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Flow of a Thermoviscous Fluid through an Annular Tube with Constriction

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ABSTRACT

The flow of thermoviscous fluid within the annulus of circular cylinders with a local constriction at the outer wall is investigated analytically, assuming that the constriction is non-symmetric wrt the radial distance. Analytical solutions for velocity and temperature fields have been obtained. The effect of the shape parameter of the constriction on velocity, temperature, Nusselt number, and flow rate are illustrated graphically and discussed.

Keywords: Thermoviscous fluid, constriction, annular region, Nusselt number, flow characteristics, flow rate, heat transfer

NOMENCLATURE

a	Radius of the inner wall
b	Thermal gradient vector
b_o	Radius of the outer wall
c	Specific heat
C_1, C_2	Nondimensional pressure and temperature gradients respectively.
d	Deformation rate tensor
$b(x)$	Radius of the tube in the constricted region
h	Nondimensional form of the radius of the tube in the constricted region
$h*$	Heat flux bivector
I_o, I_1	Modified Bessel functions of first kind
L_o	Length of the constricted region
L_1	Location of the constriction
m_1	Thermoviscous parameter
n (≥ 2)	Shape parameter of the constriction
Nu	Nusselt number
Q	Flow rate
r	Radial distance
t	Stress tensor
T_a	Temperature at the inner wall
T_b	Temperature at the outer wall
T	Temperature in the nondimensional form
T_∞	Mean mixed temperature
v_k	K^{th} component of velocity
ω	Velocity distribution
X_o	Point at which maximum constriction located
ρ	Density

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θ	Temperature field
ε	Maximum height of the constriction
ϵ_{ijk}	Permutation symbol
α_i, β_i	Scalar polynomials in *trd*, *trd*^{2}, *trb*^{2}
α_{irst}, β_{irst}	Coefficients (independent of temperature field)

1. INTRODUCTION

Curved pipe/annular configurations are of immense practical importance in almost all piping systems, the human cardiovascular system, and in several engineering devices such as heat and mass exchanges, chemical reactors, chromatography columns, and other processing equipment. Owing to the wide range of applications, the interest in the study of flow characteristics in these configurations has grown enormously during the last decades. Jakob and Rees[1] made a theoretical investigation of the problem of heat transfer through annulus space when the fluid flow is laminar and there is a uniform heating either from outside, from inside or from both. Reynolds[2], *et al.*, McCuen[3,4], *et al.*, Leung[5] *et al.* of Stanford University have made numerous theoretical and experimental studies of both laminar and turbulent heat transfer in annuli taking various types of wall temperature distributions. Reynolds[6], *et al.* have also included a bibliography of the related work on this aspect. Shigechi[7], *et al.* have made an analysis on laminar flow and heat transfer in concentric annuli with moving core to obtain the effect of relative velocity on friction factor and Nusselt number. Datta[8], *et al.* analysed the flow and heat transfer of a dusty fluid (Saffman model) within the annulus of circular cylinders under a pulsatile pressure gradient. But all the above investigations are in annular region without constriction.

Recently, Dash[9], *et al.* studied the flow of an incompressible Newtonian fluid in a curved annulus with a local constriction at the out wall. The mathematical analysis was based on a double series perturbation method for small curvature and mild constriction. Jayaraman[10], *et al.* investigated numerically the flow in curved annulus with local constriction at the outer wall. Only limited efforts have been made to hydrodynamic behaviour and none to thermal characteristics in flows through annular region with abrupt enlargement and contractions. The knowledge is nevertheless of great importance to the design of piping systems, heat exchange devices, and artificial cardiovascular systems. In this paper, the problem of thermoviscous fluid flow between two core axial circular cylinders with a local constriction at the outer wall has been presented. Closed form solutions have been obtained for axial velocity, temperature field. The influences of thermoviscous parameter (m_1) and shape parameter of the non-symmetric constriction (n) on Nusselt number and flow rate are shown graphically.

