Defence Science Journal, Vol. 57, No. 6, November 2007, pp. 787-795


# Tensile Strength of Snow using Centrifugal Technique

Agraj Upadhyay, S.K. Joshi, and Chaman Chandel

*Snow & Avalanche Study Establishment, Manali-175 103*

**ABSTRACT**

Tensile strength of snow was determined using indigenously developed automated centrifugal machine. Processed snow (sintered at –20 °C for 4 days) samples of dia: 65 mm and height:130 mm were tested using this machine.The experiments were conducted on sieved snow at four temperature levels of 0 °C, –3 °C, –6 °C and –9 °C at density ranging from 200-460 kg/m$^3$. Results of these experiments have been compared with the earlier suggested models. Probability distribution of snow strength on the basis of current experimental data has also been presented.

**Keywords:** Tensile strength, centrifugal machine, microstructural parameters, failure stress, probability distribution, snow studies

## NOMENCLATURE

| | |
|---|---|
| $r$ | Snow density |
| $R$ | Radius of cylindrical snow sample |
| $h$ | Height of snow sample |
| $N$ | rpm at the time of failure of snow sample |
| $\sigma_f$ | Failure stress |
| $\sigma_i$ | Strength of fine-grained bubble-free ice with a random orientation |
| $\sigma_m$ | Maximum stress in maximum stress envelope |
| $n$ | Porosity of snow |
| $A,B,b,n_1$ | Experimental constants |

## 1. INTRODUCTION

The strength of snow has been the area of interest from the time when snow studies started. Avalanche triggers when there is an initiation of tensile failure on the crown surface, followed by a shear failure along the snow bed in a snowpack lying on slope. Some theories suggest opposite sequence, i.e., shear failure followed by tensile failure. For assessing the failure of snowcover, the tensile strength of snow is one of the important parameters, especially in slab avalanches. The maximum stress, which can be borne by the snowpack, is a measure of its strength. Snow being an elastic-visco-plastic, highly temperature-dependent complex material, it shows a wide variation in its strength. Many parameters affect the strength of snow. The main factors are density, snow temperature, and microstructural parameters. A lot of researchers have made efforts to determine the tensile strength of snow. The centrifugal technique to determine the tensile strength of snow was first reported by Hafeli and later it was described by de Quervain in detail.

In the present study, preliminary results of the tensile strength of snow using an automated centrifugal machine for the snow have been presented.

Received 18 April 2006, revised 27 November 2006

## 2. THEORETICAL TENSILE STRENGTH OF SNOW

### 2.1 Earlier Work

A theoretical porosity-dependent expression for the tensile strength of fully age-hardened snow was developed by Ballard and Feldt[1]. This is given by the following equation:

$$\sigma_T = \sigma_i \exp\left(-\frac{2n}{1-n}\right) \tag{1}$$

A method was used to incorporate the porosity-dependent strength of snow into the theory which has resulted in a limiting strength relationship. The limiting strength relationship, in general, represents the observed strength of age-hardened snow. However, on comparison with the experimental data, it was found that this limiting curve does not satisfactorily agree with the strength data at higher porosities (lower densities). On the other hand, there is a possibility that the assumed distribution of the maximum number of bonds per particle is not valid for this higher porosity range.

Similar equation given by Mellor and Smith can be expressed as

$$\sigma_T = \sigma_i \exp\left(-\frac{bn^2}{(1-n)^2}\right), \text{ where } b = 2 \tag{2}$$

Equation for the theoretical tensile strength envelope of Ballard and McGaw[2] is given by

$$\sigma_T = \sigma_i \left(1 - \frac{n}{n_1}\right), \text{ where } n_1 = 0.544 \tag{3}$$

