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An Algorithm for Exchanging Target-Asset Pairs using the Kidney Exchange Model

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ABSTRACT

Since chemical, biological, radiological, nuclear, and high yield explosive (CBRNE) attacks can cause catastrophic damage, it is important to detect and eliminate the means of attack at the origin. In surveillance operations, efficient allocation of friendly intelligence assets and enemy targets is critical for continuous and reliable monitoring. In this research, we investigate a mathematical model for exchanging target–asset pairs when there are sudden changes in various operational environments. For this task, we refer to the kidney exchange model as a benchmark. In particular, the methods for constructing and solving the target–asset exchange problem in near real-time are presented. Additionally, we introduce the methodology and results for obtaining a feasible solution of the weapon target assignment problem using the exchange model. Our method can facilitate decisions in reconnaissance operations, especially when countless targets and assets are intricately intertwined in future battlefield scenarios.

Keywords: Target-asset matching; Weapon target assignment; Kidney exchange model; Reconnaissance operations

NOMENCLATURE

N Set of all pairs

V Set of pre-matched Target–Asset pairs $(V \subseteq N)$

V' Set of Dummy–Asset pairs $(V' \subseteq N)$

$$c_{ij} = \begin{cases} 1, \text{Asset of } i \text{ is compatible with Target of } j \\ 0, \text{otherwise} \end{cases} \quad \forall i \in N,\ \forall j \in V$$

a_i Amount of Spare Assets of I $\quad \forall i \in V'$

v_{ij} Value of exchange i to j $\quad \forall i \in N,\ \forall j \in V$

1. INTRODUCTION

Chemical, biological, radiological, nuclear, and high yield explosive (CBRNE) incidents, which have emerged and developed in modern warfare, pose a significant threat that can inflict non-linear damage to both enemy forces and friendly forces. This CBRNE attack is mainly performed from a distance by various weapon delivery systems.

On April 22, 2022, during a military parade, North Korea showcased various types of missiles, such as submarine-launched ballistic missiles (SLBMs), intercontinental ballistic missiles (ICBMs), and short-range ballistic missiles (SRBMs) code-named KN-23, KN-24, and KN-25. It continues to threaten peace on the Korean Peninsula and worldwide through missile launch tests using transporter-erector-launchers (TELs) and rail-mobile launchers.

Defense against CBRNE attacks can be divided into passive and active defense activities. Passive defense is an activity that minimizes the damage or influence from an enemy threat or attack. Military activities that are examples of passive defense include protection of bases or using interceptors such as terminal high altitude area defense (THAAD). Active defense pertains to removing the threat before an enemy attack affects the friendly forces. In regard to active defense, the most important consideration is to surveil and identify the signs of CBRNE attacks and eliminate threats. To accomplish this task, it is important to surveil the enemy's delivery system, which is the means of the CBRNE attack.

Surveilling the enemy can be abstracted as the matching of the enemy target and a friendly surveillance asset, and the matching process is an essential process of intelligence gathering operations. The development of defense science and technology has engendered diversification in the types of monitored targets and the capabilities and limitations of surveillance assets. Moreover, the number of targets and assets will increase in the future battlefield. Therefore, given the diversification and quantitative increase of targets and assets, the matching problem becomes more complicated.

If the enemy intends a CBRNE attack, various deceptive tactics are used in response to the friendly active defense. For example, when an adversary operates TEL, they will use various fake delivery systems for deception. The operation of a fake delivery system will confuse friendly forces and compel a waste of assets.

As presented above, war situations entail real-time changes,

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and the importance of the target also changes. In a situation where surveillance assets are limited, the matching of high-value surveillance assets with the targets that are no longer important or accommodating targets and assets that do not meet mutual compatibility must be exchanged. For example, if a high-value asset is monitoring a fake TEL or an asset with a low ability is monitoring a real TEL, the matching should change. Making timely decisions is essential to victory and requires real-time matching exchange. Obtaining a clear solution for an asset exchange facilitates effective asset management. The purpose of this study is to investigate a target–asset exchange model that can obtain an exchange solution considering the compatibility of targets and assets. For this purpose, we choose the kidney exchange model as a benchmark. The kidney exchange model can satisfy compatibility by exchanging incompatible pairs.