2. FORMULATION

Consider the laminar motion of a thermoviscous fluid flow past a mild constriction formed in a cylindrical tube in its annular region (Fig. 1). The flow is steady and of second-order incompressible thermoviscous fluid contained between two cylinders under a constant pressure gradient acting along its length. Let a and b_o be the radii of the inner and outer walls and the let these be kept at temperature T_a and T_b respectively. The density and the viscosity of the fluid are assumed to be constant. The profile of the non-symmetric constriction is given by Haldar[11] as

$$\frac{b(x)}{b_0} = 1 - A[L_0^{n-1}(x-L_1) - (x-L_1)^n] \quad L_1 < x < L_1 + L_0 \;=\; 1 \quad \text{otherwise} \qquad (1)$$

$$\text{where} \quad A = \left(\frac{\varepsilon}{b_0 L_0^{\,n}}\right)\left(\frac{n^{\frac{n}{n-1}}}{n-1}\right) \qquad (2)$$

Here, ε denotes the maximum height of the constriction located at $x_* = L_1 + \dfrac{L_o}{n^{1/(n-1)}}$ (3)

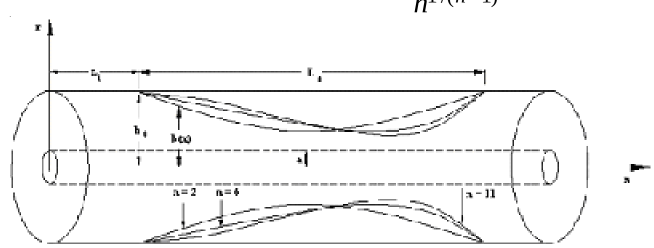

Figure 1. Geometry of constriction.

As proposed by Koh and Eringen[12], the stress tensor and heat flux bivector for the second-order thermoviscous fluids are given by

$$t = \alpha_1 I + \alpha_3 d + \alpha_5 d^2 + \alpha_6 b^2 + \alpha_8 (db - bd) \tag{4}$$

$$h^* = \beta_1 b + \beta_3 (bd - db) \tag{5}$$

$$2d_{k,m} = v_{k,m} + v_{m,k}, \tag{6}$$

$$b_{ij} = \in_{ijk} \theta_{,k} \tag{7}$$

$$\alpha_1 = \alpha_{1000} I + \alpha_{1010}\, trd + \alpha_{1020}\, trd^2 + \alpha^*_{1020} (trd)^2 + \alpha_{1002}\, trd^2 \tag{8}$$

$$\alpha_3 = \alpha_{3010} I + \alpha_{3020}\, trd \tag{9}$$

$$\alpha_5 = \alpha_{5020},\ \alpha_6 = \alpha_{6002},\ \alpha_8 = \alpha_{8011}, \tag{10}$$

$$\beta_1 = \beta_{1001} + \beta_{1011}\, trd,\ \beta_3 = \beta_{3011} \tag{11}$$

Under hypothesis of slow motion, the basic equations have been simplified by neglecting the terms, which are nonlinear in the derivatives of the velocity and temperature. In the absence of external forces, heat sources within the flow region and under the assumption that the motion is slow[13-15], the equation of momentum and energy reduces to

$$0 = -\frac{\partial p}{\partial x} + \frac{\alpha_3}{2}\nabla^2\omega - \alpha_6 \frac{\partial\theta}{\partial x}\nabla^2\theta \tag{12}$$

$$\rho c \omega \frac{\partial\theta}{\partial x} = -\beta_1 \nabla^2\theta + \frac{\beta_3}{2}\frac{\partial\theta}{\partial x}\nabla^2\omega \tag{13}$$

together with the boundary conditions

$$\omega(a) = 0;\ \omega[b(x)] = 0 \tag{14}$$

$$\theta(a) = T_a;\ \theta[b(x)] = T_b \tag{15}$$

In terms of the nondimensional quantities defined by

$$\left.\begin{aligned} & W = \frac{\alpha_3}{2\rho a}\omega,\quad R = ar,\quad \frac{\theta - T_a}{T_b - T_a} = T, \\ & \frac{\partial p}{\partial x} = \frac{\alpha_3^2}{4\rho a^3} C_1,\quad \frac{\partial\theta}{\partial x} = \frac{T_b - T_a}{a} C_2 \end{aligned}\right\} \tag{16}$$