Keeler and Weeks[3], conducted centrifugal tensile experiments on fine-grained snow and compared their data with a theoretical relation given by Ballard and Feldt[1]. It was found that there was less scatter in centrifugal tensile strength of snow and much simpler relation with density in comparison to the earlier data. A few results of centrifugal tensile experiments on natural snow were presented by Keeler[4]. The effect of sintering time on tensile strength was determined and the value of initial density of snow tested was 150 kg/m$^3$. It was observed that below 10 days of sintering time, tensile strength was <10 KPa and at 100 days sintering time, it reach up to 100 KPa. It was further deduced that the interrelation of a number of variables such as time, temperature, temperature gradient, and overburden stress is complex in a natural snowpack and there is no way in which their individual effects can be systematically studied. Sommerfeld[5] showed that maximum strength envelope for snow as a function of density can be given by the equation:

$$\log \sigma_m = A + B \log \rho \tag{4}$$

This equation characterises centrifugal tensile strength-density measurements. For fairly young snow, $A = -4.3$ and $B = -3$ when $\sigma$ is in $10^2$ N-m$^{-2}$ and $\rho$ in kg/m$^3$. The experiments were conducted on various types of snow but theses could not reveal much difference in constants. Moreover, Keeler could not observe the apparent temperature dependence at least over the range at which the measurements ($-1^\circ$C to $-15^\circ$C). Gubler[6], emphasised the use of statistical models to estimate the strength of snow together with the consistent definitions of the fundamental unit for a better understanding of brittle and ductile strength of snow in terms of structural or stereological parameters. It was also shown that using simple structural parameters instead of the snow density results in significantly improved correlations with the tensile strength.

*In situ* tensile strength of snow was determined by McClung[7], using a new method utilising large sample sizes. His data from the experiments show the effect of large sample size on tensile strength of snow indicating less scatter in the results as a function of density and lower mean values than the earlier conducted *in situ* tests. As per Sommerfeld[8], failure of snow in large volumes at low-stress rates could be predicted from tests on small volumes at high-stress rates. In the case of shear frame tests, the elements act independently and thus exhibit almost pure Daniel's behaviour, with very little Weibull behaviour. Tensile tests showed a mixture of the two types of behaviours. A problem with the tensile tests is that the appropriate volume for use in the Weibull distribution is uncertain. A comparison between Sommerfeld's and McClung's (unpublished) results indicate that it should be about $5*10^{-4}$ $m^3$. McClung's large-volume tensile strengths average about 1.5 KPa and the large-volume shear strengths of Sommerfeld and King[9] averaged about the same. Perhaps, the high ratios earlier found were an artifact of the small sample size and the statistics of snow strength.

## 2.2 Theoretical Tensile Strength of Snow

Conventional technique, to find the tensile strength of snow, is to apply a constant deformation rate on a snow sample. For this technique, special platens are made with a small heating element in it.

To fix the snow samples with platens, these platens are kept in contact with the snow sample ends under light pressure, and the power supply is given to the heating element for a short period. Platens get heated and as melting starts power supply is switched off. As cold chamber is kept at subzero temperature, melted water refreezes and an ice layer between the snow sample and platens is developed, which makes a strong bonding between the snow sample and the platens. However, if snow samples need to be tested near 0 °C or snow samples are very low density or fragile like depth hoars, then handling, mounting, and fixing of snow sample with platens on UTM becomes difficult. To overcome these difficulties, centrifugal load application technique is a good solution.

However, load application process is entirely different for both the techniques. As centrifugal force is a body force, each grain of snow experiences it due to rotation of sample. On rotation, because of mass of the snow, a centrifugal force is developed in one half of the sample and opposite half balances it. In centrifugal experiments, maximum tensile stress exists at the centre of the sample. However, in deformation rate experiments on UTM, force is applied to end surface of the snow sample and this force is transmitted to the whole sample through links of ice grains and maximum tensile stress will be in plane having minimum grain contacts (or maximum voids).