This paper is organised as follows: In Section 2, we review relevant studies. In Section 3, we describe the target–asset exchange problem and suggest solution methods. In addition, we present an algorithm to find a feasible solution to the weapon–target assignment (WTA) problem by using the proposed method. In Section 4, we provide the computational results, and Section 5 concludes.

2. LITERATURE REVIEW

In this section, we review research topics related to the target–asset exchange model. First, we describe the kidney exchange model, which is the benchmark for this study. Second, we review the research on the allocation or exchange of reconnaissance assets. We investigate whether there are any prior studies for efficient asset management and what methodologies were used. Third, we review the WTA problem. Although the WTA problem is not directly related to our study, the target–asset exchange model readily lends itself to finding a feasible solution of WTA.

2.1. Kidney Exchange

The kidney exchange problem[1] serves as motivation for further developing the idea of a target–asset exchange model. The objective of the kidney exchange problem is to maximize the transplantation of a kidney by a living donor rather than a kidney from a deceased donor. For a kidney transplant to be possible, several medical compatibility and physical conditions must be satisfied.[1] Many dialysis patients on the kidney transplant waiting list wait for deceased donors because no acquaintances are available who meet the compatibility conditions.[2] However, there are potential donors who do not meet the patient compatibility conditions but who wish to donate a kidney for the patient. In this case, the concept of kidney exchange entails exchanging donors that satisfy the compatibility between pairs. Thus, exchanging kidneys is equivalent to exchanging assets of incompatible target–asset pairs in our study. The kidney paired exchange was first proposed by Rapaport (1986),[3] and many countries and organizations have established and operated exchange transplant programs.[4-5] In general, the kidney exchange problem is known as NP-hard. What makes the problem difficult is that it constrains the maximum length of the exchange chain or cycle.[6]

Abraham, *et al*. (2007) presented the clearing algorithm of the large-scale exchange model to improve the computational speed for the constraint generation and column generation methods.[6] Anderson, *et al.* (2016)[7] proposed an algorithm inspired by the prize collecting traveling salesman problem (TSP), in which the salesman can pay a fine and exclude some cities entirely to find long exchange chains. Dickerson, *et al.* (2019)[8] presented a dynamic kidney exchange model considering the weight and failure probability of each exchange edge. Recently, Carvalho, *et al.* (2021)[9] presented a robust model that can be applied to situations in which pairs can withdraw from participating in a kidney exchange program. Mincu, *et al.* (2021)[10] studied the problem of multinational kidney exchange in European countries. Participating countries allow kidney exchange with different constraints and objectives.

Thus far, we have reviewed the literature on kidney exchange models, but few cases of the kidney exchange paradigm exist in other application domains. This study is the first military application of the kidney exchange model. Furthermore, the constraints considered in traditional kidney exchange problems (e.g., distance, importance of the exchange, probability of failure, etc.) are also important issues in the exchange process of military assets.

2.2. Military Asset Assignment and Exchange

In this section, we introduce studies related to the allocation of assets and targets in the military domain. Porto (1998) allocated unmanned aerial vehicles (UAVs) as surveillance assets using evolutionary programming that can be re-optimized when circumstances change.[11] Kappedal (2008) studied the assignment problem with stages of sensors to track and detect various types of targets with different priorities.[12] In the study, stages are assumed to be independent parallel structures, while the problem of maximizing the total mission probability is solved by using the simulated annealing (SA) method. AN (2012) applied an optimization algorithm to the surveillance asset allocation problem in response to the threat of pirates in ocean environments.[13] In a recent study[14] on arranging intelligence, surveillance and reconnaissance (ISR) assets, two processes, information requirements management and collection management, were applied as criteria for asset assignment. These criteria for allocation can also be considered in our model.

Most of the research on asset management involves preplanned assignments. Our research focuses on exchanging assets as the battlefield changes.