in which r is the radial distance, C_1 and C_2 are nondimensional pressure and temperature gradients respectively, the Eqns (12) and (13) reduce to

$$\nabla^2\omega - m_1\omega = -m_2 \tag{17}$$

$$\nabla^2\omega - b_2\nabla^2 T = b_3\omega \tag{18}$$

together with boundary conditions

$$\omega(1) = 0;\quad \omega(h) = 0 \tag{19}$$

$$T(1) = 0;\quad T(h) = 1 \tag{20}$$

in which

$$h = \frac{b(x)}{b_0}\qquad b_1 = \frac{4\rho\alpha_6 (T_b - T_a)^2 c^2}{\alpha_3^2};$$

$$\left.\begin{aligned} & b_2 = \frac{4\rho a^2 \beta_1}{\alpha_3 \beta_3 c_2}\quad b_3 = \frac{2\rho c a^2}{\beta_3} \\ & m_1 = \frac{b_1 b_3}{b_1 - b_2};\quad m_2 = \frac{b_2 c_1}{b_1 - b_2}; \end{aligned}\right\} \tag{21}$$

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)$$

From Eqns (17) and (18), one obtains the velocity and temperature distribution, subject to the relevant boundary conditions

$$\omega(r) = \frac{m_2}{m_1}\left(1 + \frac{\left[k_0(\sqrt{m_1})I_0(\sqrt{m_1}r) - I_0(\sqrt{m_1})k_0(\sqrt{m_1}r) + k_0(\sqrt{m_1}r)I_0(\sqrt{m_1}h) - I_0(\sqrt{m_1}r)k_0(\sqrt{m_1}h)\right]}{k_0(\sqrt{m_1}h)I_0(\sqrt{m_1}) - I_0(\sqrt{m_1}h)k_0(\sqrt{m_1})}\right) \tag{22}$$

$$T(r) = \frac{m_2(m_1 - b_3)}{m_1^2 b_2}\left(1 + \frac{\left[k_0(\sqrt{m_1})I_0(\sqrt{m_1}r) - I_0(\sqrt{m_1})k_0(\sqrt{m_1}r) + k_0(\sqrt{m_1}r)I_0(\sqrt{m_1}h) - I_0(\sqrt{m_1}r)k_0(\sqrt{m_1}h)\right]}{k_0(\sqrt{m_1}h)I_0(\sqrt{m_1}) - I_0(\sqrt{m_1}h)k_0(\sqrt{m_1})}\right)$$

$$+\frac{m_2 b_3}{4 m_1 b_2}(r^2 - 1) + \frac{\log r}{\log h}\left(1 + \frac{m_2 b_3}{4 m_1 b_2}(1 - h^2)\right) \tag{23}$$

The heat transfer coefficient, characterised by Nusselt number (Nu) on the tube boundary is

$$Nu = \left(\frac{m_2(m_1 - b_3)}{m_1\sqrt{m_1}b_2} \right) \left(\frac{[k_1(\sqrt{m_1}h)[I_0(\sqrt{m_1}) - I_0(\sqrt{m_1}h)] + I_1(\sqrt{m_1}h)[k_0(\sqrt{m_1}) - k_0(\sqrt{m_1}h)]}{k_0(\sqrt{m_1}h)I_0(\sqrt{m_1}) - I_0(\sqrt{m_1}h)k_0(\sqrt{m_1})} \right) + \frac{m_2 b_3}{2m_1 b_2}h + \frac{1}{h\log h}\left(1 + \frac{m_2 b_3}{4m_1 b_2}(1-h^2)\right) \quad (24)$$

