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Conceptual Lanchester-type Decapitation Warfare Modelling

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ABSTRACT

Decapitation operation has existed for a long time in military history; however, it was not until March 2003 'decapitation attack' became a well known term in the mass media. This paper is based on the connotation of decapitation based on historical study and refines the term into military strategic concept of decapitation strategy. Ideas derived from detailed studies on Lanchester-type combat models are used to describe the effectiveness of conventional regular forces under decapitation warfare, which includes asymmetric, nonlinear, stand-off and special operation forces (SOF) operations. A conceptual model is presented to describe the effects of the decapitation strategy on the regular battlefield. With extensive coverage of operational factors such as robustness of forces, time difference between combats, undermining effects, breakpoints, attrition rates, total force level and force allocation, the model is suitable to analyse complex scenario with different types of military operations consisting of decapitation strategy. An illustrative example is provided to demonstrate the application of the model. The conceptual model is built based on hypotheses, assumptions, and criteria. In the absence of historical data, no data analysis and parameter estimation are involved.

Keywords: Decapitation, Lanchester, square law, combat fire effectiveness, decapitation attack, decapitation strategy, military strategy, decapitation warfare combat models, military operations

NOMENCLATURE

$A_b(A_r)$	Blue (Red) regular army force strength
$D_b(D_r)$	Blue (Red) garrison (decapitation) force strength
$K_{Ab}(K_{Ar})$	Blue (Red) regular army attrition-rate coefficient
$K_{Db}(K_{Dr})$	Blue (Red) garrison (decapitation) force attrition-rate coefficient
$m(n)$	Fraction of Blue (Red) regular army which is affected by the undermining operation and decapitation combat
$p(q)$	Undermining effect factor of Blue (Red) regular army
$f_{Db} = \frac{D_b}{D_{b0}} (f_{Dr} = \frac{D_r}{D_{r0}})$	Surviving portion of Blue garrison (Red decapitation) force and subscript '0' denotes the initial condition.

DEFINITIONS

Decapitation Strategy: It is the strategy adopted by the attacker with the intention to weaken or degrade the combat potential of the defender by means of undermining operation and ground combat.

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Undermining Operation: It is the military operation other than ground combat, including kidnapping or assassination of key leaders (the so called Political Decapitation), sabotage, precision strike, air bombardment, missile attack, etc. with the purpose to undermine the military ability of the target country.

Political Decapitation: It is the annihilation by physical elimination of part or all of the key governmental players of a country[1].

Ground Combat: It consists of decapitation and regular combat; the purpose of the former is to seize the capital of the target country and through the seizure to degrade the combat effectiveness of the defender's regular forces; the latter is to defeat the defender's entire regular army.

Decapitation Combat: It is the combat in which the attacker's decapitation troops fight the defender's capital guarding garrison forces.

Regular Combat: It is the combat in which the attacker's regular forces clamp and fight the defender's regular army.

1. INTRODUCTION

In 2003, the US-led coalition forces defeated Iraq with superior military technology. During the first phase of the operation, decapitation attack became a highly visible military term in the mass media. The attempt of the operation phase may be told from the speech of US President G.W. Bush given on the night of March 19, 2003: "…, coalition forces have begun striking selected targets of military importance to undermine Saddam Hussein's ability to wage war." Decapitation strategy is different from 'confronts the tough with toughness' way of war fighting; but rather it is a combination of precision blitzkrieg, nonlinear, asymmetric, stand-off and air-land-sea-space operations.

During the decapitation operation on Iraq, the coalition forces with overwhelming technology superiority not only widen the gap between allied and Iraqi forces but also minimise allied losses[2]. Moreover, from historical military observations it is noted that assassination attempt is part of the decapitation operation[3], even the core of it[1].

Decapitation operation itself is not a new type of warfare, according to Serge Walder[1] and Hyder's[4] historical review; there have been eight political decapitations operations in addition to 9/11 attack and wars against Afghanistan and Iraq since 1901, as shown in Table 1. Walder[1] also argued that political decapitation is the most cost-effective way to languish, remove, and reshape a regime.

Similarly, decapitation strategy can be used in other scenario. Across the pacific, some military analysts warn of an eminent crisis across the Taiwan Strait[5,6]. They assume that PRC (People's Republic of China) might adopt Decapitation Strategy to disable command and control system, to sabotage key military facilities and sites in order to capture Taipei -- ROC's (Republic of China) Capital and occupy the country.