2.3. Weapon–Target Assignment Problem

As mentioned in Section 2, we briefly review the WTA problem as a prelude to proposing an algorithm that finds a feasible solution for WTA. The WTA problem is a traditional research topic in military operations research that assigns available weapons to enemy targets. The WTA problem was proposed by Flood in 1957,[15] and the formulation of the problem was established by Manne in 1958.[16] The WTA problem can be divided into several categories. First, depending on the choice of the objective function, it can be divided into target-based WTA[16] and asset-based WTA.[17-18] In other classifications, the problem can be classified into static WTA and dynamic WTA based on whether there is a stage of assignment.[18-20]

A general WTA problem formulated as a mixed integer non-linear programming (MINLP) problem was proven to be NP-complete.[21] It is known that the exact algorithm does not exist even in a small size with 20 targets and assets.[22] In a recent WTA study, a method was implemented to find a solution by approximating the objective function, which is a non-linear function, with a piecewise convex linear function.[23] However, when applied to a large-scale WTA problem, it cannot provide a solution within a short time. In operations, the WTA problem can be applied in situations where available time is limited. Therefore, various heuristic algorithms including meta-heuristics have been studied to find a feasible solution with good objective function values within a limited time.[24] It is important to find various initial feasible solutions in a meta-heuristic algorithm. In this paper, we present the process of finding the initial feasible solution of the WTA problem using the proposed exchange algorithm.

The contributions of this study are as follows.

- The previously known assignment or matching is a problem of allocating available agents to tasks in a situation where there is no initial assignment. In contrast, the proposed algorithm can be applied to already matched pairs, unlike the assignment problem. When the existing matching becomes inappropriate due to a real-time change in the situation, the algorithm finds an appropriate matching plan through the exchange of assets between pairs. Notably, it may be more useful to perform exchanging as needed rather than reallocating everything.
- Using the proposed algorithm, a feasible solution (i.e., integer solution) to the WTA problem can be obtained. This method can be used as a way to provide good bounds for WTA by combining it with existing methods to find feasible solutions.

3. METHODOLOGY

3.1 Target–Asset Exchange Problem

In this section, we introduce the target–asset exchange problem. As previously discussed, the target–asset exchange model is given an existing assignment. This consists of finding an exchange solution between pairs so that compatibility can be satisfied by exchanging the existing assignment. Compatibility is an abstract concept indicating whether the asset can achieve an appropriate desired effect on the target when the asset is assigned to the target. For example, when surveilling a target with an asset, it is a compatible case if the desired level of information can be obtained; otherwise, it is an incompatible case.

The formulation of the target–asset exchange (hereafter referred to as TAE) model is as follows.

$$\max \quad \sum_{i \in N} \sum_{j \in N} v_{ij} x_{ij} \tag{1}$$

$$s.t. \quad \sum_{j \in N} x_{ij} \le 1 \qquad \forall i \in V \tag{2}$$

$$\sum_{j \in N} x_{ij} = \sum_{j \in N} x_{ji} \qquad \forall i \in V \tag{3}$$

$$x_{ij} \le c_{ij} \qquad \forall i \in N,\ \forall j \in N \tag{4}$$

$$\sum_{j \in V} x_{ij} \le a_i \qquad \forall i \in V' \tag{5}$$

$$x_{ij} \in \{0,\ 1\} \qquad \forall i \in N,\ \forall j \in N \tag{6}$$

Before describing the TAE model, we describe the set, parameters, and decision variables of the model. Set N is the set of all pairs participating in the exchange. Set V is the set of target–asset pairs that are currently matched with each other. Operational units may have assets that are not yet assigned to the target. To include the spare asset in the exchange model, we generate dummy–asset pairs and denote the set of dummy–asset pairs as V'. The number of dummy–asset pairs is less than the number of spare assets but equivalent to the number of asset types, and the holding quantity for each type is defined as parameter a_i. C_{ij} is an important parameter in the model. It takes 1 if the asset of pair *i* and the target of pair *j* are compatible and 0 otherwise. v_{ij} denotes the value of exchanging pair *i* to pair *j*. The decision variable x_{ij} is a binary variable that has a value of 1 if the asset of pair *i* is exchanged with pair *j* and 0 otherwise. For example, suppose that Pair 1 is a pair consisting of Target 1 and Asset 1 and Pair 2 is a pair consisting of Target 2 and Asset 2. In this situation, '$x_{12} = 1$' means that Asset 1 of Pair 1 is matched with Target 2 of Pair 2. Therefore, Pair 2 is changed to Target 2-Asset 1.

The objective function is to maximize the number of exchanges considering the value of the exchange. Inequality (2) means that the asset of pair *i* can only be exchanged with at most one pair, and Eqn. (3) is a restriction such that if a pair exchanges an asset for another pair, the asset should be exchanged from the other pair. Inequality (4) indicates that the feasibility of the exchange between pair *i* and pair *j* is constrained by C_{ij}. Inequality (5) means that when additional assets are assigned to a pair, they must be assigned within the available quantity a_i.

