

# Simulation Model for Studying the Effect of Function Distribution on the Evaluation of Building Damage Caused by Missile Attack

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## ABSTRACT

When a building is hit by a missile, the most important parts are usually destroyed first to achieve maximum damage to the functions of the building. To accurately quantify the damage to a building, a function distribution density is constructed to describe the importance of different parts, and is applied to the probability damage calculation of a building under a missile strike. Based on the objective characteristics, the building is divided into several modules. The importance of the different modules is calculated using the damage tree. The distribution densities of the physical and system functions are constructed separately and combined into the function distribution density of the building. Meanwhile, the landing points of the missile are simulated using the Monte Carlo method, and depending on whether the function distribution density is considered, a probability damage calculation is performed. In comparison, the calculation results considering the function distribution density have a larger irregular shape, which can describe the damage to the building more accurately. This study can provide support for improving the physical protection of buildings and ensuring the operational reliability of their functions.

**Keywords:** Modular quantization; Function distribution density; Probability damage calculation; Reliability analysis

## 1. INTRODUCTION

Conventional missiles are among the main weapons used on modern battlefields. Its damage to targets has always been an important issue in damage evaluation. Shock waves, which are the main damage element in conventional missiles, are seriously destructive to buildings that occupy a large proportion of military facilities. A building is an entity that carries certain expected functions, which can be divided into physical and system functions. The physical function is the objective function derived from material properties, and the system function is the function attached to the physical basis. Therefore, studying the damage caused by shock waves to buildings is of great significance. Our group has conducted studies on damage to buildings, in which the functional setting of the buildings is uniform<sup>1-2</sup>. However, in reality, most buildings have various functional layouts. Therefore, we address this problem in the present study.

Different parts of a building carry different functions, which result in different levels of importance in different areas. The building damage caused by a missile is not only affected by the landing point of the missile, but also by the area importance within the building. Therefore, describing the importance of these areas and combining it with probability damage calculation has become urgent problems. Several studies have investigated this issue.

In terms of damage to buildings, simulations and experiments are primarily used to evaluate the degree of damage to buildings. In relevant studies, some scholars evaluated the damage to buildings by comparing the data obtained from monitoring or experimental testing with real situations<sup>3-4</sup>. Only a few studies involved the impact of function areas, in which only the core part or facility is considered<sup>5-6</sup>. Although the damage assessment considering the core improved the accuracy of predicting physical damage, there is still a large deviation from the actual situation. This occurs because the description of the function distribution is insufficient. Moreover, large-scale simulations and experiments can accurately determine the damage to buildings, but the use of simulations or experiments inevitably has problems, such as low efficiency and high cost. These problems make it difficult to widely apply these methods. Previous studies have often employed data analysis or empirical formulas to judge the damage to a large target system or discuss the failure mechanism of buildings<sup>7-8</sup>, but they are insufficient for small target damage and data collection.

In terms of area importance, only the importance of different areas or systems has been investigated for ships and aircraft<sup>9-10</sup>, there is a lack of functional analysis of buildings. At the same time, most studies stop at the importance calculation of different areas or systems, which makes it impossible to obtain the damage to the buildings. In importance calculation studies, some scholars have quantitatively analyzed various factors based on internal relations to obtain the corresponding importance<sup>11-13</sup>. Importance calculation is the basis for

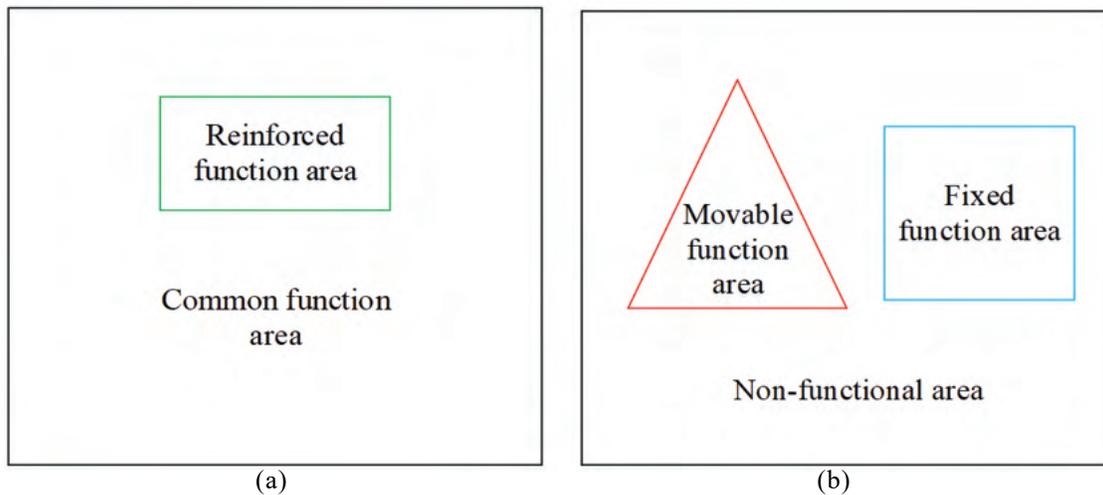


Figure 1. Modular diagram of function areas: (a) Physical function areas, and (b) System function areas.

an accurate description of system characteristics and the improvement of system reliability.