Flow rate is defined by

$$Q = 2\pi \int_a^{b(x)} \omega \, r dr \quad (25)$$

Incorporating Eqn (22) in Eqn (25) and on rearrangement

$$Q = \left(\frac{2\pi m_2}{m_1}\right) \times \left[\frac{1}{k_0(\sqrt{m_1}h)I_0(\sqrt{m_1}) - I_0(\sqrt{m_1}h)k_0(\sqrt{m_1})}\right] \times \left[\frac{h}{\sqrt{m_1}} \left[\begin{array}{l} I_1(\sqrt{m_1}h)\left(k_0(\sqrt{m_1}) - k_0(\sqrt{m_1}h)\right) + \\ k_1(\sqrt{m_1}h)\left(I_0(\sqrt{m_1}h) - I_0(\sqrt{m_1})\right) \end{array} \right] - \frac{1-h^2}{2}[k_0(\sqrt{m_1}h)I_0(\sqrt{m_1}) - I_0(\sqrt{m_1}h)k_0(\sqrt{m_1})] - \frac{1}{\sqrt{m_1}} \left[\begin{array}{l} I_1(\sqrt{m_1})\left(k_0(\sqrt{m_1}) - k_0(\sqrt{m_1}h)\right) \\ + k_1(\sqrt{m_1})\left(I_0(\sqrt{m_1}h) - I_0(\sqrt{m_1})\right) \end{array} \right] \right] \quad (26)$$

Mean mixed temperature can be obtained from

$$T_* = \frac{2\pi \int_a^{b(x)} T\omega \, r dr}{2\pi \int_a^{b(x)} \omega \, r dr} \quad (27)$$

3. DISCUSSIONS

The problem of flow of a thermoviscous fluid through an annular tube with constriction is considered. To get the physical insight of the problem, velocity, temperature field, flow rate, Nusselt number have been discussed by assigning numerical values to various parameters like thermoviscous parameter (m_1) and shape parameter (n) of the constriction which characterise the flow phenomena. The influences of these parameters on the velocity, temperature, flow rate and Nusselt number have been studied and are presented graphically.

The profiles of the velocity and temperature field are illustrated in Figs 2 and 3 for different values of m_1. The velocity profiles become more flat as the thermoviscous parameter decreases. It can be noticed that there is an increase in the velocity of the fluid with the increase of n. The decrease of the temperature with increase can be noticed from Fig. 3. Further there is a slight increase in temperature profile (for a fixed m_1) with the increase of n.

Effect of n on the flow rate is shown in Fig. 4, flow rate decrease with the increase of n. From Fig. 4, it can be noticed that the mass flow decreases with the increase of thermoviscous parameter. The heat-transfer coefficient, characterised by Nusselt number (Nu) on the boundary is depicted in Fig. 5. It is observed that Nusselt number increases with the increase of n. From Fig. 5 it can be noticed that the Nusselt number decreases with the increase of thermoviscous parameter.

To make a comparative study, in Figs 6 and 7 the variation of the mass flow and Nusselt number with for the case of constricted tube (without inner cylinder). It is observed that mass flow decreases with increases of n [Fig. 6]. Further, mass flow decreases with increase of m_1 as noticed in the annular tube with constriction. Figure 7 shows that the Nusselt number increases with the increase of m_1 in contrast to the case of annular tube with constriction.

The results of the present study will hopefully enable a better understanding of the clinical applications