Constructing and updating C_{ij} affect both the quality and complexity of the algorithm. When judging the compatibility in the exchange of surveillance assets, various factors can be considered, such as the properties of assets and targets and the phase or aspect of operations. General considerations include asset capacity, target value, the movement required for an exchange, and the improvement of inefficient asset management. C_{ij} has a value of 1 that indicates an exchange is permitted when all of the criteria are satisfied and has a value of 0 when one or more criteria are not satisfied. There are several approaches to determining C_{ij}, and the following method is presented as an example. Assume that the following three criteria are to be considered when deciding whether to allow an exchange.

Let C be the set of criteria. Suppose $C=\{C_1, C_2, C_3\}$.
C_1: Exchange efficiency (Is the target–asset assignment appropriate in terms of cost and value?)
C_2: Minimum requirement of detection probability (Is the detection probability higher than the specific threshold after the exchange?)
C_3: Improvement of detection probability (Does the detection probability increase after the exchange?)

Suppose $c_{ij}^{k} \in \{0, 1\}$ indicates compatibility with respect to criterion $k \in C$. Then, the final compatibility C_{ij} is obtained as $\prod_{k \in C} c_{ij}^{k}$. As an example, consider the initial assignment pair P1,...,P4, as shown in Fig. 1.

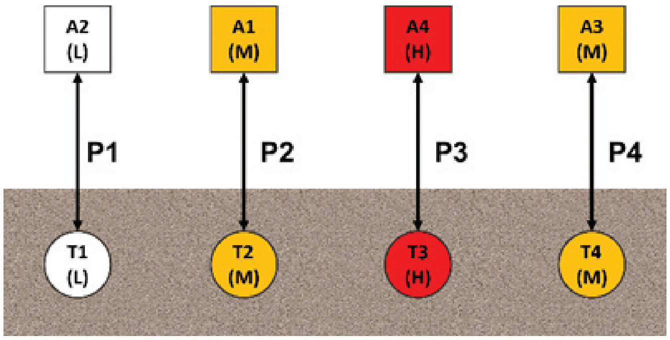

Figure 1. Example of target–asset pairs.

In this example, there are 4 types of targets (circles) and assets (rectangles). The target and asset have unique threat and capacity levels, respectively, as shown in Table 1. The detection probability varies according to different combinations of the level of targets and assets, as shown in Table 2. For example, if A4 with the high capacity level is matched to T1 with the low threat level, the detection probability is 0.99. These values are all given as parameters.

Table 1. Example of types and properties of targets and assets

Target	Threat level	Asset	Capacity level
T1	Low	A1	Middle
T2	Middle	A2	Low
T3	High	A3	Middle
T4	Middle	A4	High

Table 2. Example of detection probability

A \ T	T1	T2	T3	T4
A1	0.95	0.85	0.5	0.8
A2	0.9	0.5	0.1	0.4
A3	0.95	0.8	0.3	0.9
A4	0.99	0.95	0.9	0.95

Suppose that the judgment regarding the target has changed through information analysis, as shown in Figure 2. In this figure, the dotted line in P1 means that this pair is no longer compatible.

Consider Criterion 2 in the situation of Figure 2. Table 3 shows the detection probability after exchange when pair-to-pair exchange occurs in a changed situation. The value in parentheses in Table 3 is $c_{ij}^{c_2}$ and can be obtained by applying the C_2 condition (threshold value of 0.8) to each element. The meaning of the first row of Table 3 is as follows. Assets of P1 are not compatible with the targets of P1, P2, and P3 but are compatible with the targets of P4. Similar to applying C_2, we can obtain the c_{ij}^{k} value for each criterion $k \in C$, and C_{ij} can be obtained as $\prod_{k \in C} c_{ij}^{k}$. The decision maker can determine C_{ij} by applying only the criterion necessary for the current situation. The experimental results for this model are presented in Section 4.

