Defence Science Journal, Vol. 56, No. 2, April 2006, pp. 251-257

SHORT COMMUNICATION

Evaluating the Main Battle Tank Using Fuzzy Number Arithmetic Operations

Deng Yong

Shanghai Jiao Tong University, Shanghai–200 030, *China*

and

Shen Cheng

The Command of the General Logistics Department of the PLA, Beijing–100 854, *China*

ABSTRACT

Since the descriptions and judgements on weapon systems are usually linguistic and fuzzy, it is more realistic to evaluate weapon systems in the framework of fuzzy sets theory. In this paper, a new method to evaluate the best main battle tank is proposed. It can be seen that the proposed method is more efficient due to the fact that, by canonical representation of arithmetic operation on fuzzy numbers, simple arithmetic operations on crisp numbers are used, instead of complicated fuzzy numbers operations. In addition, the final scores of each alternative can be represented as crisp numbers. As a result, the order of alternatives can be determined without the procedure of ranking fuzzy numbers. Finally, a numerical example to evaluate the best main battle tanks is used to illustrate the efficiency of the proposed method.

Keywords: Fuzzy numbers, multi-criteria decision-making, linguistic variables, weapon systems evaluation, fuzzy sets theory

1. INTRODUCTION

Weapon evaluation is very complex. For example, in a weapon evaluation situation, a good weapon system requires good weapon performance and minimal cost; the performance and cost depend on improvement of science and technology and economic resources; technology depends on ideas and resources; ideas depend on politics for their acceptance and support; and so on. Mon[1], *et al*. presented a method for evaluating weapon systems using fuzzy analytic hierarchy process (AHP) based on entropy weights. However, in complex systems, the experiences and judgements of experts are represented by linguistic and vague patterns, and these are not quantitatively digital. Therefore, by fuzzy sets theory, one can give a much better representation of these linguistic data, and thus, further refine the evaluation methods. A number of authors have provided interesting results on weapon systems evaluation using fuzzy sets theory[2-5]. Recently, Cheng[6], *et al*. proposed a method to evaluate the best main battle tank with linguistic variables. In their method, the experts' opinions were described by linguistic terms, which could be expressed in trapezoidal (or triangular) fuzzy numbers. The fuzzy ratings were used to construct the fuzzy decision matrix. Then, the aggregate fuzzy number

Revised 24 August 2005

by multiplying the fuzzy decision matrix with the corresponding fuzzy attribute weights, could be derived. Finally, fuzzy numbers were ranked to obtain the final decision.

In general, existing methods to calculate the multiplication of fuzzy weight matrix and fuzzy decision matrix are time-consuming. Owing to the classical fuzzy numbers arithmetic operations, the final scores are often fuzzy numbers. Hence, it is inevitable to rank fuzzy numbers in final decision, while it is well-known that how to rank fuzzy numbers correctly is still an open issue. In this paper, a simple and effective method is proposed to address the above problem.

2. PRELIMINARY INFORMATION

Some important definitions have been briefly reviewed in this section.

Definition of a Fuzzy Set

Let X be a universe of discourse, $\tilde{A}$ is a fuzzy subset of X iff for all $x \in X$, there is a number $\mu_{\tilde{A}}(x) \in [0,1]$ assigned to represent the membership of x to $\tilde{A}$, and $\mu_{\tilde{A}}(x)$ is called the membership[7] of $\tilde{A}$.

Definition of a Fuzzy Number

A fuzzy number $\tilde{A}$ is a normal and convex fuzzy subset of X. Here, the normality implies that[7]

$$\exists x \in R, \bigvee_{x} \mu_{\tilde{A}}(x) = 1 \tag{1}$$

and convex means that

$$\forall x_1 \in X\ ,\ \ x_2 \in X\ ,\ \ \forall \alpha \in [0,1]$$

$$\mu_{\tilde{A}}(\alpha x_1 + (1-\alpha)x_2) \geq \min[\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)] \tag{2}$$

Definition of a Trialgular Fuzzy Number

A triangular fuzzy number $\tilde{A}$ can be defined by a triplet (a,b,c) shown in Fig. 1. The membership function[7] is defined as