Free-body diagram of snow sample under centrifugal force is shown in Fig. 1. At any distance ($r$) from the axis of rotation, centrifugal force on a snow disc of thickness ($dr$) can be given by

$$dF = \rho\pi\left(R^2 dr\right)\left(\frac{2\pi N}{60}\right)(r) \qquad (5)$$

Hence centrifugal force on half of the snow sample can be determined by integrating Eqn (5) in the limits of $r$ from 0 to $h/2$. Final equation used for failure load calculation is given by

$$F = \frac{1}{2}\rho\pi\left(R^2 h\right)\left(\frac{2\pi N}{60}\right)^2 \frac{h}{4} \qquad (6)$$

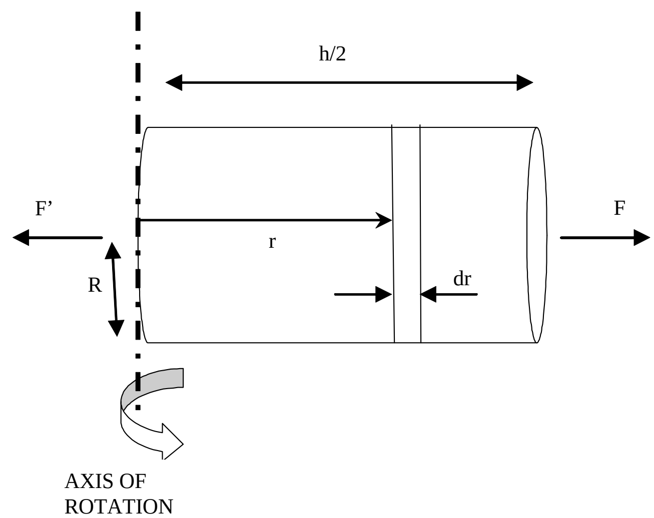


**Figure 1. Line diagram of snow sample under centrifugal loading.**

To determine the tensile strength of snow, stress in the sample needs to be increased. For this purpose, speed of rotation of snow sample is increased linearly by controlling the motor voltage, however, centrifugal force increases parabolically. Effects of variation in speed of rotation with time and density on input centrifugal force are plotted in Fig. 2.

# 3. EXPERIMENTATION

## 3.1 Sample Preparation

To conduct the experiments during winter at SASE, cylindrical samples were made by sieving

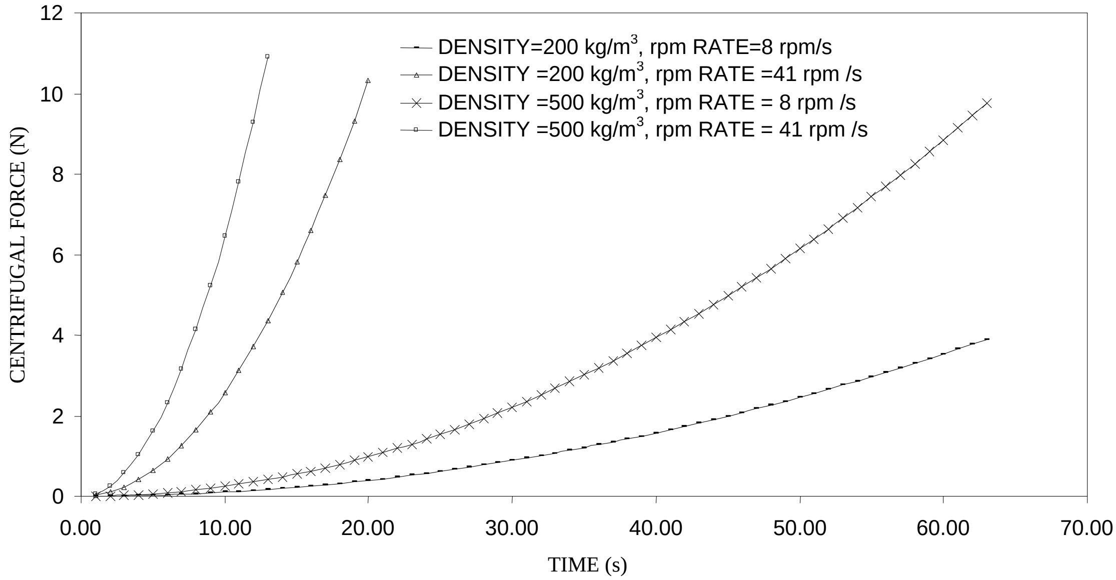


**Figure 2. Dependence of input centrifugal force on sample density and ramp rate of rotation.**

natural snow collected from Patseo (3800 m), and stored in cold rooms at –20 °C. Snow was sieved to get grain size of range 0.5 – 1.0 mm. These snow samples were kept at –20 °C for 4 days for sintering. All the tests were conducted under constant temperature conditions. However, variation in cold chamber temperature was observed to be in the range ± 0.5 to 1°C. All samples were kept at experimental temperature for 3–4 h before conducting the experiments.