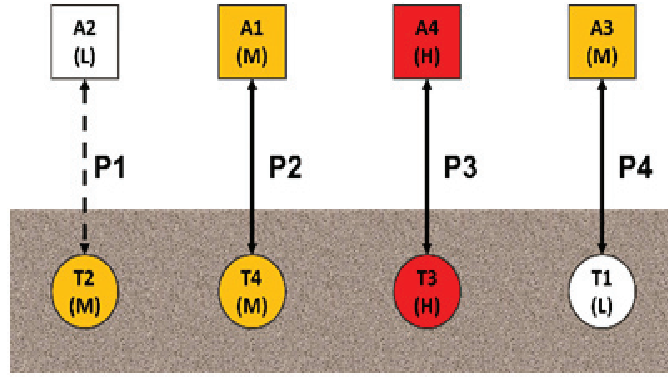

Figure 2. Change of judgment for targets of P1, P2, and P4.

Table 3. Detection probability ($c_{ij}^{c_2}$) when exchanging pairs

i \ j	P1	P2	P3	P4
P1	0.5(0)	0.4(0)	0.1(0)	0.9(1)
P2	0.85(1)	0.8(1)	0.5(0)	0.95(1)
P3	0.95(1)	0.95(1)	0.9(1)	0.99(1)
P4	0.8(1)	0.9(1)	0.3(0)	0.95(1)

3.2. Finding a Feasible Solution of the Weapon–Target Assignment Problem

As discussed in Section 2, TAE can be used for finding a feasible solution of WTA. Suppose T is a set of targets and W is a set of weapons. Let p_{ij} be the probability of killing the target $j \in T$ by weapon $i \in W$ and v_j be the value of target j. For each weapon *i*, it has μ_i, the number of available units. The decision variable x_{ij} indicates that how many weapons *i* are assigned to target *j*. WTA problems are generally modeled as the following formulation.

$$\min \quad \sum_{j \in T} v_j \prod_{i \in W} (1 - p_{ij})^{x_{ij}} \tag{7}$$

$$s.t. \quad \sum_{j \in T} x_{ij} \le \mu_i \qquad \forall i \in W \tag{8}$$

$$x_{ij} \in Z^{+} \qquad \forall i \in W, \ \forall j \in T \tag{9}$$

TAE has the advantage of a simple mixed integer programming (MIP) model. The key idea to apply the algorithm is to make an initial assignment and repeat the process of finding an exchange solution in the direction of reducing the survival probability of the target.

The algorithm requires proper conversion of parameters from WTA to TAE. Specifically, the type and quantity of weapons and targets and the values of target v_j and p_{ij} are necessary. An algorithm for obtaining a feasible solution of the WTA problem by applying the target–asset exchange algorithm is given as follows.

Algorithm 1. Finding a feasible solution for WTA

Obtain Target & Asset information, v_j, p_{ij} , and μ_i , $\forall i \in W,\ \forall j \in T$ from WTA

Initialization: $k = 1$

Generate initial assignment for TAE

Calculate Z_1 (i.e. ,objective function value of WTA)

Loop Pairwise Comparison $\forall i \in N,\ \forall j \in N$

Calculate d_{ij} (i.e., a surrogate function for estimating the improvement of the exchange)

if $d_{ij} > 0$ then

$c_{ij} = 0$ (i.e., incompatible case)

Else

$c_{ij} = 1$

End Loop

Solve the TAE with updated c_{ij}

Calculate Z_{k+1}

if $Z_{k+1} < Z_k$ then

Go to line 5 and $k \leftarrow k+1$

else

STOP

The objective function value of WTA, Z, is calculated outside the TAE algorithm. d_{ij}, which appears in the algorithm, is an evaluation function that considers the change in the survival probability of pair *i* and *j*. If the value of all pairs is the same, only the change in survival probability should be considered, but if the value is different, d_{ij} should be calculated by considering the values of the targets. The exchange occurs greedily in a direction in which the sum of the survival probability multiplied by the value decreases. Therefore, this guarantees a non-increasing Z. However, as an additional consideration, the WTA problem and the TAE problem have different assumptions. TAE is a one-to-one match between the target and asset, whereas WTA allows scenarios where no assets are assigned or multiple assets are assigned to one target.

To obtain a feasible solution of WTA under the assumption of one-to-one matching, conceptual elements, *clones*, can be created for pairs of TAE problems and included in the algorithm, as shown in Fig. 3.