Successful implementations of decapitation strategy will soft-up[7] the military strength of the adversary. From the military point of view, Decapitation Strategy may consist of two operations, undermining and ground combat, which could be in sequence or overlapped. To minimize the casualties and costs, the undermining operation of the offensive is to degrade the combat effectiveness of the defender, as shown in Fig.1. The ground combat is divided into decapitation combat and regular combat.

The model focuses on the regular combat and attempts to illustrate the effects of undermining operation and decapitation combat on the regular forces' combat effectiveness.

Figure 2 shows the systematic diagram of the model; 'Effect 1' is the effect of undermining operation on decapitation combat which is integrated into the 'Effect 2', the effect of decapitation combat on the regular combat.

The purpose of the research is to provide a mathematical tool to aid military conceptual analysis and evaluation. The process of model building is based on given hypotheses, assumptions, model

Table 1. Decapitation operations in history

Time	Target	Attacker	Defender	Operation type
1901#	Emilio Aguinaldo	US	Philippines	Unconventional war
1916#	Francisco Pancho villa	US	Mexico	Multiple missions
1943#	Admiral Isoroku Yamamoto	US	Japan	Single mission in world war II
1956.10*	Ben Bella	French	Algerian	Single mission
1973.09*	Allende	US (CIA)	Chile	Coup
1979.12*	Amin	Soviet Russia	Afghanistan	**War**
1989.12*#	Noriega	US	Panama	**War** Operation Just Cause
1993#	Pablo Escobar	US and Colombia	Drug Dealer	Multiple missions by Colombia authorities
2001.09*	World Trade Center	Al Qaeda	US	Terrorist attack
2001.10*	Osama bin Laden	US	Afghanistan	**War**
2003.03*	Saddam Hussein	US Aillied	Iraq	**War**

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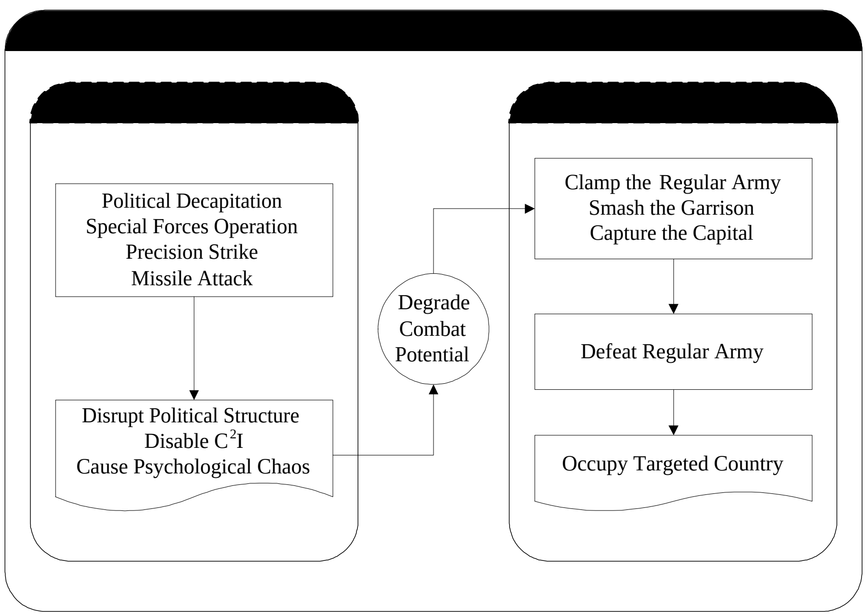


Figure 1. Decapitation strategy.

criteria and scenario. There is no data analysis and parameter estimation involved by now.