### 3.2 Experimentation on Centrifugal Tensile Testing Machine

Once samples were ready, centrifugal tensile testing machine's cover was opened and after measuring the density of the snow, sample was pushed (using a plunger) into the sample holder (Fig. 3). The sample holder was 130 mm in length, while the sample was 150 mm; therefore extra snow was removed from the sample. Snow sample was held in place by U-shaped clips made of copper tubes. This sample-holder was rotated about an axis that was perpendicular to the axis of cylinder. Speed of rotation was slowly increased (using a software program) to increase the centrifugal force. When developed centrifugal force exceeded the strength of the snow sample, snow sample failed. As soon as failed snow sample came out of sampler and blocked the light coming to the photosensors, it was sensed by the 16 photosensors placed in a plane below the plane of rotation of snow sample. These sensors sent this information of failure to the software, which recorded the failure speed and stopped the machine.

## 4. RESULTS AND DISCUSSION

Effect of various parameters on tensile strength of snow has been discussed. In centrifugal experiments, applied force on snow sample is due to snow density and rotation speed of snow sampler. In these experiments, applied stress as well as stress rate increases as speed of rotation is increased linearly. Speed of rotation was increased at two different rates for two different sets of samples. Failure stress of snow with density for two different speed ramp rates has been plotted in Figs 4 and 5. In Fig. 4, a lot of scatter in data is visible, however it can be seen that there is an increase in failure stress with density. For density above 300 kg/m$^3$, failure stress of majority of snow samples is found to be above 20 KPa. For density below 300 kg/m$^3$ very few data points are available and the value of failure stress has been found to be below 20 KPa. As all the samples were kept at experimental temperature for 3-4 h only, hence the effect of experimental temperature on failure stress was

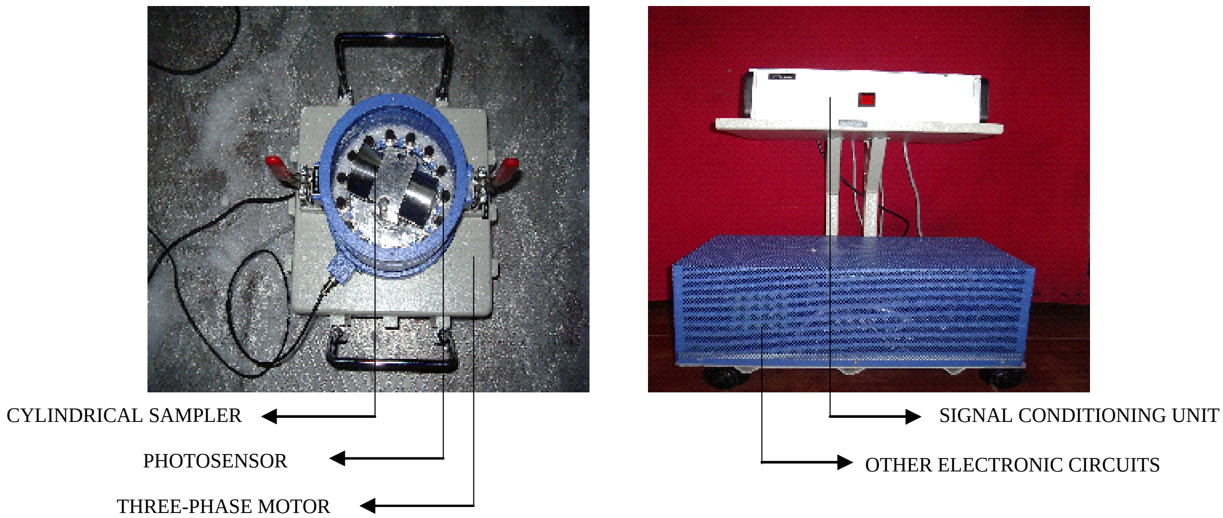


**Figure 3. Experimental setup for centrifugal experiments in cold laboratory.**

difficult to make out. Similar kind of results were found for speed ramp rate of 8 rpm/s as shown in Fig. 5. However, upper limit of failure stress for 41 rpm/s speed ramp rate data was found to be around 40 KPa, while for a lower speed ramp rate, this limit is little higher, up to 55 KPa.