In this study, aiming at the probability damage calculation of a building that considers different function areas, function distribution densities are constructed to characterize the importance of the functions of the building. Their effects on the probability damage are analysed. In the analysis process, we first build a damage calculation model for the building based on the damage element parameters. Subsequently, by combining the probabilities of landing points simulated by the Monte Carlo method, a damage probability model is constructed. Finally, to determine the effect of considering or not the function distribution density, the damage caused by the missile to the building is calculated separately. Quantitative results can indirectly reflect the importance distribution in a building. The importance distribution can guide the functional layout and structural protection, which are important references for ensuring the operational reliability of functions.

## 2. MODULARITY OF FUNCTION AREAS OF BUILDINGS

Because a building is a physical entity with some system functions, its degree of damage after being attacked cannot be accurately described solely at the physical level or functional level. In addition, the system functions depend on the physical foundation, and they are non-uniformly coupled to each other. To quantify the damage more accurately, this study introduces a modularized processing method for function areas to characterize the physical and system functions of building.

### 2.1 Modular Analysis of Function Areas

A modular analysis is performed on a building, and the importance of the function areas is characterized by the function distribution density. The specific steps are as follows:

- The characteristics of the physical foundation and system functions are determined
- The relationship between the common and reinforced areas of physical foundation are determined
- The importance of each system function area is calculated

- Function distribution densities are constructed for the physical foundation and system functions
- The target probability damage is calculated in combination with the function distribution density
- The effect of the function distribution density on the target probability damage is contrastively analysed.

The steps for constructing the distribution density of the physical function are (1), (2), (4), (5), and (6). The importance is not zero as long as there is a physical foundation. The steps for constructing the distribution density of the system function are (1), (3), (4), (5), and (6). The number, type, and location of the system function areas are not fixed.

### 2.2 Modular Diagram of Function Areas

The division of function areas should be based on the distribution of specific functions. Physical function areas are classified into reinforced and common areas, and they have multiple strength relationships. The system function areas are divided into multiple function areas, and no direct correspondence is observed between the function areas. To simplify the calculations, the physical and system function areas are represented as regular geometries, and the functions in each area are uniform. A modular diagram is shown in Fig. 1.

Figure 1 shows a simple illustration. The function areas can be flexibly divided according to the needs of the actual situation.

## 3. DETERMINATION OF THE FUNCTION AREAS OF BUILDINGS

The premise for dividing the function areas is to determine the system functions, reinforced parts, and corresponding distribution information. After the function areas are identified, the importance of each function area is determined.

### 3.1 Function Areas

The buildings in this study are targets whose heights have little effect on the probability damage calculation. This type of target usually has a larger area and lower height, and is widely distributed in military facilities.

The physical function area corresponds to the physical foundation, which is divided into reinforced and common areas based on strength. Strength is defined as the resistance of various modules within buildings to overpressure and impulses. The system function area must be determined according to the specific system functions. They can be classified as fixed, movable, and non-functional areas.

- The fixed function area corresponds to the immovable system within the building
- The movable function area is the area occupied by the equipment in the building, which changes with the availability and location of the equipment.
- The non-functional area is the area other than (1) and (2) within the building without any system function.

The division of physical and system function areas in a modular manner is independent of each other.

### 3.2 Importance Calculation of Function Areas

The importance of the physical and system function areas must be calculated separately. Because the distribution rules and relationships between function areas are different, importance calculations are performed in different ways.

#### 3.2.1 Importance Calculation of Physical Function Areas

The common area can be determined based on the building area. If the physical strength of some areas is significantly greater than that of the surrounding areas, the reinforced area can be determined. Because physical strength is used as a criterion of numerical quantity, the relationship between reinforced areas and common areas can be characterized each other by integers or decimals. The importance can be obtained from the strength relationships among the various areas.

#### 3.2.2 Calculation of Importance of the System Function Areas

Although a building may include multiple system function

areas and the importance of each function area is different, the system functions are interrelated. Owing to this feature, the damage tree<sup>14</sup>, which is the most appropriate method, is selected to calculate the importance of the system function areas.

The damage tree is developed from the fault tree<sup>15</sup>, which determines the importance of each event according to logical relationships. The coefficients that characterize the importance of basic events include structure importance, probability importance, and critical importance<sup>16</sup>. Based on the meanings of the coefficients and the characteristics of the object, the structure importance is adopted to characterize the importance of each area. Methods for solving the structure importance include the minimum cut set method, structure importance coefficient method, and probability importance property method<sup>17</sup>. Considering that the number of basic events examined in this study is not excessive and the principle of accuracy first, the structure importance coefficient method is used to solve the structure importance.