2. LANCHESTER-TYPE COMBAT MODELS

It has been 90 years since Lanchesterian combat model was first developed. The model is widely used not only in military field but also in electoral and commercial fields[8]. Lanchester-type models are divided into two branches which are deterministic and stochastic. The former uses the mean to model competition phenomenon; the latter focuses on the random characteristic of competition, and these both complement each other[9]. Each branch can be

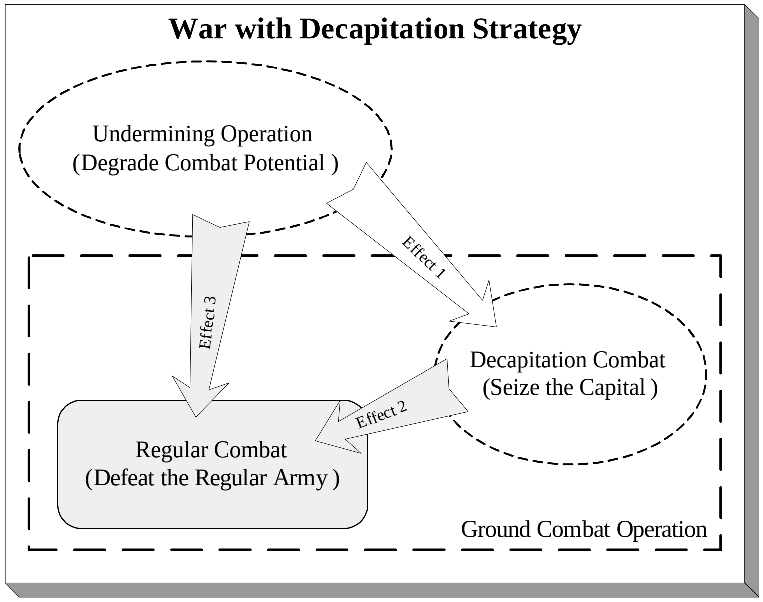


Figure 2. Systematic diagram of decapitation strategy.

divided into two types, homogeneous and heterogeneous forces, based on the levels of resolution. Some of the deterministic-homogeneous Lanchester-type combat models are listed in Table 2.

Except well known laws based on different attrition processes, such as square law, linear law, mixed law, logarithmic law, etc.[10], tremendous efforts have been made to bring the combat model more realistic and more applicable. Among all the efforts made, some of these focus on combat effectiveness; for example, Helmbold[11] modified Lanchester's equations with fire-effectiveness-modification factor and unified square, linear and logarithmic laws as shown in Eqn (1).

$$\frac{dx}{dt} = -a(t) \times y \times h\left(\frac{x}{y}\right)$$
$$\frac{dy}{dt} = -b(t) \times x \times h\left(\frac{y}{x}\right) \quad (1)$$

Taylor[10] integrated survivors' effectiveness with the function of breakpoint and losses, along with the existence of permanently ineffective fraction of force, $f_{I,X}$, $f_{I,Y}$ into the form of Eqn (2).

$$\frac{dx}{dt} = -a \times \left(1 - f_{I,Y}\right) \times \left[1 - \left(\frac{y_0 - y}{y_0 - y_{BP}}\right)^v\right] \times y$$
$$\frac{dy}{dt} = -b \times \left(1 - f_{I,X}\right) \times \left[1 - \left(\frac{x_0 - x}{x_0 - x_{BP}}\right)^u\right] \times x \quad (2)$$

in which, u and v are force casualty-degradation parameters.

Equations (1) and (2) show that ineffectiveness does exist and it is caused by the inefficiencies of force scale, attrition or permanently ineffective portion of force. This paper attempts to extend the discussions on effectiveness to decapitation strategy and build conceptual mathematical model to describe the degrading course of fire effectiveness on a regular battlefield.

3. MODELLING BASES

3.1 Hypotheses

Hypotheses 1: The undermining operation has effects on both sides regarding regular combat.

Hypotheses 2: The course of decapitation combat has influence on both sides regarding regular combat.