A large scatter in data can be explained up to some extent by Griffith theory. As per Griffith theory for brittle failure, which is the mode of failure mostly in snow, stress concentration near flaws, is not relieved. The strength of any particular sample is determined not only by its bulk properties but also by the weakest flaws (microstructure parameters) which are included in the sample, hence large scatter in strength data is obtained. Hence, even at the same density of snow, there may be large variation in microstructure parameters, which can lead to large scatter in snow strength.

Present data of snow strength was compared with a porosity-dependent theoretical model for fully age-hardened snow given by Ballard and Feldt[1]. This relation gives the maximum strength (after $t = \infty$ sintering duration) of any snow sample with assumption that porosity of the sample does not change with time. Present experimental data was fitted with this relation and value of $\sigma_i$ was found

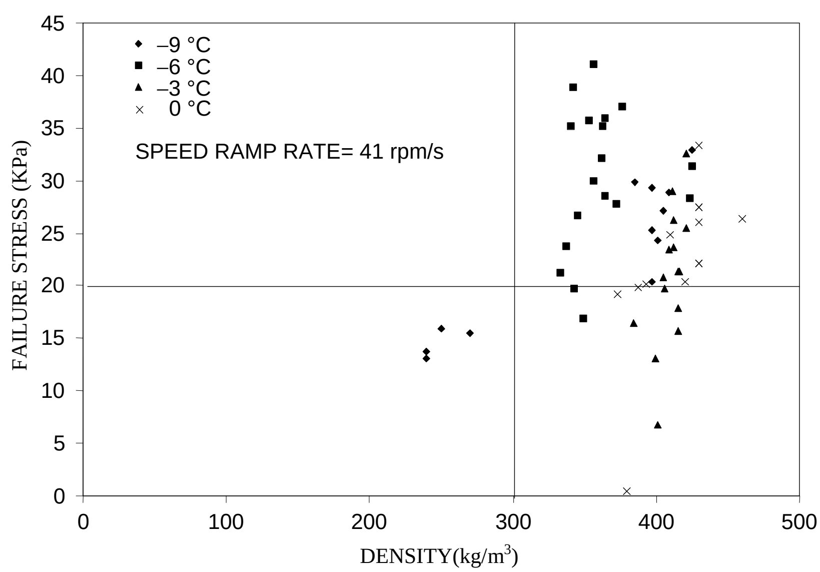


**Figure 4. Dependence of tensile strength of snow on temperature and density for speed ramp rate of 41 rpm/s.**

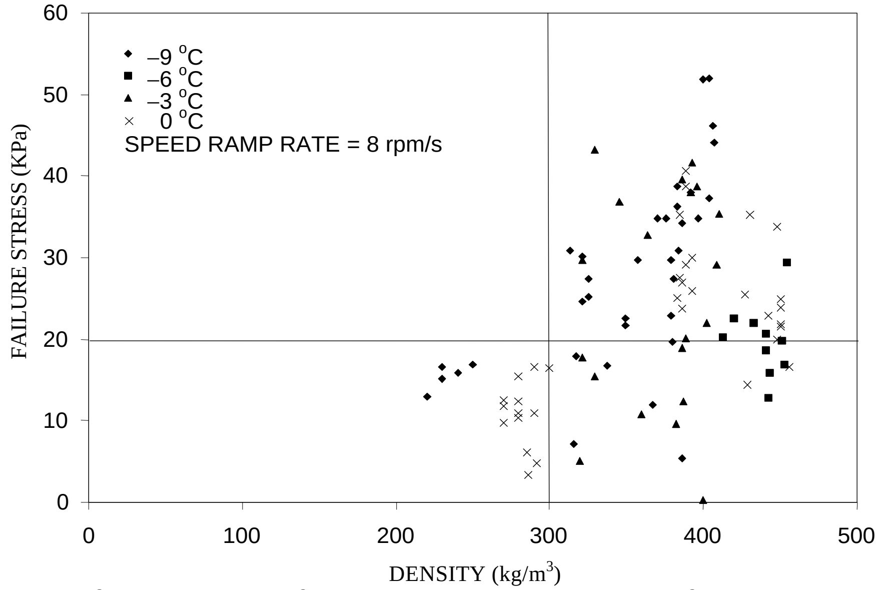


**Figure 5. Dependence of tensile strength of snow on temperature and density for speed ramp rate of 8 rpm/s.**

to be 0.312 MPa with a correlation coefficient of 0.94 (Fig. 6). Keeler and Weeks[3] also used this relation to compare with their centrifugal tensile strength data. Temperature range for their fine-grained dry natural snow samples was 0 to –10 °C with a density range 100 to 500 kg/m$^3$. However, the age of that particular snow layer was not mentioned. On fitting with this relation, it was found that the value of $\sigma_i$ is 27.3 kg/cm$^2$ or 2.68 MPa, which is a very high value in comparison to present experimental data (Fig. 6). The value of $\sigma_i$ from Keeler and Weeks[3] data is very close to the strength of ice. Ballard and Mcgaw[2] found the value of $\sigma_i$ to be 28.3 kg/cm$^2$ from the ring tensile test data of Butkovich[11] using the theoretical linear relationship. However, his relation is not very useful as it gives negative values of tensile strength of snow for density below 418 kg/m$^3$.