This method is based on the changes in the state of basic events. When a basic event  $X_i$  changes from the normal state (0) to the damage state (1), and the states of other basic events remain unchanged, the state of result event changes from  $\varphi(0_i, X)$  to  $\varphi(1_i, X) = 1$ . This implies that the state change of the basic event plays a role in the occurrence of the result event. The total number of incompatible combinations of the two states of  $n$  basic events is  $2^n$ . When the basic event  $X_i$  is taken as the change object, the remaining  $n-1$  basic events remain unchanged in the control group with a total of  $2^{n-1}$ . The ratio of the number of basic events that cause a change of the result event to  $2^{n-1}$  is the structure importance of  $X_i$ . The formula is as follows:

$$I_{\varphi(i)} = \frac{1}{2^{n-1}} \sum [\varphi(1_i, X) - \varphi(0_i, X)] \tag{1}$$

where,  $I_{\varphi(i)}$  is the structure importance of basic event  $X_i$ . The construction of a damage tree is a rigorous analysis process, in which logical relationships are generally established by deduction. The symbols for events and logic gates are shown in Fig. 2.

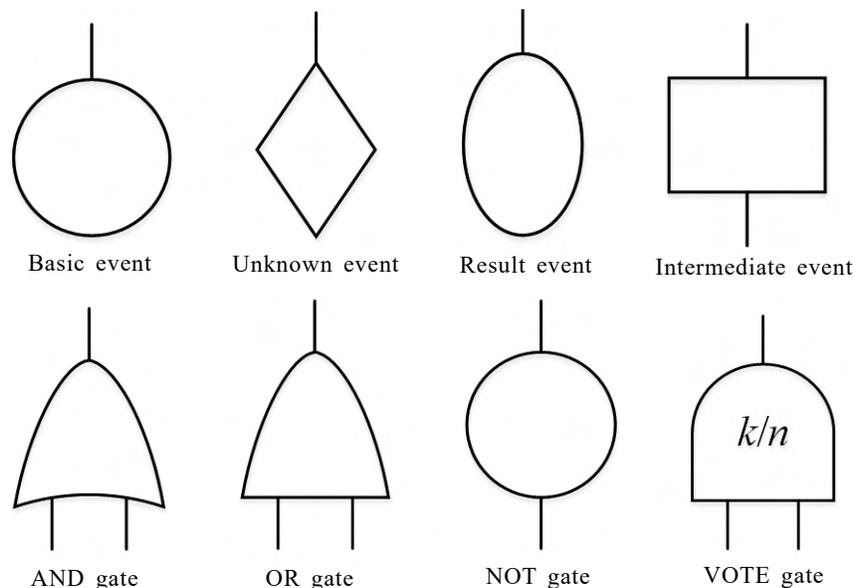


Figure 2. Symbols of the damage tree.

Symbols representing various areas (systems) are combined through logical relationships, which results in the damage tree of the building.

Because the types and layouts of functions in different buildings differ, the division of function areas should be based on the specific functions of the buildings. In addition, for convenience of analysis and calculation, each function area is represented by regular geometry. The functions carried by each function area are unique and there are no overlapping areas. This relationship is expressed as follows:

$$A = \bigcup_{j=1}^m A_j, A_k \cap A_l = \emptyset (k \neq l) \tag{2}$$

where,  $A$  is the area of a building;  $A_k$  and  $A_l$  are the  $k^{\text{th}}$  and  $l^{\text{th}}$  function areas, respectively;  $m$  is the number of function areas.

#### 4. CONSTRUCTION OF FUNCTION DISTRIBUTION DENSITY

##### 4.1 Construction of Distribution Density of Physical Functions

The physical function of a building refers to a series of physics-based functions such as support, occlusion, and storage, which belong to the nature of the physical foundation. Once the physical foundation is destroyed, its physical functions disappear accordingly. The ratio of disappearance is proportional to the damaged area of the building.

The internal structures of different buildings are different. However, owing to the small differences in physical strength, except for the reinforced areas, a uniform distribution is used to characterize the common area. The reinforced area is still characterized by a uniform distribution and multiple relationship with the common area. The specific form of the distribution density of the physical functions is:

$$f_w = \begin{cases} \iint_{Q_1} f(x, y) dx dy & (x, y) \in Q_1 \\ a \cdot \iint_{Q_2} f(x, y) dx dy & (x, y) \in Q_2 \end{cases} \tag{3}$$

where,  $f_w$  is the distribution density of the physical functions,  $x$  and  $y$  are the coordinates in the horizontal plane,  $f(x, y)$  is the density function of the common areas,  $Q_1$  and  $Q_2$  represent the common and reinforced areas, respectively, and  $a$  is a multiple.

##### 4.2 Construction of Distribution Density of System Functions

The system function refers to the additional functions included in a building, such as communication, control, and protection. The distribution density of the system functions must be constructed based on these system functions. A system is composed of different function areas. The importance within each function area is equal, and the sum of the importance is 1.

Because the importance of each function area is set to be continuous within the area, a piecewise function is used for the partitional processing of the distribution density of system functions, and the corresponding expression form is:

$$f_g = \begin{cases} \iint_{Z_1} f_i(x, y) dx dy & (x, y) \in Z_1 \\ \vdots & \vdots \\ \iint_{Z_i} f_i(x, y) dx dy & (x, y) \in Z_i \\ \vdots & \vdots \\ \iint_{Z_c} f_c(x, y) dx dy & (x, y) \in Z_c \end{cases} \tag{4}$$

where,  $f_g$  is the distribution density of the system functions;  $c$  is the number of system function areas;  $f_i(x, y)$  is the density function of the  $i^{\text{th}}$  system function area;  $Z_1, Z_2, \dots,$  and  $Z_c$  are different system function areas.