Table 2. Some deterministic-homogeneous Lanchester-type attrition models

Type	Defender	Attacker	Remarks
Aimed fire[10,15]	$\frac{dx}{dt}=-ay$	$\frac{dy}{dt}=-bx$	$b(x_0^2-x^2)=a(y_0^2-y^2)$
Area fire[10,15]	$\frac{dx}{dt}=-axy$	$\frac{dy}{dt}=-bxy$	$b(x_0-x)=a(y_0-y)$
Ambush[10]	$\frac{dx}{dt}=-ay$	$\frac{dy}{dt}=-bxy$	$\frac{b}{2}(x_0^2-x^2)=a(y_0-y)$
Operational losses[10]	$\frac{dx}{dt}=-ax$	$\frac{dy}{dt}=-by$	$b\ln\frac{x_0}{x}=a\ln\frac{y_0}{y}$ Early stage of small unit engagement
Fire support[10]	$\frac{dx}{dt}=-ay-\lambda x$	$\frac{dy}{dt}=-bx-\mu y$	supporting fire not subject of attrition
Fire effectiveness[10,15]	$\frac{dx}{dt}=-a(t)\times y\times h\left(\frac{x}{y}\right)$	$\frac{dy}{dt}=-b(t)\times x\times h\left(\frac{y}{x}\right)$	h: inefficiencies of scale[9]
Historical attrition law[12,13,14]	$\frac{dx}{dt}=-e^{C}x^{D}y^{G}$	$\frac{dx}{dt}=-e^{F}x^{G}y^{D}$	D, G: universal constants C, F: varied coefficients
Attrition and break point[10,15]	$\frac{dx}{dt}=-a(1-f_I^Y)(1-\frac{y_0-y}{y_0-y_{BP}})^{\nu}y$	$\frac{dy}{dt}=-b(1-f_I^X)(1-\frac{x_0-x}{x_0-x_{BP}})^{\mu}x$	f_I , x_{BP}, y_{BP} : the inherent ineffective fraction and break-point of x, y
Reinforcement[15]	$\frac{dx}{dt}=-ay+X$	$\frac{dy}{dt}=-bx+Y$	X, Y: reinforcements rate
	$\frac{dx}{dt}=-ay-cx+X$	$\frac{dy}{dt}=-bx-dy+Y$	
Range dependent kill-rate[10]	$\frac{dx}{dt}=-a_0(1-\frac{r}{r_\alpha})^{\nu}y$	$\frac{dy}{dt}=-b_0(1-\frac{r}{r_\beta})^{\mu}x$	r_β, r_α; ν, μ : maximum effective range of weapons; range dependence parameters r_0, v : opening range of battle, attack speed
Attrition rate coefficients and range (Mobile attack)[10,15]	$\frac{dx}{dt}=-k_a(t+\frac{r_\alpha-r_0}{\mathrm{v}})^{\nu}y$	$\frac{dy}{dt}=-k_b(t+\frac{r_\alpha-r_0}{\mathrm{v}}+\frac{r_\beta-r_\alpha}{\mathrm{v}})^{\mu}x$	
Vulnerable area[10,14] Small arm	$\frac{dx}{dt}=-\frac{\nu_Y a_{V_X}}{A_X}xy$	$\frac{dy}{dt}=-\frac{\nu_X a_{V_Y}}{A_Y}xy$	ν, a_V, a_L, A : fire-rate, vulnerable area, lethal area and presented area
Large lethality	$\frac{dx}{dt}=-\frac{\nu_Y a_{L_Y}}{A_X}xy$	$\frac{dy}{dt}=-\frac{\nu_X a_{L_X}}{A_Y}xy$	
Guerrilla warfare[10,15]	$\frac{dx}{dt}=-a(t)y$	$\frac{dy}{dt}=-[b_1(1-e^{-\gamma t})+b_2ye^{-\gamma t}]x$	γ : shift rate from area fire to aimed fire

i. Force-on-force attrition modelling, 1980[10]
ii. Warfare modeling, 1995[13]
iii. Military operation research, 1997[14]
iv. Mathematical methods on defense analysis, 2000[15]

3.2 Criteria of Model

The conceptual model should be able to describe the following contents

Criterion 1: The effects of undermining operation on regular combat;

Criterion 2: The influence of decapitation combat on regular combat;

3.3 Assumptions

Assumption 1: The effects of undermining operation on the decapitation combat are integrated to the attrition-rate coefficients already.

Assumption 2: Both of the battlefields follow Helmbold Lanchesterian model[11], which is similar to Eqn (3)[15]. Other Lanchester laws might be applied to the modelling concept

Assumption 3: The course of decapitation combat is expressed by the attrition of decapitation troops and garrison forces.

