (2-4), respectively.

$$T=\frac{1}{2}I_z\dot{\psi}^2+\frac{1}{2}I_x\dot{\delta}^2+\frac{1}{2}M_s\dot{z}_s^2+\frac{1}{2}M_u\dot{z}_t^2 \quad (2)$$

$$U=\frac{1}{2}K_\psi\psi^2+\frac{1}{2}K_\delta\delta^2+\frac{1}{2}K_s\left(z_s-z_t\right)^2+\frac{1}{2}K_t(z_t-z_g)^2 \quad (3)$$

$$D=\frac{1}{2}C_\psi\dot{\psi}^2+\frac{1}{2}C_\delta\dot{\delta}^2+\frac{1}{2}C_s(\dot{z}_s-\dot{z}_t)^2+\frac{1}{2}C_t(\dot{z}_t-\dot{z}_g)^2 \quad (4)$$

3.1 Gyroscopic Moment

Generalised forces and moments in the equations of motion are due to tire lateral forces, a vertical force of the wheel center, and due to gyroscopic couple. Various gyroscopic couple experienced by rolling wheel is caused by tilting of the wheel, swivelling of a wheel and vertical deformation of a tire. Eqns. (5-7) represents equations corresponding to various gyroscopic couples as described in reference 11.

3.1.1 Due to Tilting of Wheel

If the entire wheel assembly tilts at an angular velocity $\dot{\psi}$ gyroscopic couple of magnitude given as:

$$M_{z\psi}=-\frac{v}{R}I_{pw}\dot{\psi} \quad (5)$$

Where, v is rolling speed of wheel, R is radius of the wheel, I_{pw} is polar moment of inertia of wheel.

3.1.2 Due to Swivelling of Wheel

If the entire wheel assembly swivels at an angular velocity $\dot{\delta}$ gyroscopic couple of magnitude given as:

$$M_{z\delta}=-\frac{v}{R}I_{pw}\dot{\delta} \quad (6)$$

3.1.3 Due to Vertical Deformation of Tire

Gyroscopic couple due to rolling of a wheel with vertical deformation of the tire at the ground is expressed as:

$$M_{z_{zt}}=-\frac{v}{R}\frac{I_{pt}}{B}\dot{z}_t \quad (7)$$

where, *v* is rolling speed of wheel, *R* is radius of the wheel, B is vertical length from the wheel center to the strut attachment, I_{pt} is polar moment of inertia of tire.

3.2 Torsional Dynamics

Dynamic equations of the mathematical model are derived using the second Lagrange equation. By summing the moments about the vertical axis, the equation of motion corresponding to torsional mode is expressed by the Eqn. (8).

$$I_z\ddot{\psi}+C_\psi\dot{\psi}+K_\psi\psi+\frac{I_{pt}}{B}\frac{v}{R}\dot{z}_t+\frac{v}{R}I_{pw}\dot{\delta}+F_R\left(R\tau+n\right)-F_vA\tau=0 \quad (8)$$

where, A is lateral length from wheel center to strut attachment, τ is rake angle, F_R is tire lateral force as given in Eqn. (9) and (10). F_V is the vertical resultant force acts at wheel center as expressed in Eqn. (11). Equations corresponding to lateral tire force and vertical resultant force are referred from[1]:

$$F_R=\left[c_1\alpha+c_2\left(F_{zo}-F_z\right)\right]\alpha+c_3\alpha^3 \quad (9)$$

$$F_R=K_1\alpha+K_2\left(A\tau\psi+x_t-q\right)\alpha+K_3\alpha^3 \quad (10)$$

$$K_1=c_1+c_2F_{zo}, K_2=c_2K_t, K_3=c_3$$

c_1 - c_3 are tire coefficients, *α* is slip angle of the tire, F_{zo} is the nominal load of the tire,

$$F_v=F_z+F_s$$

Vertical force component due to runway roughness

$$F_z=K_t\left(z_g-A\tau\psi-z_t\right)$$

Vertical force component due to suspension

$$F_s=C_s\left(\dot{z}_s-\dot{z}_t\right)+K_s\left(z_s-z_t\right)$$

Total vertical force expressed as Eqn (11).