The distribution density of the system functions is simplified. Each part of the piecewise function is set to a constant value. However, this can be set according to specific requirements.

##### 4.3 Function Distribution Density of the Building

The combination of distribution densities forms the function distribution density of the building; however, the respective weights must be defined. Different weights directly affect the value of the function distribution density of the building, and the sum formula is as follows:

$$f = W_w \cdot f_w + W_g \cdot f_g \tag{5}$$

where,  $f$  is the function distribution density of the building;  $W_w$  and  $W_g$  are the weight coefficients, and  $W_w + W_g = 1$ .

#### 5. TARGET PROBABILITY DAMAGE CALCULATION

##### 5.1 Calculation of Building Damage

Physical and system functions are based on the physical foundation. Once the physical foundation is damaged, both physical and system functions disappear. Therefore, the basis of damage calculation is to quantify the damage to the physical foundation. Because the height of the buildings is not considered, the percentage of damaged area is an index that intuitively measures the damage to the physical foundation.

###### 5.1.1 Characterization of Damage Relationship

The damage caused by missiles to buildings is mainly due to shock waves, and overpressure and impulse are the main parameters for measuring the damage. Premising the percentage of damaged area is used to measure the damage, we derive how the percentage of damaged area is related to the overpressure and impulse.

The PROBIT equation<sup>18</sup> is used to comprehensively describe serious damage to buildings considering overpressure and impulse. By combining the explosion data and TNO (The Netherlands Organization) data provided by Lee<sup>19-20</sup>, a specific relationship is obtained<sup>2</sup>.

$$100P_d = 50.05 - 7.31 \ln \left[ \left( \frac{40000}{P_s} \right)^{7.4} + \left( \frac{460}{I} \right)^{11.3} \right] \tag{6}$$

where,  $P_d$  is the percentage of damaged area;  $P_s$  is the overpressure, Pa;  $I$  is the impulse, Pa·s.

Once the composition and mass of the explosives in a missile are identified, the overpressure and impulse at any location on the plane can be determined. Therefore, Eqn. (6)

can be converted into a relationship between the percentage of damaged area and the distance from the explosion center.

Based on the relationship curves between the overpressure-scaled distance and scaled impulse-scaled distance of TNT (Trinitrotoluene) under a hemispherical explosion given by I.Chem.E., the overpressure and scaled impulse are characterized<sup>21-22</sup>. By substituting the corresponding relationship into Eqn. (6), it can be used to obtain the relationship of how the percentage of damaged area is related to the distance from the explosion center and the mass of the explosive.

(1) In the interval of  $1 \leq l' < 10$

$$100P_d = 50.05 - 7.31 \ln \left[ 1.83 \times 10^{-11} \left( \frac{l^3 Q_{vTNT}}{\alpha m_w Q_{vi}} \right)^{4.96} + 10338 \left( \frac{l^{1.43} Q_{vTNT}}{\alpha m_w Q_{vi}} \right)^{7.19} \right] \quad (7)$$

(2) In the interval of  $10 \leq l' \leq 200$

$$100P_d = 50.05 - 7.31 \ln \left[ 1.30 \times 10^{-5} \left( \frac{l^3 Q_{vTNT}}{\alpha m_w Q_{vi}} \right)^{2.86} + 36 \left( \frac{l^{1.54} Q_{vTNT}}{\alpha m_w Q_{vi}} \right)^{7.76} \right] \quad (8)$$

where,  $m_w$  is the mass of the explosive, kg;  $Q_{vi}$  is the explosive heat of explosive, kJ/kg;  $Q_{vTNT}$  is the explosive heat of TNT, kJ/kg, which is 4187 kJ/kg;  $l$  is the distance from the explosion center, m;  $\alpha$  is the correction factor, the value of infinite space explosion is 1, the value of rigid ground explosion is 2, and the value of ordinary ground explosion is 1.8.

### 5.1.2 Characterisation of Damage Degree

Based on the classification of building functions and the definition of damage modes in this study, the damage to physical and system functions is characterised.

#### 5.1.2.1 Damage to Physical Functions

The damage to physical functions is proportional to the total damaged area of the building. Therefore, when characterizing the reduction of physical functions, a univariate function of the total damaged area is adopted.

$$F_w = \frac{S_w}{S_z}, S_w = \iint_D f_w(x, y) d\sigma \quad (9)$$

where,  $S_w$  is the total damaged area;  $S_z$  is the total area of the building;  $D$  is the integral area;  $F_w$  is the percentage of total damaged area for physical functions.

#### 5.1.2.2 Damage to System Functions

Because a system function is a whole, its destruction mode differs from that of physical functions. The reduction of system function is that the corresponding system function disappears once the physical foundation of the function area is partially damaged. Therefore, when measuring the damage to system functions, the actual damaged area of the physical foundation should be obtained using an accurate equivalent method<sup>2</sup>.

$$F_g = \frac{S_g}{S_t}, S_g = \iint_D f_g(x, y) d\sigma \quad (10)$$

where,  $S_g$  is the damaged part of the system functions;  $S_t$  is the

overall system functions;  $F_g$  is the percentage of damage to the system functions.