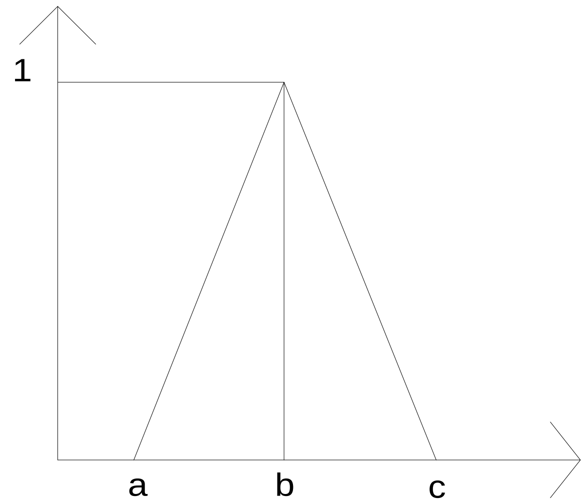

Figure 1. Triangular fuzzy number *(a,b,c)*

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a \\ \dfrac{x-a}{b-a} & a \leq x \leq b \\ \dfrac{c-x}{c-b} & b \leq x \leq c \\ 0, & x > c \end{cases}$$

Definition of Graded Mean Integration Representation of a Triangular Fuzzy Number

Given a triangular fuzzy number $\tilde{A} = (a_1, a_2, a_3)$, the graded mean integration representation of a triangular fuzzy number $\tilde{A}$ is defined as[8]

$$P(\tilde{A}) = \frac{1}{6}(a_1 + 4 \times a_2 + a_3) \tag{4}$$

Given a trapezoidal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$, the graded mean integration representation of a triangular fuzzy number $\tilde{A}$ is defined as[8]

$$P(\tilde{A}) = \frac{1}{6}(a_1 + 2 \times a_2 + 2 \times a_3 + a_3) \tag{5}$$

Let $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ be the two triangular fuzzy numbers. By applying Eqn (4), the graded mean integration representation of a triangular fuzzy numbers $\tilde{A}$ and $\tilde{B}$ can be obtained, as follows:

$$P(\tilde{A}) = \frac{1}{6}(a_1 + 4\times a_2 + a_3)$$

$$P(\tilde{B}) = \frac{1}{6}(b_1 + 4\times b_2 + b_3)$$

The representation of the addition operation on triangular fuzzy numbers $\tilde{A}$ and $\tilde{B}$ can be defined as[8]

$$P(\tilde{A}\oplus\tilde{B}) = P(\tilde{A})\times P(\tilde{B}) = \frac{1}{6}(a_1 + 4\times a_2 + a_3 + b_1 + 4\times b_2 + b_3) \quad (6)$$

The canonical representation* of multiplication operation on triangular fuzzy numbers $\tilde{A}$ and $\tilde{B}$ is defined as[8]

$$P(\tilde{A}\oplus\tilde{B}) = P(\tilde{A})\times P(\tilde{B}) = \frac{1}{6}(a_1 + 4\times a_2 + a_3)\times\frac{1}{6}(b_1 + 4\times b_2 + b_3) \quad (7)$$

3. PROPOSED METHOD

In previous studies about weapon systems evaluation, the general steps can be described as follows:

First, the decision-makers use the linguistic weighting variables to assess the importance of the criteria $\tilde{W}$ and utilise the linguistic rating variables $\tilde{R}$ to evaluate the rating of alternatives wrt each qualitative criterion. Then, by fuzzy numbers multiplication operation $(\times)$, the final scores of each alternative are calculated as

$$\tilde{S} = \tilde{W}\times\tilde{R}$$

This step is time-consuming owing to the complexity of fuzzy numbers multiplication operation. In addition, the final scores are obtained as fuzzy numbers. Therefore, ranking fuzzy numbers is inevitable to determine the final order. To address these problems, a new method has been proposed based on the canonical representation of fuzzy numbers multiplication. In this proposed method, the importance of the criteria $\tilde{W}$ and the linguistic rating variables $\tilde{R}$ are translated into crisp numbers by applying Eqn (4). As a result, the complicated multiplication operation on fuzzy numbers is converted into the simple arithmetic operation on crisp numbers [Eqn (6)] and the final scores are also obtained as crisp numbers, from which the final order of each alternative can be easily determined without ranking fuzzy numbers.

The proposed method is described as follows:

Step 1.