$$F_v = K_t\left(z_g - A\tau\psi - z_t\right) + C_s\left(\dot{z}_s - \dot{z}_t\right) + K_s\left(z_s - z_t\right) \qquad (11)$$

In Eqn. (8), the first term corresponds to inertia, second and third terms belong to torsional damping and torsional stiffness, respectively. The fourth term is gyroscopic couple due to tire deflection as given in[1]. The fifth term is gyroscopic couple due to wheel bending as presented in[12], the sixth term represents moment due to the tire's lateral force, and the seventh term is due to vertical force from the airframe.

3.3 Lateral Dynamics

Similarly, by summing the moments about the longitudinal axis of the airplane, the equation of motion for lateral bending mode is expressed as in Eqn. 12.

$$I_x \ddot{\delta} + C_\delta \dot{\delta} + K_\delta \delta - \frac{I_{pt}}{B}\frac{v}{R}\left(\dot{z}_t\right) - \frac{v}{R} I_{pw}\dot{\psi} - F_v e_{eff}\sin\theta + l_g F_R \cos\theta\cos\tau = 0 \qquad (12)$$

where, θ is swivel angle, τ is rake angle, lg is the length of landing gear, e_{eff} is effective caster length as described in[12] is expressed in Eqn. (13).

$$e_{eff} = e\cos\tau + \left(R + e\sin\tau\right)\tan\tau \qquad (13)$$

$$\theta = \psi cos\tau$$

In Eqn. (12) first term corresponds to inertia, the second and third terms belong to torsional damping and torsional stiffness moment, respectively, the fourth term is gyroscopic couple due to tire deflection as discussed in[1], the fifth term is gyroscopic couple due to wheel lateral tilting as presented in[12], the sixth term represents moment due to vertical force and the seventh term due to lateral force of tire.

3.4 Vertical Dynamics of Tire

Tire vertical dynamics due to unsprung mass can be expressed as Eqn. (14),

$$M_u \ddot{z}_t - C_s\left(\dot{z}_s - \dot{z}_t\right) + C_t\left(\dot{z}_t - \dot{q}\right) - K_s\left(z_s - z_t\right) + K_t\left(z_t - q\right) - \frac{I_{pw}}{B}\frac{v}{R}\dot{\psi} - \frac{I_{pw}}{B}\frac{v}{R}\dot{\delta} + (K_t A\tau\psi) = 0 \qquad (14)$$

3.5 Vertical Dynamics of Strut

Landing gear strut vertical dynamics due to sprung mass can be expressed as Eqn. (15),

$$M_s \ddot{z}_s + C_s\left(\dot{z}_s - \dot{z}_t\right) - K_s\left(z_s - z_t\right) = 0 \qquad (15)$$

3.6 Elastic Tire Deformation

When the vehicle taxiing, the tire starts rolling along with lateral and yaw motions. The lateral deformation λ of the tire as presented in[5] is expressed as in Eqn. (16),

$$\dot{\lambda} = -\frac{v}{\sigma}\lambda + v\psi + a\dot{\psi} + l_g\dot{\delta} = 0 \qquad (16)$$

where, a is contact patch length, l_g length of landing gear, σ is relaxation length.

From λ, an equivalent deformation angle α is obtained using the Eqn. (17),

$$\alpha = \frac{\lambda}{\sigma} \qquad (17)$$

Eqn. (18) represents the deformation angle which depends on lateral angular velocity $\dot{\delta}$, shimmy angle ψ and shimmy angular velocity $\dot{\psi}$ can be expressed as:

$$\dot{\alpha} = -\frac{v}{\sigma}\alpha + \frac{v}{\sigma}\psi + \frac{a}{\sigma}\dot{\psi} + \frac{l_g}{\sigma}\dot{\delta} = 0 \qquad (18)$$

4. NUMERICAL ANALYSIS

The landing gear and tire parameters relevant for analysis are taken from Table 1, as given in[13]. Formulated dynamic equations solved in MATLAB. The disturbance of the yaw angle is given as ψ = 0.001 radians. The effect of runway roughness excitation is neglected by assuming aircraft ride over the smooth runway. In this work, two simulations were carried out. First simulation studies the model behavior by introducing the gyroscope effect. Next simulation deals with the effect of gyroscope couple parameters on shimmy oscillations.