Assumption 4: The information regarding the losses of troops on decapitation battlefield will be transmitted to both sides on regular battlefield freely.

$$\frac{dx}{dt} = -e^{C} x^{D} y^{G}$$
$$\frac{dy}{dt} = -e^{F} x^{G} y^{D} \quad (3)$$

3.4 Scenario

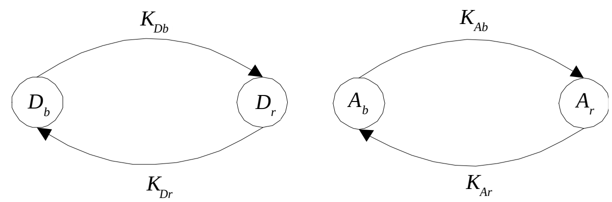
To occupy the target country Blue, the attacker Red seizes air and naval superiority and launches forces entry into target country by air landing, dropping and amphibious operation following undermining operation as part of its decapitation strategy. The purpose of the landed forces is to execute two missions: The decapitation troops (D_r) is to defeat the capital garrison forces (D_b) and to seize the capital while the regular troops (A_r) are to clamp the Blue's regular forces (A_b) from rushing to rescue the capital, and then to defeat the regular forces following the capturing of the capital.

The decapitation and regular battlefield are of conventional military combat; the war is terminated when the combat potential of one side on the regular battlefield is eliminated.

4. MODEL BUILDING

Considering that the combat power of a given unit equipped with specific weapons is transformed from combat potential, traditionally, the combat power is derived directly from combat potential and is treated as the same[17]. The homogeneous Lanchester model describes combat potential with force strength and attrition-rate coefficient. The former integrates units, arms of services, and types of weapon systems into an aggregated number; the latter is the inherent capability of decreasing the antagonistic force strength. When several operations or battlefields are treated isolated or independently and there are no connections/ influences among/between them, the situation could be illustrated as in Fig. 3, square law as explanatory example.

With decapitation strategy, the transformation efficiency of combat potential to combat power is determined by the proposed decapitation effect multiplier which integrates the effects of undermining operation and decapitation combat on regular forces' combat potential. Once the decapitation effect exists, the efficiency of the combat power transformation



$$\frac{dD_r}{dt} = -K_{Db} \times D_b \qquad \frac{dA_b}{dt} = -K_{Ar} \times A_r$$
$$\frac{dD_b}{dt} = -K_{Dr} \times D_r \qquad \frac{dA_r}{dt} = -K_{Ab} \times A_b$$

Figure 3. Independent battlefields.

will be degraded; that is, only portion of the combat potential will be transformed to combat power and cause attrition to the opposing forces. The modelling concept is shown in Fig. 4, also square law is taken as example.

Based on the modelling concept and assumption 2, the decapitation warfare model for regular combat is given in Eqn (4).

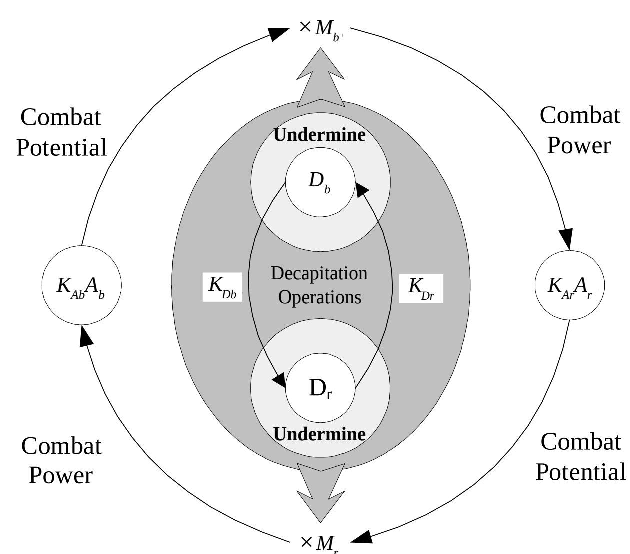


Figure 4. Decapitation effects on regular combat.

$$\frac{dA_b}{dt} = -M_r \times e^C \times A_b^D \times A_r^G$$
$$\frac{dA_r}{dt} = -M_b \times e^F \times A_b^G \times A_r^D \quad (4)$$

4.1 Decapitation Effect Multiplier

Basically, the multipliers are designed to integrate effects caused by decapitation strategy-undermining, capital decapitation-and robustness of the regular forces themselves. Eqn (5) gives the proposed form of the multipliers.