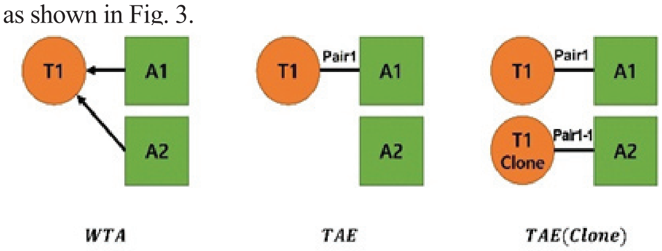

Figure 3. Comparison of WTA, TAE and TAE (clone).

In the case of exchanges including clones, the sum of the overall survival probabilities may increase as a result of the greedy action of each pair. In practice, the objective function value could be improved in some cases when iterating until the final exchange without creating a clone and then creating and exchanging the clone.

4. COMPUTATIONAL RESULTS

To verify the effectiveness of the exchange algorithm, we conduct experiments with various instances for the TAE and the WTA problems. The algorithm was coded using Python (version 3.10).[25] The TAE problem mentioned in Section 3.1 is solved using IBM CPLEX[26] with the General Algebraic Modeling System (GAMS) (version 34.3.0).[27] All experiments are conducted using CPU(Intel i7-1165G7) with 8 GB of RAM.

4.1 Target–Asset Exchange Problem

We conducted experiments to determine how much compatibility can be satisfied through pair-to-pair exchange for random instances. The experiment was conducted by changing the number of targets, assets, and existing pairs, and the levels were 10, 30, 50, 100, and 200.

The name of the instance denotes the '(number of targets and assets)-(number of pairs)'. An instance has a detection probability for each combination of targets and assets. The detection probability for each pair is a uniformly distributed number between 0 and 1. In Table 4, n_1 means the number of incompatible pairs before the exchange, n_2 is the number of exchanges made, n_3 is the number of compatible pairs after the exchange, the ratio is the proportion of n_3 to the number of pairs, which denotes the ratio of pairs that satisfy compatibility among all pairs after the exchange, time is the computation time (s) needed to solve the TAE problem and density is the proportion of non-zero elements in the C_{ij} matrix. For example, regarding instance 10-200 in Table 4, 155 pairs need to be exchanged. Through the TAE algorithm, it is possible to find the compatibility of 182 pairs, including the exchange of 173 different pairs. The density of this instance is 0.257, which

Table 4. Computational results of TAE with various instances

instances	n_1	n_2	n_3	ratio	time(s)	density
10-10	9	3	4	0.4	1.844	0.22
10-30	28	21	21	0.7	1.516	0.214
10-50	42	25	28	0.56	1.187	0.188
10-100	83	64	68	0.68	1.328	0.162
10-200	155	173	182	0.91	1.641	0.257
30-10	8	5	6	0.6	1.094	0.21
30-30	25	27	27	0.9	2.797	0.199
30-50	39	42	46	0.92	2.297	0.211
30-100	87	91	93	0.93	1.922	0.197
30-200	171	187	194	0.97	4.016	0.185
50-10	8	3	4	0.4	1.093	0.14
50-30	28	27	27	0.9	2.828	0.177
50-50	44	45	46	0.92	1.062	0.196
50-100	83	97	100	1	2.047	0.205
50-200	171	197	199	1	2.796	0.203
100-10	8	4	6	0.6	1.125	0.17
100-30	25	26	28	0.93	1.219	0.198
100-50	40	48	49	0.98	2.922	0.186
100-100	79	100	100	1	2.734	0.197
100-200	162	198	200	1	2.312	0.195
200-10	8	6	7	0.7	1.64	0.21
200-30	25	28	30	1	1.594	0.207
200-50	42	49	50	1	1.844	0.21
200-100	88	100	100	1	2.078	0.205
200-200	165	199	200	1	7.875	0.195

indicates the ratio of non-zero elements in the C_{ij} matrix.

In the experiment, an exchange solution was obtained within a relatively short time regardless of the instance size. The remarkable achievement indicated by the experimental results is that the greater the number of pairs is, the greater the ratio satisfying the compatibility after the exchange regardless of the quantity of assets and targets. In the table, the density is an indicator of how many compatible pairs there are. If the density is high, the number of possible exchanges increases. Considering the military situation, having enough various kinds of assets is beneficial to becoming compatible through exchanges.