The decision-makers use the linguistic weighting variables to assess the importance of the criteria, and utilise the linguistic rating variables to evaluate the rating of alternatives wrt to each qualitative criterion. The linguistic variables and the correspondinglinguistic ratings are shown in Tables 1 and 2. Determine the importance weight of the linguistic criterion and the rating of the decision-makers under linguistic criteria.

Table 1. Linguistic variables of the importance weight and corresponding graded mean integration representation

Linguistic variables for importance weights	Fuzzy numbers	Graded mean integration representation
Very low (VL)	(0,0,0.1,0.2)	0.0667
Low (L)	(0.1,0.2,0.2,0.3)	0.2000
Medium-low (ML)	(0.2,0.3,0.4,0.5)	0.3500
Medium (M)	(0.4,0.5,0.5,0.6)	0.5000
Medium-high (MH)	(0.5,0.6,0.7,0.8)	0.6500
High (H)	(0.7,0.8,0.8,0.9)	0.8000
Very high (VH)	(0.8,0.9,1,1)	0.9333

Table 2. Linguistic variables of the ratings and corresponding graded mean integration representation

Linguistic variables for ratings	Fuzzy numbers	Graded mean integration representation
Very poor (VP)	(0,0,1,2)	0.6667
Poor (P)	(1,2,2,3)	2.0000
Medium poor (MP)	(2,3,4,5)	3.5000
Fair (F)	(4,5,5,6)	5.0000
Medium good (MG)	(5,6,7,8)	6.5000
Good (G)	(7,8,8,9)	8.0000
Very good (VG)	(8,9,10,10)	9.3333

*For more details about canonical representation of arithmetic operations on triangular fuzzy numbers, please refer to[8].

Step 2.

Calculate the final scores of each alternative by canonical representation of multiplication operation on the importance weight and the rating.

Step 3.

Determine the order of each alternative by final scores, which can be represented by crisp numbers.

4. EVALUATING THE BEST MAIN BATTLE TANK

The example illustrated by Chang[4,6], *et al.* has been used to show the efficiency of the proposed method. The alternatives are the three best main battle tanks, namely, M1A1 (USA), Challenger 2 (UK), and Leopard 2 (Germany). The basic performance of each alternative can be obtained from the article by Christopher[9] and listed in Table 3. A committee of three decision-makers D_1, D_2, and D_3 has been formed to select the best main battle tank. The hierarchical structure of this decision problem is shown in Fig. 2.

First, the decision-makers used the linguistic weighting variables to assess the importance of the criteria, and utilise the linguistic rating variables to evaluate the rating of alternatives wrt each qualitative criterion. The linguistic weight variables and the linguistic rating variables have been shown in Tables 1 and 2. The graded mean of fuzzy rating and weighting is calculated and the results are shown in Tables 1 and 2.

Second, the importance weight of the linguistic criteria and the rating of the three decision-makers under linguistic criteria were obtained by fuzzy Delphi method and shown in Tables 4 and 5. The average graded mean of fuzzy rating and weighting was calculated. The results have been shown in Tables 4 and 5.

Then, construct the decision matrix as follows:

$$\begin{bmatrix} 7.944 & 8.000 & 6.5000 & 7.2500 \\ 7.0000 & 5.7500 & 6.5000 & 7.2500 \\ 7.9444 & 7.6250 & 5.7500 & 7.2500 \end{bmatrix}$$

and calculate the final scores of each alternative as follows:

Table 3. Basic performance data for three types of main battle tanks

Item	Type		
	M1A1 (USA)	Challenger 2 (UK)	Leopard 2 (Germany)
Armament	1×120.00 mm gun	1×120.00 mm L30 gun	1×120.00 mm gun
	2×7.62 mm MG	2×7.62 mm MG	2×7.62 mm MG3
	1×12.70 mm MG		
Ammunition	40	Up to 50 projectile stowage	42
	1000	Positions (7.62 mm)4000	4750
	11400		
Smoke grenade dischargers	2×6	2×5	2×8
Power-to-weight ratio	27 hp/t	10.2 hp/t	25.12 hp/t
Max. road speed	72 km	56 km/h	72 km
Max. range	498 km	450 km	500 km
Fording	1.219 m	1.07 m	1 m
Gradient	60 %	60 %	60 %
Vertical obstacles	1.244 m	0.90 m	1.10 m
Trench	2.743	2.43	3.00
Armour protection	Good	Excellent	Fair
Acclimatisation	Good	Fair	Good
Communication	Fair	Fair	Fair
Scout	Medium	Medium	Medium