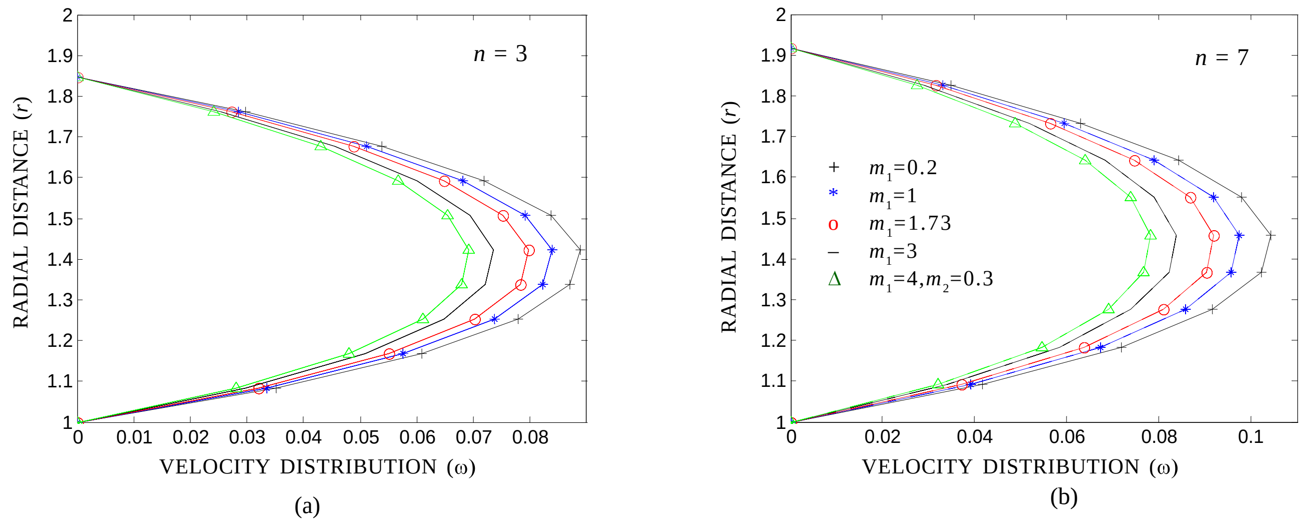

Figure 2. Velocity distribution.

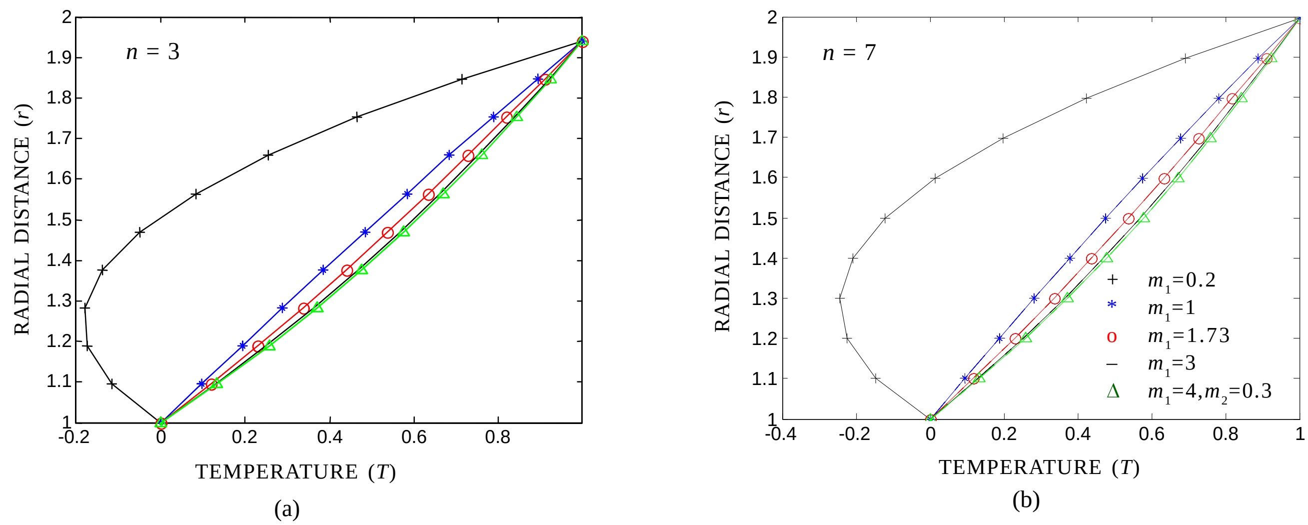

Figure 3. Temperature distribution.