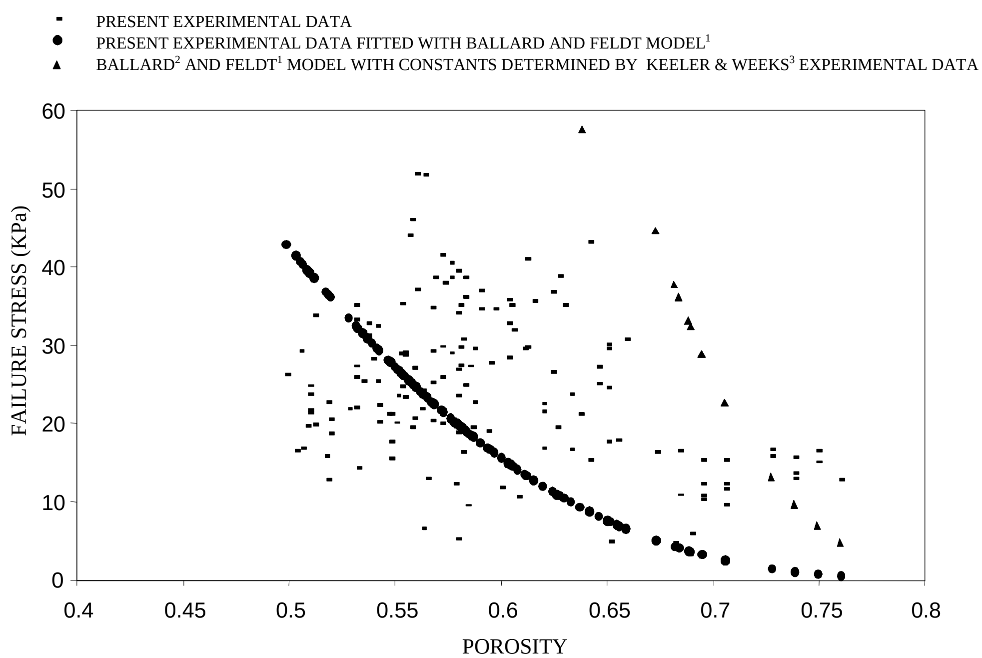


**Figure 6. Comparison of experimental data with few porosity-dependent models.**

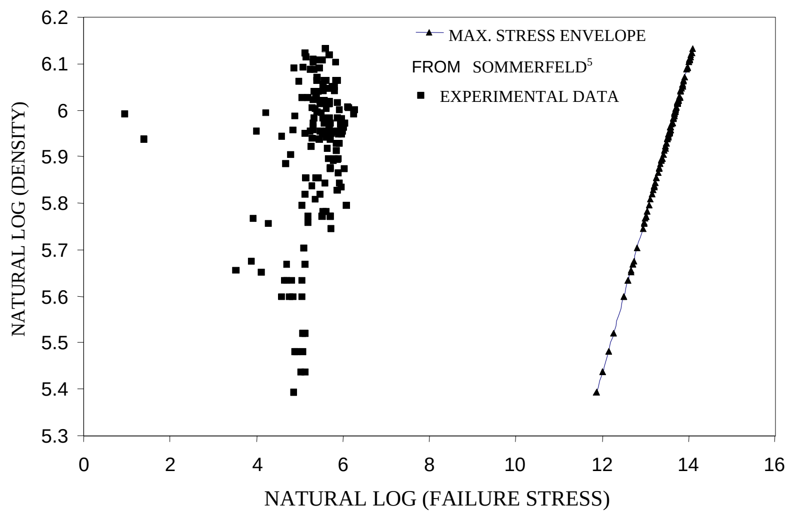


**Figure 7. Comparison of present experimental data with maximum strength envelope given by Sommerfeld[5].**

Though, it is difficult to predict the tensile strength of snow for a particular value of density but Sommerfeld[5] found a maximum strength envelope as a function of density given by the Eqn (4). Maximum strength envelope of Sommerfeld has been plotted in Fig. 7 against the present experimental data. From Fig.7, it can again be inferred that the failure stress of snow for a particular density obtained experimentally is well below what one gets from an equation given by Sommerfeld[5] .

A probability distribution of failure stress of processed snow has been determined using present experimental data. The whole data has been divided into five density ranges from 200–460 kg/m$^3$. Failure stress is also divided into six subgroups in each density range starting from 0–60 KPa with an interval of 10 KPa. Total number of data points available in each density group has been shown in brackets with legends in Fig. 8. For each density group, probability of failure of a snow sample in a particular stress range has been determined and plotted in Fig. 8. However, since number of data points is not quite large, it is therefore sufficient to infer from Fig. 8 that below density 300 kg/m$^3$, most of snow samples fail below 20 KPa and maximum probability of failure lies in the stress range 10–20 KPa. Contrary to this, for densities above 300 kg/m$^3$, most of snow samples fail above 20 KPa and maximum failure probability lies in the stress range 20–30 KPa.

## 5. CONCLUSIONS

Tensile strength of age-hardened snow (allowed to sinter at –20 °C for 4 days) found to depend on density with a lot of scatter in data. However, approximate value of tensile strength of snow can be found using theoretical porosity dependence relations given by Ballard and Feldt[1], using appropriate constants depending on sintering conditions. For present snow type, below density 300 kg/m$^3$ maximum probability of failure lies in the stress range 10–20 KPa and for densities above 300 kg/m$^3$, maximum failure probability lies in the stress range 20–30 KPa. Using a large data set under different snow conditions, a better probabilistic model can be developed to predict the tensile strength of snow. These models can further be used to predict the tensile strength of natural snow in field.

To determine the tensile strength of snow accurately, study of microstructural parameters of samples is also important. But this is not a practical approach to predict the snow strength as far as avalanche forecasting is concerned. By conducting large number of *in situ* experiments and by finding probability distribution of a particular snow layer, most probable failure range of failure stress can be found. Further, effect of sintering temperature and duration on tensile strength of snow is an important study that needs to be done.