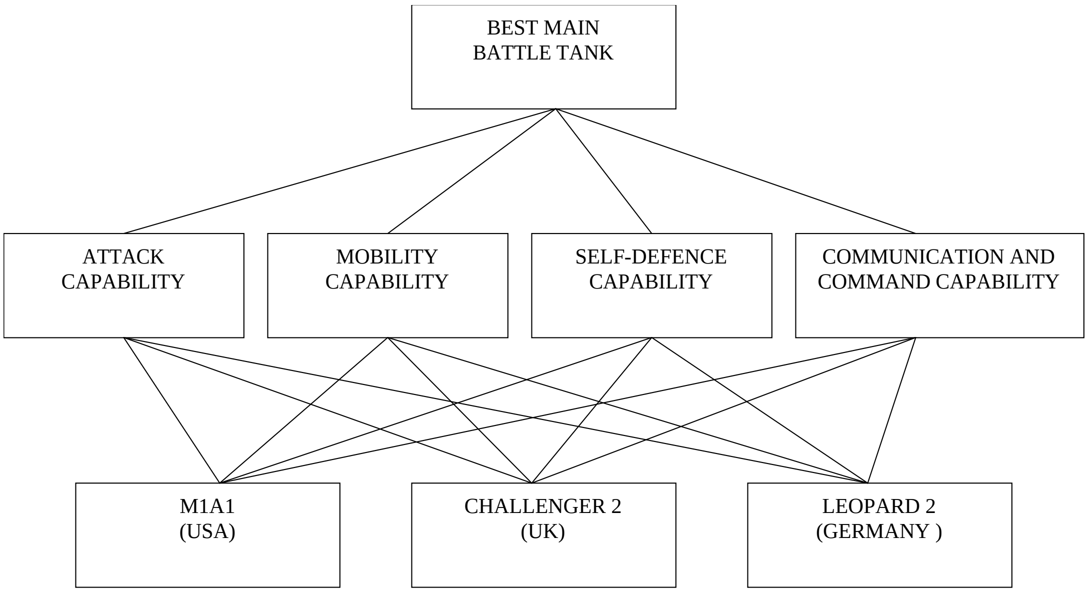

Figure 2. Hierarchical structure of evaluating three types of main battle tanks

Table 4. Importance weight of the linguistic criteria and its average graded mean

	D1	D2	D3	Average graded mean
Attack	Very high	High	High	0.8444
Mobility	Very high	High	Very high	0.8889
Self-defence	Medium	Very high	Medium-high	0.6500
Communication and command	Medium	Medium	Medium	0.5000

Table 5. Ratings of attribute performance for three types of main battle tanks

Criteria	Item	Type		
		M1A1 (USA)	Challenger 2 (UK)	Leopard 2 (Germany)
Attack	Armament	Medium-good	Good	Good
	Ammunition	Very Good	Medium-good	Medium-good
	Smoke grenade dischargers	Good	Medium-good	Very Good
	Average graded mean	7.9444	7.0000	7.9444
Mobility	Power-to-weight ratio	Good	Fair	Good
	Max. road speed	Good	Fair	Good
	Max. range	Good	Medium-good	Good
	Passing trench/obstacle	Good	Medium-good	Medium-good
	Average graded mean	8.0000	5.7500	7.6250
Self-defence	Armour protection	Medium-good	Good	Fair
	Acclimatisation	Medium-good	Fair	Medium-good
	Average graded mean	6.5000	6.5000	5.7500
Communication and command	Communication	Good	Good	Good
	Scout	Medium-good	Medium-good	Medium-good
	Average graded mean	7.2500	7.2500	7.2500

$$\begin{bmatrix} 7.9444 & 8.0000 & 6.5000 & 7.2500 \\ 7.0000 & 5.7500 & 6.5000 & 7.2500 \\ 7.9444 & 7.6250 & 5.7500 & 7.2500 \end{bmatrix}$$

$$\begin{bmatrix} 0.8444 & 0.8889 & 0.6500 & 0.5000 \end{bmatrix}^t$$

$$= \begin{bmatrix} 21.6695 \\ 18.8720 \\ 20.8486 \end{bmatrix}$$

Finally, determine the order of each alternative by the final scores.