5. RESULTS AND DISCUSSION

The response of the model depends on velocity, vertical load, system parameters, and gyroscopic couple. The influence of any particular combination of landing gear parameters and velocity on vehicle shimmy is studied to check whether the system is stable or unstable. Influence of gyroscopic moment and vehicle parameters on stable and unstable velocity range presented.

5.1 Effect of Gyroscope Couple on Shimmy

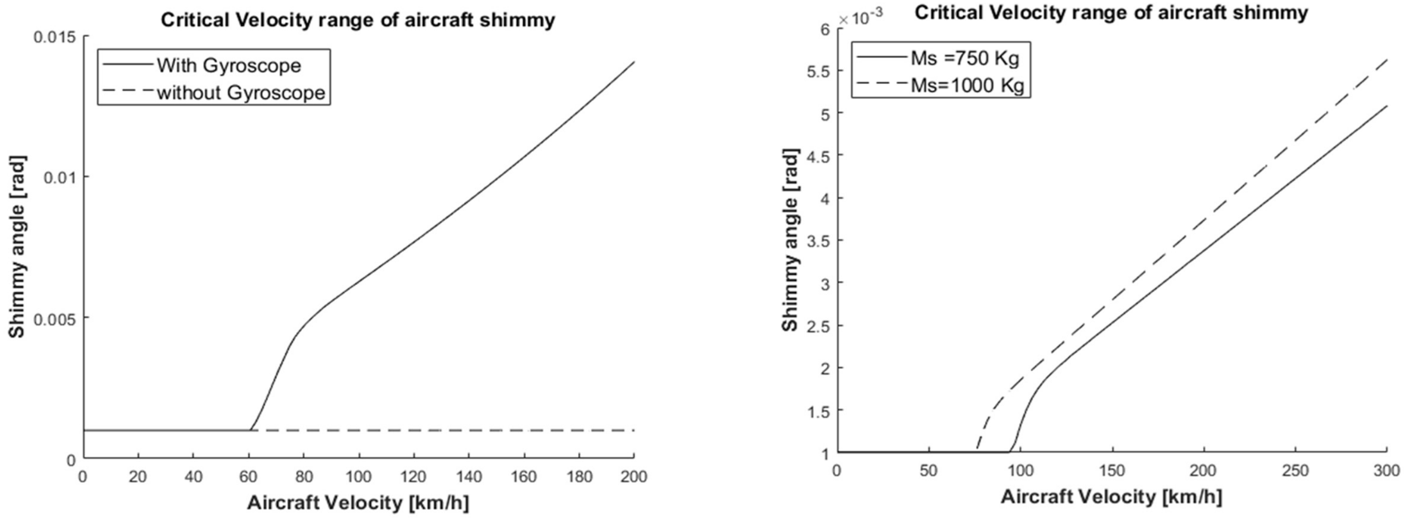
In Fig. 2(a), it can be observed that by excluding the gyroscopic couple on the model, the amplitude of shimmy oscillation is hardly velocity dependent and gives infinite critical velocity similar to discussion given in[14]. If a gyroscopic couple is introduced in the model, the onset of instability is observed at critical velocity. Figure 2(a) shows that the critical velocity is 61 km/h for chosen landing gear and tire parameters. Increasing aircraft forward velocity above critical velocity increases shimmy amplitude dramatically and the system loses its stability. Results are compared to the studies in the literature[14] as reported that at higher velocity system is unstable.