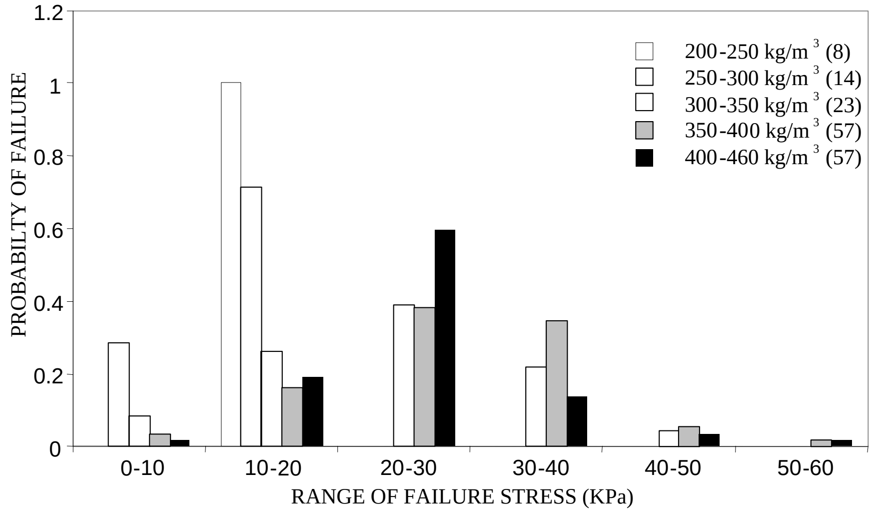


**Figure 8. Probability distribution of tensile strength of snow in different density ranges.**

## ACKNOWLEDGEMENTS

Authors are thankful to Sarvashri D.N. Sethi and P.K. Stayawali for providing constant encouragement to publish this work. Authors thankfully acknowledge the technical assistance provided by the Cold Laboratory staff.

## REFERENCES

1. Ballard, G.E.H. & Feldt E.D. A theoretical consideration of the strength of snow. *Journal of Glaciology*, 1965, **6**(43), 159-70.
2. Ballard, G.E.H. & McGaw, R.W. A theory of snow failure. US Cold Regions Research and Engineering Laboratory, 1965. Research Report 137.
3. Keeler, C.M. & Weeks, W.F. Investigations into the mechanical properties of alpine snowpacks. *Journal of Glaciology*, 1968, **7**(50), 253-71.
4. Keeler, C.M. The growth of bonds and the increase of mechanical strength in a dry seasonal snowpack. *Journal of Glaciology*, 1969, **8**(54), 441-50.
5. Sommerfeld, R.A. The relationship between density and tensile strength in snow. *Journal of Glaciology*, 1971, **10**(60), 357-62.
6. Gubler, H. An alternate statistical interpretation of the strength of snow. *Journal of Glaciology*, 1978, **20**(83), 343-57.
7. McClung, D.M. *In-Situ* Estimates of the tensile strength of snow using large sample sizes. *Journal of Glaciology*, 1979, **22**(87), 321-29.
8. Sommerfeld, R.A. Statistical models of snow strength. *Journal of Glaciology*, 1980, **26**(94), 217-23.
9. Sommerfeld, R.A. & King, R.M. A recommendation for the application of the Roch index for slab avalanche release. *Journal of Glaciology*, 1979, **22**(87), 402-04.
10. Gow, A.J. & Ramseier, R.O. Age hardening of snow at South Pole. *Journal of Glaciology*, 1963, **4**(35), 521-36.
11. Butkovich, T.R. Strength studies of high density snow, 1956. SIPRE Research Report No. 18.

## Contributors

**Mr Agraj Upadhyay** obtained BSc (Mech Engg) from Dayalbagh Educational Institute, Agra, and MTech (Cold Region Sci and Engg) from GB Pant University of Agriculture and Technology, Pantnagar. He joined DRDO at the Snow & Avalanche Study Establishment (SASE), Manali in 2001. His areas of research are snow mechanics studies and stress analysis of snow pack using finite element technique.

**Mr S.K. Joshi** obtained his MSc (Physics) from Kumaon University, Nainital, and MTech (Cold Region Sci and Engg) from GB Pant University of Agriculture and Technology, Pantnagar. Currently, he is doing PhD from IIT Delhi, New Delhi (Applied Mechanics Dept). His topic of research is finite element simulation of avalanche initiation mechanism. His areas of interest are: Snow microstructure, snow mechanics, and fractography. Currently, he is working as Dy Director in Directorate of Missiles, New Delhi.

**Mr Chaman Chandel** obtained his BTech (Mech Engg) and MTech (Cold Region Sci & Engg) from GB Pant University of Agriculture & Technology, Pantnagar. He joined DRDO at the SASE, Manali in 2002. His area of research is snow mechanics studies and stress analysis of snowpack.