It is obvious that the order can be determined as

M1A1 (USA) > Leopard 2 (Germany) > Challenger 2 (UK)

The result coincides with the one presented by Chang[6], *et al*.

5. DISCUSSION & CONCLUSION

In this paper, a new multi-attribute decision-making (MADM) method has been proposed for evaluating the best main tanks in fuzzy environment. In real application systems, the attributes of each alternative and their relative weights can often be represented by linguistic variables given by domain experts. Hence, it is reasonable to model the attributes and weights by fuzzy numbers to deal with uncertain information in decision-making. Generally, the final score of each alternative can be obtained by multiplication of attributes and weights and the scores are still fuzzy numbers. It causes two problems. The one is that the heavy computation load due to the great numbers of multiplication operation of fuzzy numbers, if the system is very complex, in that a lots of attributes and weights should be taken into consideration. The other is that the final step should be ranking fuzzy numbers, which is still an open issue to be solved.

Our proposed method can model the uncertain information by fuzzy numbers. In addition, the computation procedure is very simple compared with classical fuzzy multi-attribute decision-making. From the illustrated numerical example, one can see that the proposed method is more efficient due to the fact that

- By canonical representation of arithmetic operation on fuzzy numbers, simple arithmetic operations on crisp numbers are used, instead of complicated fuzzy numbers operations in the previous work.
- The final scores of each alternative are obtained as crisp numbers. As a result, the order of alternatives can be determined without the procedure of ranking fuzzy numbers.

The above merits of the proposed method can deal with other multi-attribute decision-making in fuzzy environment.

ACKNOWLEDGEMENTS

The authors would like to express their thanks for the anonymous referees for their valuable comments and suggestions, which helped them to make improvement in the paper.

This work was supported by National Natural Science Foundation of China through Grant No. 30400067, Shanghai Nature Foundation through Grant No. 03ZR14065, and China Postdoctor Foundation through Grant No. 200436273, National Defence Key Laboratory through Grant No. 51476040105JW0301, Key Laboratory of Education Ministry for Image Processing and Intelligent Control through Grant No. TKLJ0410, Aerospace Basic Support Foundation through Grant No. 2004-JD-4.

REFERENCES

1. Mon, D.L.; Cheng, C.H. & Lin, J.C. Evaluating weapon system using fuzzy analytic hierarchy process based on entropy weight. *Fuzzy Sets Syst.*, 1994, **62**, 127-34.
2. Chen, S.M. Evaluating weapon systems using fuzzy arithmetic operations. *Fuzzy Sets Syst.*, 1996, **77**, 265-76.
3. Cheng, C.H. Evaluating naval tactical missile system by AHP based on the grade of membership function. *European J. Oper. Res.*, 1997, **96**(2), 343-50.

4. Cheng, C.H. & Mon, D.L. Evaluating weapon system by analytic hierarchy process based on fuzzy scales. *Fuzzy Sets Syst.*, 1994, **63**, 1-10.

5. Chen, S.M. A new method for evaluating weapon systems using fuzzy set theory. *IEEE Trans. Syst. Man Cyber.*, 1996, **26**, 493-97.

6. Cheng, C.H.; Yang, K.L. & Hwang, C.L. Evaluating attack helicopters by AHP based on linguistic variable weight. *European J. Oper. Res.*, 1996, **116**, 423-35.

7. Kaufmann, A & Gupta, M.M. Introduction to fuzzy arithmetic: Theory and applications. Van Norstrand Reinhold, New York. 1985.

8. Chou, C.C. The canonical representation of multiplication operation on triangular fuzzy numbers. *Comp. Math. Appl.*, 2003, **45**, 1601-610.

9. Christopher, F. Foss. Jane's armour and artillery-1996-1997, Ed. 17. Janes Information Group Inc, 1996.

Contributor

Dr Deng Yong received his Bachelor (Physics) from the Shanxi Normal University, Xi'an, China, in 1997, the Master (Instrument Engg) from the Hunan University, Changsha, China, in 2000, and PhD (Precise Instrument and Mechanic Engineering) from the Shanghai Jiao Tong University, Shanghai, China, in 2003. He is currently Associate Professor at the School of Electronics & Information Technology, Shanghai Jiao Tong University. His research areas include: Information fusion, fuzzy logic, and decision analysis. He is a member of IEEE.