5.2 Effect of Aircraft Mass

It can be noted from Fig. 2(b) that the occurrence of shimmy for low mass configuration is delayed than higher mass as presented in[13]. Increasing mass leads to increase of vertical load which in turn reduces critical velocity for occurrence of shimmy.

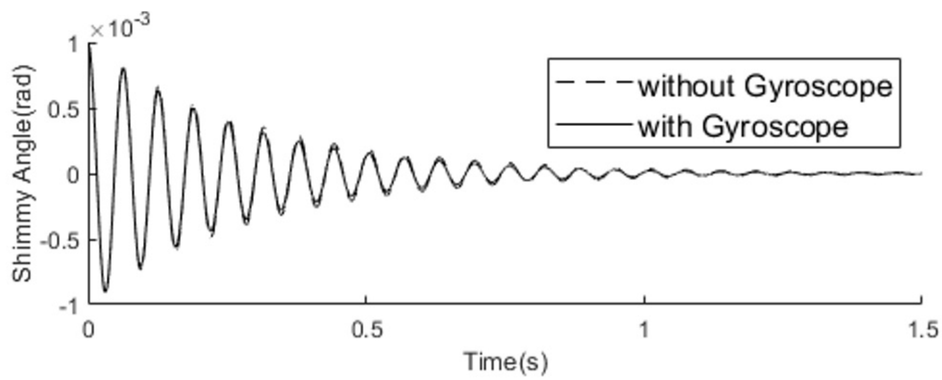
5.3 Effect of Velocity on Vehicle Shimmy

The influence of forward aircraft velocity due to the contribution of gyroscopes will become more dominant on shimmy amplitude. Figures 3(a) and 3(b) compare the model's time history with and without gyroscopic effects at stable and unstable velocity regions. It can be noted from Fig. 3a that torsional angle decays and exhibits a similar trend at lower velocity irrespective of whether the model is with or without

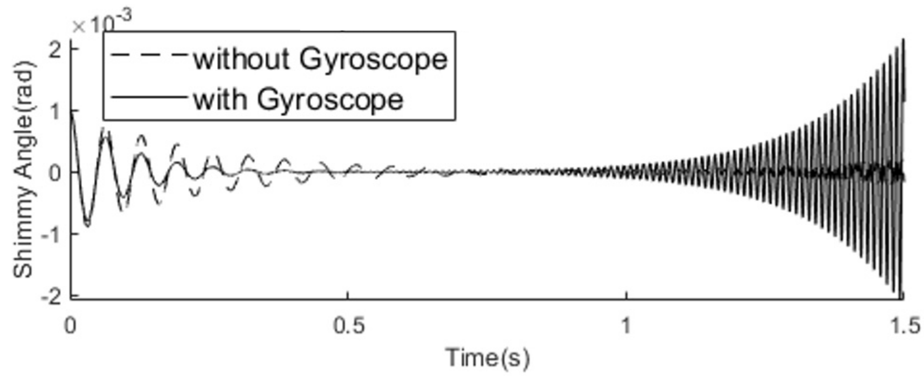


(a) (b)

Figure 2. (a) Effect of velocity, and (b) Effect of mass.



(a) At velocity 30 Km/h



(b) At velocity 100 Km/h

Figure 3. Effect of gyroscopic couple on shimmy oscillation at stable and unstable velocities.