## 5.2 Calculation of the Landing Point Probability

In the process of striking targets, the missile is inevitably affected by self-factors and external factors, which produce corresponding errors and cause it to deviate from the ideal landing point. The Monte Carlo method<sup>23</sup> is employed to simulate the distribution of landing points, and the distribution model adopts the two-dimensional normal distribution that is commonly used<sup>24</sup>. It is assumed that the transverse and longitudinal distributions are independent of each other. When  $(x_0, y_0)$  is considered as the ideal landing point, the distribution density function of the landing points is:

$$p(x, y) = \frac{1}{2\pi\sigma_x\sigma_y} \exp \left\{ -\frac{1}{2} \left[ \frac{(x-x_0)^2}{\sigma_x^2} + \frac{(y-y_0)^2}{\sigma_y^2} \right] \right\} \quad (11)$$

where,  $\sigma_x$  and  $\sigma_y$  are the standard deviations in the  $x$  and  $y$  direction, respectively.

Simplifying Eqn. (11), when the standard deviations in the  $x$  and  $y$  directions are equivalent, the probability of landing points in a circle of radius  $C$  is:

$$P = 1 - \exp\left(-\frac{C^2}{2\sigma^2}\right) \quad (12)$$

The CEP (circular error probable)<sup>25</sup> is the most commonly used index for measuring the strike accuracy of a missile. This is defined as the value of  $C$  during  $P = 0.5$ . The smaller the CEP, the higher is the strike accuracy. Substituting  $P = 0.5$  into Eqn. (12) gives:

$$CEP = C_{max} = \sqrt{2\sigma^2 \ln 2} \approx 1.177\sigma \quad (13)$$

By applying Eqn. (13) into the simulation of the landing points, the probability of landing points under any degree of damage can be obtained.

## 6. IMPACT ANALYSIS OF PROBABILITY DAMAGE CALCULATION

### 6.1 Information of a Missile and a Building

To analyse the impact of the probability damage, the

Table 1. Parameters of a missile

Mass	Explosive heat	CEP
1000 kg	5673 kJ/kg	85 m

parameters of a missile are first presented, as listed in Table 1.

A hypothetical command center is attacked by the missile. The specific distributions of the physical and system function areas are shown in Fig. 3. There are no reinforced areas, and the red areas represent important system function areas.

### 6.2 Importance Calculation of Function Areas

Only the importance of the system function areas must be addressed because there are no reinforced areas. A damage tree is constructed according to the logical relationship between the various systems of the command center, as shown in Fig. 4.

The basic events of the damage tree are X1–X26, and

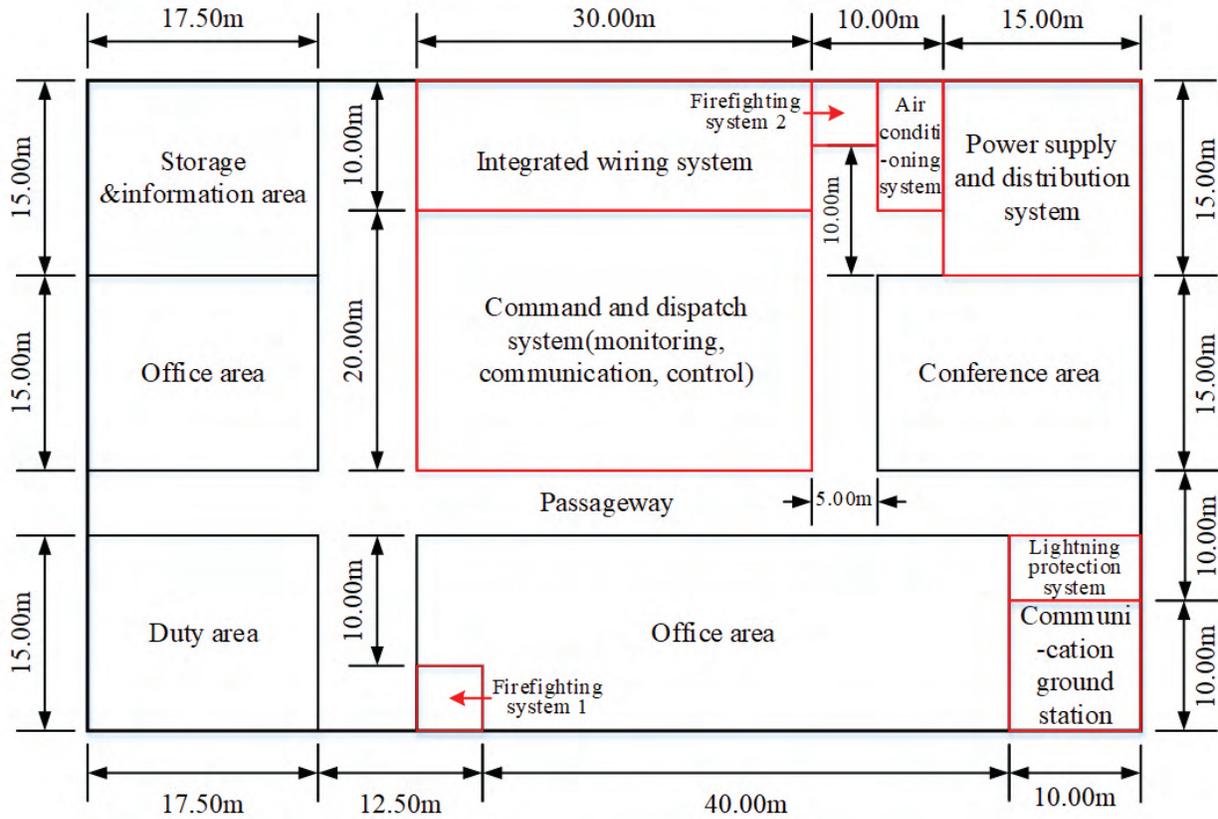


Figure 3. Distribution of function areas of the command center.