4.2. Weapon-Target Assignment Problem

In this section, we present the experimental results to find

Table 5. Results of finding the WTA feasible solution

Instance	iteration	value	computation time(s)	#of exchange
WTA-100-1	initial	25.40506		
	1st	11.344162	3.109	83
	2nd	9.311302	3.453	22
	3rd	8.393934	3.234	15
	4th	7.969868	3.375	9
	5th	7.691145	3.391	5
	6th	7.688678	3.172	1
	WTA(MINLP)	1.224	281.19	
WTA-100-2	Initial	30.65726		
	1st	15.01639	1.375	100
	2nd	7.02865	1.5	95
	3rd	3.40384	1.453	79
	4th	2.57981	1.359	24
	WTA(MINLP)	1.748	221.64	
WTA-100-3	initial	21.73079		
	1st	10.86233	1.672	98
	2nd	5.69887	1.547	85
	3rd	3.70978	1.64	65
	4th	2.87281	1.266	14
	WTA(MINLP)	1.08	189.94	
WTA-150-1	initial	34.91911		
	1st	17.51089	2.141	148
	2nd	8.89802	2.031	143
	3rd	4.6597	2.078	115
	4th	2.82881	1.907	68
	5th	2.74136	1.907	5
	WTA(MINLP)	6.005	116	
WTA-150-2	initial	42.64522		
	1st	21.5578	1.937	147
	2nd	12.2577	2.141	142
	3rd	6.269	2.032	111
	4th	4.2025	1.984	65
	5th	3.79948	1.953	7
	WTA(MINLP)	4.872	213.19	
WTA-150-3	initial	38.51309		
	1st	18.46247	2.062	150
	2nd	8.66938	1.844	141
	3rd	4.94561	1.86	114
	4th	3.22926	1.969	64
	5th	2.73641	1.938	21
	WTA(MINLP)	4.66023	326.67	
WTA-150-4	initial	37.611859		
	1st	18.4932	1.922	146
	2nd	9.79515	1.985	139
	3rd	5.039	1.985	117
	4th	2.68646	2	72
	WTA(MINLP)	5.09527	300.55	

a feasible solution to the WTA problem by iteratively applying the TAE algorithm. The algorithm improves the objective function value from the initial random target–asset assignment until no further improvement occurs. We compared the time to solve the WTA of the general MINLP model for each instance. The WTA problem was solved using the GAMS[27] solver BARON[28] with the default setting. In the table of results, the values and times of the original WTA problem are shown for each instance when an integer that is a feasible solution to the WTA problem is found. The feasible solution of WTA through exchange was found with any target–asset assignment as the initial pair, and it was iteratively improved. Compared to the computation time of MINLP, it was possible to obtain a fair feasible solution in a short time.

5. CONCLUSIONS

In this section, we summarize the study, discuss the military findings, and present directions for future research.

This study presented a TAE model that satisfies target–asset compatibility through asset exchange with inappropriate or inefficient matching. The TAE model can obtain an exchange solution within a short time (within approximately 3 seconds) even when the number of target–asset pairs is 200. In addition, we proposed an algorithm to find a good feasible solution to the WTA problem by applying the TAE model. As the size of the problem in the experiment increases, it has an advantage in computation time compared to solving the WTA with the MINLP formulation.

The findings regarding military perspectives derived from this study are as follows.

- With the rapid development of drones and unmanned system technologies, more reconnaissance assets will form a complex network on the future battlefield. Hence, various measures are required to control reconnaissance assets, including those on the ground as well as in the air. These control measures will include more efficient operations beyond simply monitoring videos or images taken by reconnaissance assets. It is expected that our TAE algorithm can be used as one of such measures.
- In considering how to apply an optimal exchange using the TAE model in practical military applications, commanders and staff should be concerned about incompatibility occurrences. These scenarios may be due to erroneous judgments using the initially available information or may arise as intentional obstructions perpetrated by the enemy. To prevent this incompatibility from thwarting the success of reconnaissance operations, it is important to have a standard that can quickly determine incompatibility and to have sufficient assets.

The TAE model has a limitation in that it does not reflect the physically required movement time when exchanging assets. Considering the suggested limitations, future research should be directed toward ascertaining whether the time required to exchange assets can be reflected in the model. In regard to operational aspects, when an asset exchange occurs sequentially, a study on the determination of priorities and schedules is also required.

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