$$M_b = 1 - m + m\left[e^{(1/p-1)}e^{(1-1/f_{Db})}\right]$$
$$M_r = 1 - n + n\left[e^{(1/q-1)}e^{(1-1/f_{Dr})}\right] \quad (5)$$

The exponential terms inside the square brackets represent the effects caused by the undermining and capital combat; details will be discussed in the following paragraphs. The overall mathematical behaviour of Eqn (5) is described as follows:

1. The multipliers are symmetrical, that is when m=n, p=q, and f_{Db}=f_{Dr} then M_b=M_r.
2. 0≤m (n)≤1 acts as a weight between 1 and terms inside the square brackets.
3. Since $p(q)$≥1 and 0≤$f_{Db}(f_{Dr})$≤1, square bracketed terms are ≤1.
4. The values of the multipliers, 0≤$M_b(M_r)$≤1.
5. When $p(q)$=1 means there is no undermining operation against Blue (Red).
6. When f_{Db}=f_{Dr}=1, means there is no capital decapitation or capital combat not initiated yet.

4.2 Weight or Robustness: Parameters *m,n*

Given the scale of undermining operation (which might be measured with air raid sorties, rounds of precision bombs, number of ballistic/cruise missiles, etc.) or attrition level of capital combat, it is suggested that no unit suffers exactly the same inefficiency of combat power transformation. The decline of efficiency of combat power transformation changes from nation to nation, service to service, forces to forces, even unit to unit.

Reasons that cause such differences in transforming efficiency could be training, organisation, doctrines, technology, will to win, etc. The proposed model refers such differences as robustness of the regular forces, the more robust the unit is the less decline in combat power transformation efficiency. And the robustness could be referred as weight, mathematically.

When the robustness is extremely low, $m(n)$ = 1, the transformation efficiency is totally determined by the square bracketed terms in Eqn (5). On the other hand, if the robustness is extremely high, $m(n)$ = 0, the combat potential is completely transformed to combat power, regardless of the scale of undermining and the attrition of capital garrison/decapitation forces.

4.3 Undermining Factor: *p,q*

$e^{(1/p-1)}$ and $e^{(1/q-1)}$ express the effects of undermining operations against Blue and Red sides. The idea to generate factors p and q could be based on the accumulative scales of observable objective quantities, such as air raid sorties, numbers of precision bombs or missiles, equivalent SOF (Special Operation Forces) missions, etc.

Since factor generation is suitable to express the decreasing marginal effect. From operational point of view, the targets subject to undermining operation are prioritised, the targets with high priority will be neutralised first to cause the most tremendous damage on the opposing side. But the undermining operation may not able to eliminate the combat power of the enemy.

As $p(q)$ approaches infinity, the minimum value of the exponential function approaches to $e^{-1} \approx 0.36788$; as $p(q)$ = 1, the maximum value of the exponential function = 1, which indicates that no such effect exists.

4.4 Decapitation Combat Status

The second exponential term, $e^{(1-1/f_{Db})}$ or $e^{(1-1/f_{Dr})}$, aggregates the second effect of decapitation strategy, decapitation combat. In general, combat outcomes could be divided into three categories: mission accomplishment, spatial effectiveness, and casualty, among which casualty[18] is the most objective and easy category to be evaluated.

By applying assumption 3, the decapitation combat status is expressed in the form of surviving fraction of the force strength. Before the initiation of decapitation combat, $f_{Db}=f_{Dr}=1$, there is no effect on regular forces at this time, the exponential terms are = 1; once the decapitation combat begins, $f_{Db}\leq 1$ and $f_{Dr}\leq 1$, the terms are < 1. The terms could be equal to zero mathematically, but the surviving portions are bounded by breakpoints of garrison/decapitation forces, that is, the term might be > 0 unless the forces are eliminated.