Figure 4. Damage tree of the command center.

the corresponding structure importance is calculated using the methods described in Section 3.2.2. The results are summarized in Table 2.

Basic events represent the components within the system function areas. While quantifying the importance of the system function areas, the structure importance of the components (basic events) contained in each area is accumulated to obtain the importance of the corresponding system function area. Based on the damage tree of the command center and the structure importance of the basic events, the importance of each system function area can be obtained. The importance of the function areas is listed in Table 2, and that of the non-functional areas is 0. Because the firefighting system includes two sub-regions, the corresponding importance is divided equally.

### 6.3 Function Distribution Density of the Command Center

#### 6.3.1 Distribution Density of Physical Functions

Because there are no reinforced areas within the command center, the physical function is uniform throughout the area. To be consistent with the quantification of the distribution density of the system functions, the integration sum of the distribution density of the physical function is set to 1.

#### 6.3.2 Distribution Density of System Functions

According to the definition of the distribution density of system functions and the destruction mode of system function areas, the importance of each system function area solved in

Section 6.2 is the overall importance, which does not have to be expressed in an integral form. The distribution density of the system functions is:

$$f_g = \begin{cases} 0.205 & (x, y) \in Z_1 \\ 0.141 & (x, y) \in Z_2 \\ 0.103 & (x, y) \in Z_3 \\ 0.192 & (x, y) \in Z_4 \\ 0.153 & (x, y) \in Z_5 \\ 0.0512 & (x, y) \in Z_6 \\ 0.154 & (x, y) \in Z_7 \\ 0 & (x, y) \in Z_8 \end{cases} \quad (14)$$

where,  $Z_1$  is the communication ground station area;  $Z_2$  is the integrated wiring system area;  $Z_3$  is the power supply and distribution system area;  $Z_4$  is the command and dispatch system area;  $Z_5$  is the air conditioning system area;  $Z_6$  is the lightning protection system area;  $Z_7$  is the firefighting system area;  $Z_8$  is the non-functional area of the command center.

#### 6.3.3 Function Distribution Density of the Command Center

The function distribution density of the command center is obtained by combining the distribution densities of the physical and system functions. The Delphi method<sup>26</sup> is used to determine the weight coefficients, and the values of the physical and system functions are 0.3 and 0.7, respectively.

The system function areas of the command center are

Table 2. Structure importance and normalised values of X1–X26

System	Basic event	Structure importance	Normalised value	System	Basic event	Structure importance	Normalised value
Communication ground station	X1	1	0.0512	Command and dispatch system	X14	1	0.0513
	X2	1	0.0513		X15	1	0.0513
	X3	1	0.0513		X16	0.25	0.0128
	X4	1	0.0513		X17	0.25	0.0128
Integrated wiring system	X5	0.25	0.0128	X18	0.25	0.0128	
	X6	0.25	0.0128	Air conditioning system	X19	1	0.0513
	X7	0.25	0.0128		X20	1	0.0513
	X8	1	0.0513		X21	1	0.0513
Power supply and distribution system	X9	1	0.0513	Lightning protection system	X22	0.5	0.0256
	X10	0.5	0.0256		X23	0.5	0.0256
	X11	0.5	0.0256	X24	1	0.0513	
	X12	1	0.0513	Firefighting system	X25	1	0.0513
	X13	1	0.0513		X26	1	0.0513

divided into eight categories. To ensure a uniform expression of the distribution densities of the physical and system functions, the combined expression is as follows:

$$f = \begin{cases} 0.3 \cdot \iint_z \frac{1}{4000} dx dy + 0.7 \cdot 0.205 & (x, y) \in Z \subseteq Z_1 \\ 0.3 \cdot \iint_z \frac{1}{4000} dx dy + 0.7 \cdot 0.141 & (x, y) \in Z \subseteq Z_2 \\ 0.3 \cdot \iint_z \frac{1}{4000} dx dy + 0.7 \cdot 0.103 & (x, y) \in Z \subseteq Z_3 \\ 0.3 \cdot \iint_z \frac{1}{4000} dx dy + 0.7 \cdot 0.192 & (x, y) \in Z \subseteq Z_4 \\ 0.3 \cdot \iint_z \frac{1}{4000} dx dy + 0.7 \cdot 0.153 & (x, y) \in Z \subseteq Z_5 \\ 0.3 \cdot \iint_z \frac{1}{4000} dx dy + 0.7 \cdot 0.0512 & (x, y) \in Z \subseteq Z_6 \\ 0.3 \cdot \iint_z \frac{1}{4000} dx dy + 0.7 \cdot 0.154 & (x, y) \in Z \subseteq Z_7 \\ 0.3 \cdot \iint_z \frac{1}{4000} dx dy + 0.7 \cdot 0 & (x, y) \in Z \subseteq Z_8 \end{cases} \quad (15)$$

As the destruction modes of the physical and system function areas are different, the distribution density of the physical function is expressed in an integral form, and the distribution density of the system function is expressed in an overall importance form in Eqn. (15). In addition, for clarity, Eqn. (15) is not simplified.