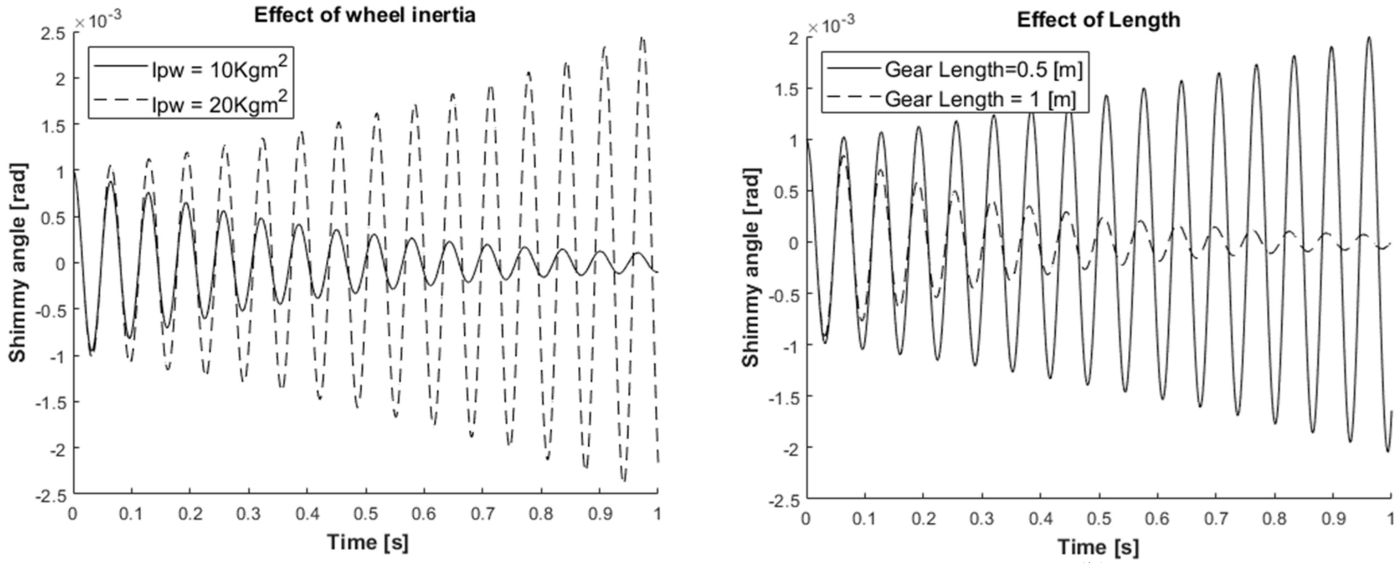
gyroscopic effect. But in Fig. 3(b), it can be observed that at higher velocity (above critical velocity) torsional angle diverging with time. It shows that the system exhibits severe unstableness at higher speeds due to the gyroscopic effect.

5.4 Effect of Wheel Inertia

At higher velocities, tire inertia changes tire stiffness, so gyroscopic couples are introduced into the model to predict shimmy oscillation accurately. From Fig. 4(a), it can be noted that by reducing wheel inertia, shimmy amplitude decays and exhibits stable nature. Obtained results validated with literature studies depicted in[15,12].

5.5 Effect of Gear Length

Simulations were carried out for two different landing gear lengths with initial disturbance of shimmy excitation with 0.001 rad on landing gear and the forward aircraft velocity of 30 m/s. Figure 4(b) shows that increasing landing gear



(a) (b)

Figure 4. (a) Effect of wheel inertia and (b) Effect of gear length.

length shimmy oscillation can be reduced by keeping all other parameters fixed.

6. CONCLUSION

A mathematical model of the landing gear with 5dof has been developed to investigate the combined effects of the gyroscopic couple and rolling velocity on vehicle shimmy. The simulation results of the model demonstrate that:

- If the gyroscopic couple is excluded, the shimmy response is hardly velocity dependent and theoretically value of critical velocity is infinite.
- If the gyroscopic couple is introduced into the model, the shimmy response is velocity dependent and critical velocity is determined as 60 Km/h.
- Increasing aircraft forward velocity above critical velocity increases shimmy response dramatically and the system loses its stability.
- The occurrence of shimmy for low mass configuration is delayed than higher mass.
- Shimmy oscillation increases as aircraft mass and wheel inertia increases.
- Shimmy oscillation decreases as landing gear length increases.
- Future work to study the effect of other landing gear parameters on aircraft shimmy and to perform stability analysis.

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This research was carried out under his supervision. He provided many valuable suggestions and feedback for the applicability of the gyroscopic couple concept. He also helped in reviewing the manuscript.

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In the current study, he developed mathematical model, carried out simulations, interpreted results and prepared manuscript.