5. DISCUSSION

Equation (6) summarises the model building results for regular combat under decapitation strategy which integrates the so called Helmbold relationship[12] with effects of undermining operation and capital decapitation combat, and altogether is tuned with robustness of regular forces. Based on the assumptions, 1 to 4, the presented model satisfies criteria 1 and 2; and is able to interpret hypotheses 1 and 2.

$$\begin{aligned} \frac{dA_b}{dt} &= -\left\{1-m\left[1-e^{(1/p-1)}e^{(1-1/f_{Db})}\right]\right\}\times e^{C}\times A_b^{D}\times A_r^{G} \\ \frac{dA_r}{dt} &= -\left\{1-n\left[1-e^{(1/q-1)}e^{(1-1/f_{Dr})}\right]\right\}\times e^{F}\times A_b^{G}\times A_r^{D} \end{aligned} \quad (6)$$

It is obvious that when regular forces of both side have extremely low robustness, $m=n=1$, a much more concise form of Eqn (7) is obtained and it could be easily handled mathematically.

$$\begin{aligned} \frac{dA_b}{dt} &= -e^{(1/p-1)}e^{(1-1/f_{Db})}\times e^{C}\times A_b^{D}\times A_r^{G} \\ \frac{dA_r}{dt} &= -e^{(1/q-1)}e^{(1-1/f_{Dr})}\times e^{F}\times A_b^{G}\times A_r^{D} \end{aligned} \quad (7)$$

The effects of undermining and decapitation combat are time-dependent variables. When various operations were conducted in separated stages, capital decapitation combat starts after the completion of undermining operation and is followed by regular combat. When this is the case, it is easy to evaluate the regular combat result, since the decapitation effect multipliers hold constant during the regular combat. The overlapping part among these three types of operations complicates the model and a numerical example is provided in the next section to illustrate a discrete time simulation approach to solve the issue.

6. NUMERICAL EXAMPLE

6.1 Initial Conditions

The total strength of both sides' ground forces is equal and is divided evenly in half to commit capital and regular combat. Both sides have the capability to launch undermining operations against each other and the ground combat starts at the end

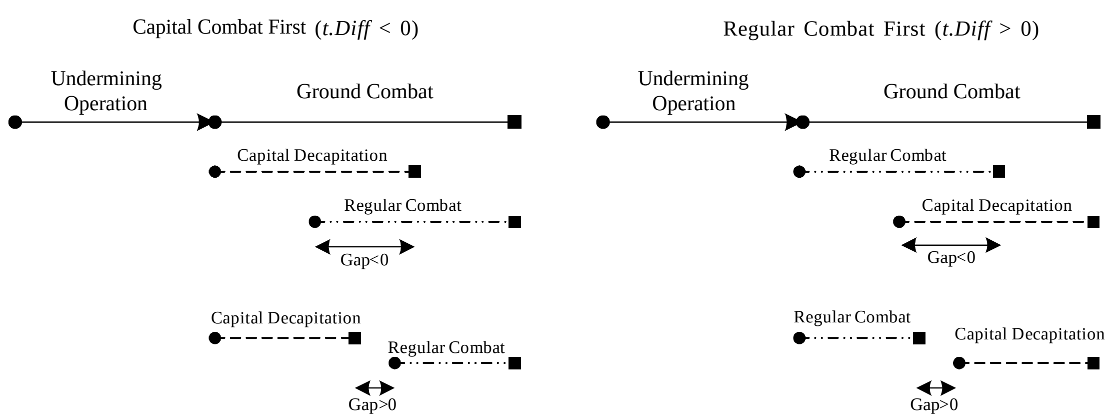


Figure 5. Time difference and combat gap.

of undermining operations. The capital decapitation and regular combat may be overlapped or in sequence as shown in Fig. 5.

All ground forces have the same attrition rates, and both ground battles are fixed-force-level-breakpoint battle, the victory is determined by the result of regular combat, that is which side (regular forces) reaches the breakpoint first loses the war, if both sides reach the breakpoint at the same time, then they are in a tie (parity). To provide deep understandings of the possible outcomes of the war, the Blue operation planners assume a set of data for assessing, as shown in Table 3.

With Table 3, the decapitation strategy model is rewritten in the form of Hartley's work[12] as Eqn (8).

$$\text{Decapitation:}\begin{cases} \dfrac{dD_b}{dt} = -0.0302 D_b^{0.75} D_r^{0.4} \\ \dfrac{dD_r}{dt} = -0.0302 D_r^{0.75} D_b^{0.4} \end{cases}$$

$$\text{Regular:}\begin{cases} \dfrac{dA_b}{dt} = -0.0302\left[1 - n + ne^{(1/q-1)} e^{(1-1/f_{Dr})}\right] A_b^{0.75} A_r^{0.4} \\ \dfrac{dA_r}{dt} = -0.0302\left[1 - m + me^{(1/p-1)} e^{(1-1/f_{Db})}\right] A_r^{0.75} A_b^{0.4} \end{cases} \quad (8)$$