**6.4 Damage Calculation of the Command Center**

The TNT equivalent method is used to calculate the damaged area of the command center. Based on the parameters listed in Table 1, the corresponding radii in Eqn. (7) and (8) are 13.94 to 139.42m and 139.41 to 2788.32m in the case of the

rigid ground explosion.

The minimum distance corresponding to a 1% difference is used as the side length of the damage grid. According to the calculation results using Eqn. (7) and (8), the closer to the explosion center, the smaller is the distance. The minimum distance is 0.4695 m.

From the corresponding applicable ranges of Eqn. (7) and (8), the missile is applicable only to Eqn. (7). By substituting the parameters of the missile and explosion type into Eqn. (7), the percentage of damaged area of any grid can be obtained.

**6.5 Comparative Analysis of Probability Damage Calculation**

The essence of probability damage is to solve the probability of landing points. Under the same strike conditions, the landing point at which the same missile causes the same damage to the same building is not sole. Landing points with the same degree of damage are extracted to form the damage contour lines. By combining the damage contour lines of the building and Monte Carlo simulation of the landing points, the probability of any landing point can be obtained.

Based on the distribution information shown in Fig. 3 and the damage relationship, the damage contour lines of the command center with and without consideration of the function areas are obtained, as shown in Fig. 5.

By substituting the CEP of the missile into Eqn. (13), the standard deviation of the two-dimensional normal distribution is 72.19. The coordinate system is established centering on the ideal landing point, and the expectation of the two-dimensional normal distribution is 0m. The ideal landing point where the missile causes the largest percentage of damaged area to the command center is taken as the center of the Monte Carlo

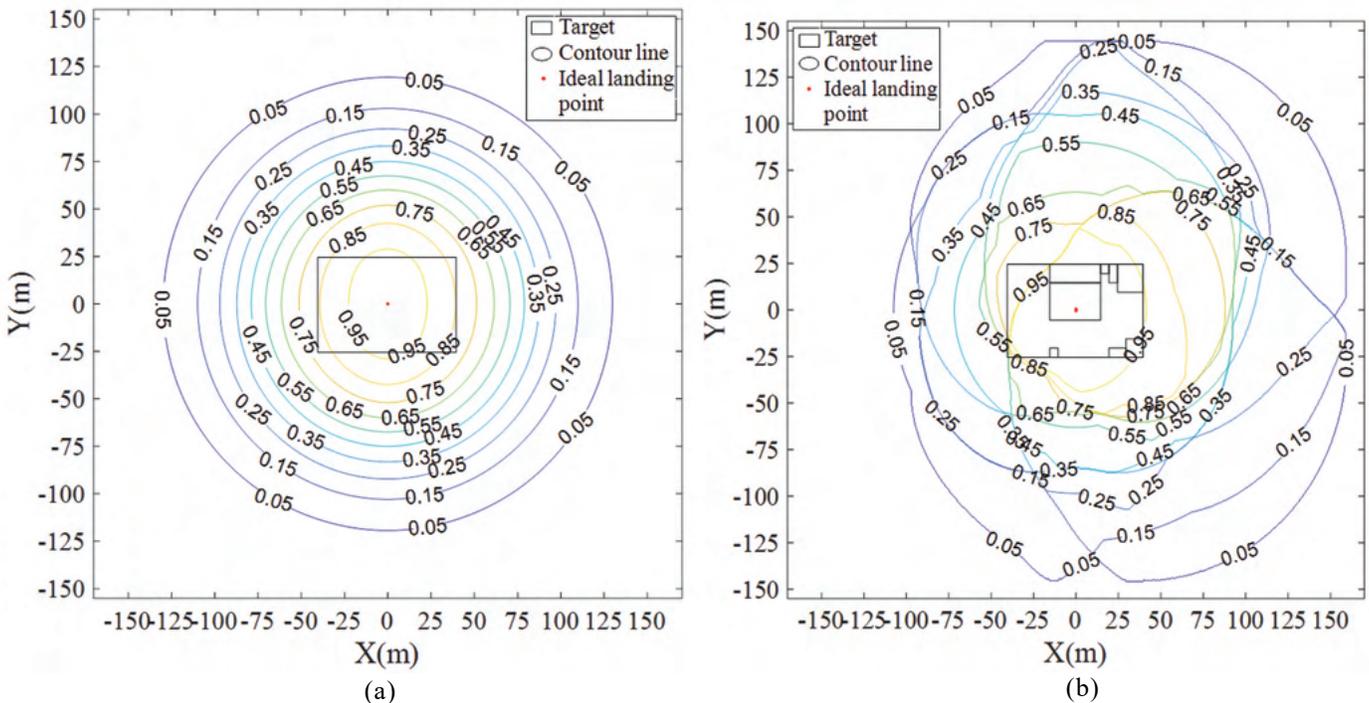


Figure 5. Damage contour lines of the command center, (a) Without consideration of the function areas, and (b) Considering the function areas.