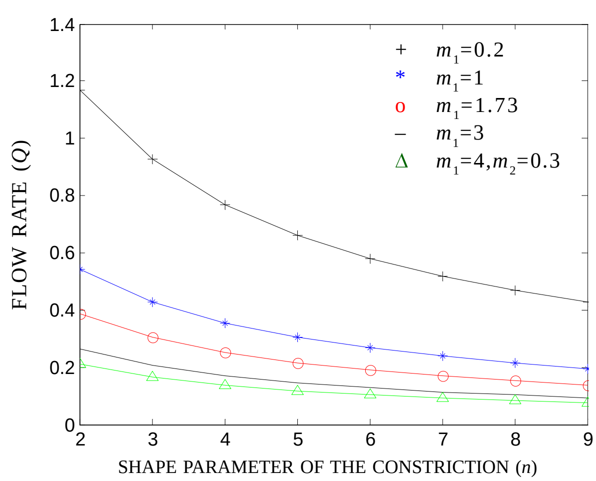

Figure 4. Flow rate versus shape parameter of the constriction.

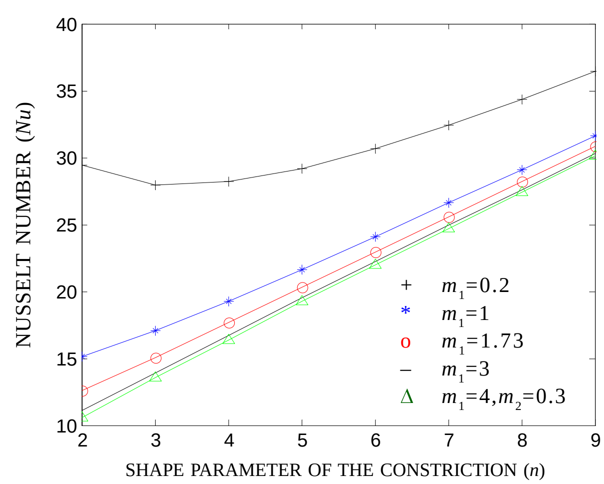

Figure 5. Nusselt number versus shape parameter of the constriction.

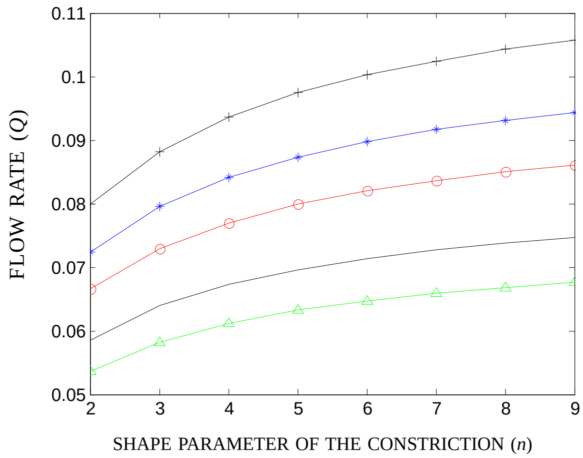

Figure 6. Flow rate versus shape parameter of the constriction.

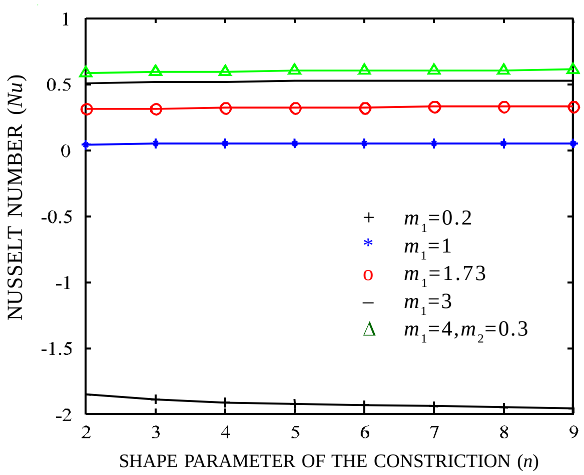

Figure 7. Nusselt number versus shape parameter of the constriction.

such as endoscope problem and blood flow in a catheterised artery.

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