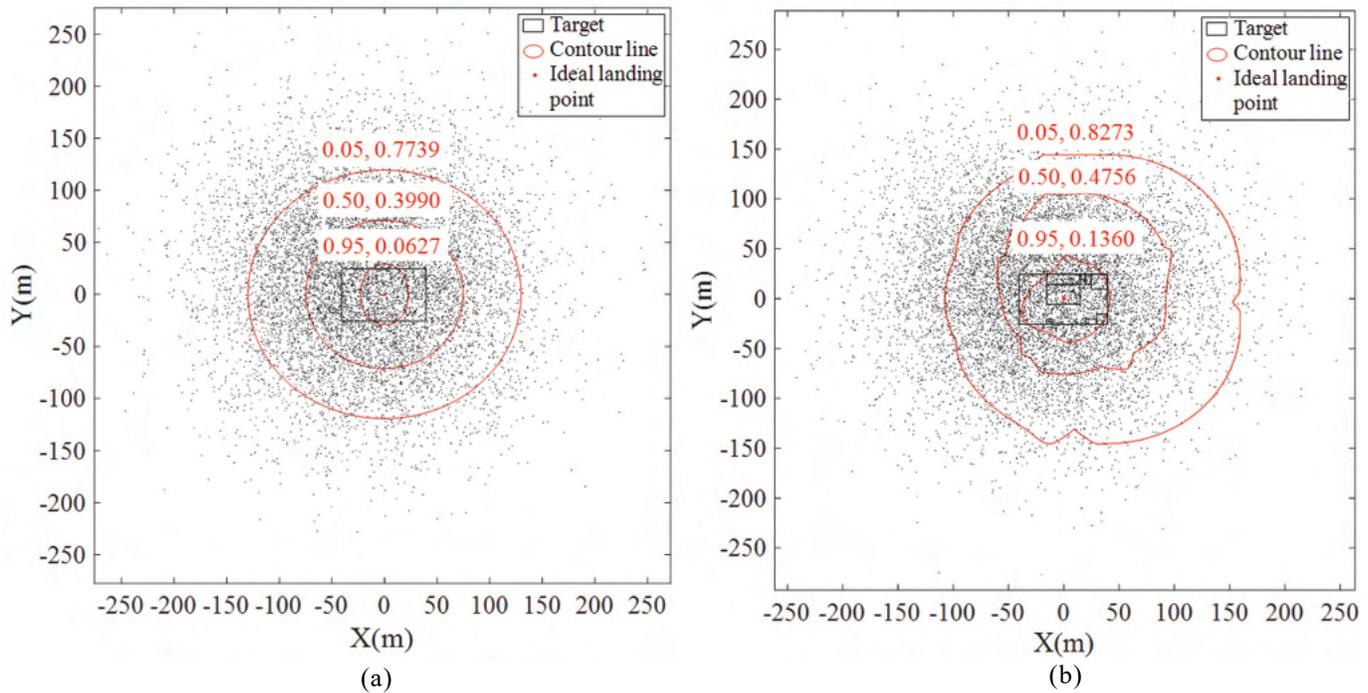


Figure 6. Probability damage of the command center: (a) Without consideration of the function areas, and (b) Considering of the function areas.

simulation, and 10,000 landing simulations are performed to obtain the distribution of landing points. By combining the damage contour lines, the damage probability of the command center at any degree of damage can be obtained, as shown in Fig. 6.

Taking the command center as an example, the probabilistic damage in the two cases, whether the function areas are considered or not, is calculated. The following differences can be found by analysing Fig. 5 and Fig. 6:

- The damage contour lines considering the function distribution density are not only affected by the shape of the command center but also by the distribution and importance of its function areas, which make the damage contour lines more irregular
- Under the same degree of damage to the command center, the result obtained by considering the function distribution density has a greater damage probability than that obtained without considering it. Under the same degree of physical damage, a greater damage effect is produced when the importance of the function areas is considered.

## 7. CONCLUSIONS

To accurately describe a building, the method of dividing areas is adopted to model typical functions. The importance of the physical and system function areas is measured by the strength-relationship and damage tree, respectively. Subsequently, a function distribution density composed of the physical and system components is established.

Based on the damage characteristics, the damage modes of the physical and system functions are classified, and the corresponding equations representing the damage degree of a building are constructed.

Through Monte Carlo simulation of the landing points,

the damage probability of a building is accurately determined based on the damage relationship and function distribution density. The calculation results considering the function distribution density exhibited greater irregularities, that are closer to the actual situation.

For simplicity and efficiency, regular geometries are adopted to represent the function areas, and a uniform distribution is adopted for the probability damage calculation. However, both the shape and distribution can be adjusted according to the requirements, which ensures the applicability of the function distribution density proposed in this study.

This study can describe the damage distribution of buildings more accurately, which not only provides a reference for protective improvement measures but also optimizes the layout of functions. Additionally, this model can help to conduct quantitative evaluations of functionally related building groups and provide strategic support for their reliable operation.

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