Table 3. Possible combat data

Category/side	**Blue**	**Red**
Robustness of regular forces	m = 0 or 0.5	n = 0 or 0.5
Undermine Effect	p = 1 or 2	q = 1 or 2
Force level on capital battlefield	D_{b0} = 0.5	D_{r0} = 0.5
Force level on regular battlefield	A_{b0} = 0.5	A_{r0} = 0.5
Attrition rate capital combat forces	0.0302	
Attrition rate regular forces		
Breakpoint of capital combat forces	0.4	
Breakpoint of regular forces	0.6	
Time difference between capital and regular combat	$-25 \le t.Diff \le 25$ @ 5	
Constants in Eqn (6)[12]	D = 0.75, G = 0.4	

6.2 Solution Approach

A simple discrete time simulation is adopted to solve the military problem. Using Eqn (8), the attrition of capital combat forces is continually calculated at every fixed time step and the decapitation effect multipliers are calculated based on the updated capital forces' levels, then the attrition of regular force in the time step is obtained by Eqn (8) also. This approach is suitable to solve situations with overlapping combats or sequential combats.

SimScript II.5 simulation language is used to construct simulation program. The functional flowchart is demonstrated in Fig. 6.

6.3 Simulation Results

A total of 176 options/combinations are simulated and the results are given in Figs 7 and 8. The line of RWV (rated winner's value) represents the winner of the war (the winner of regular combat), when Blue wins the war, RWV=30; when Red wins the

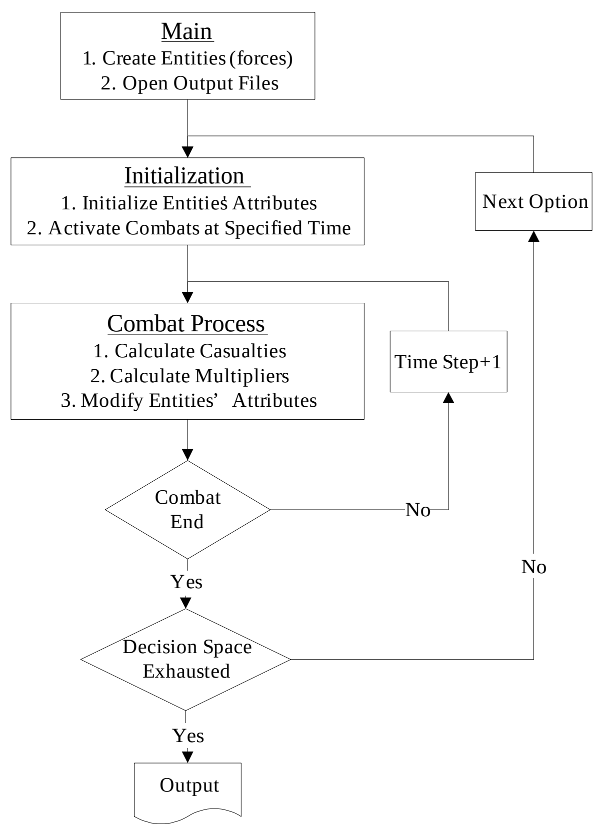


Figure 6. Simulation flow chart.

war, RWV = –30; RWV = 0 means parity. The line of *t*. Gap represents the overlapping time of combats and it is illustrated in Fig. 5. The term 'Duration' means the time span of ground combat (undermining operation is not taken into account).

Both figures can be divided into four sections based on (*m*, *n*) values; from left to right, these are (0,0), (0, 0.5), (0.5, 0) and (0.5, 0.5). On each section of *t. Diff*, there are 11 segments representing time differences between capital combat and regular combat, from –25 to 25 at interval of 5 time units; every segment contains four sample points representing (*p*, *q*) values: (1,1), (1,2), (2, 1) and (2,2) from left to right.

Some observations from the results are:

1. When (*m*, *n*)=(0, 0) decapitation strategy has no effect on regular forces and when both sides are of equal combat power they reach the breakpoint simultaneously, thus resulting in parity. The same situation happens in capital decapitation battlefield which is not shown in figures.

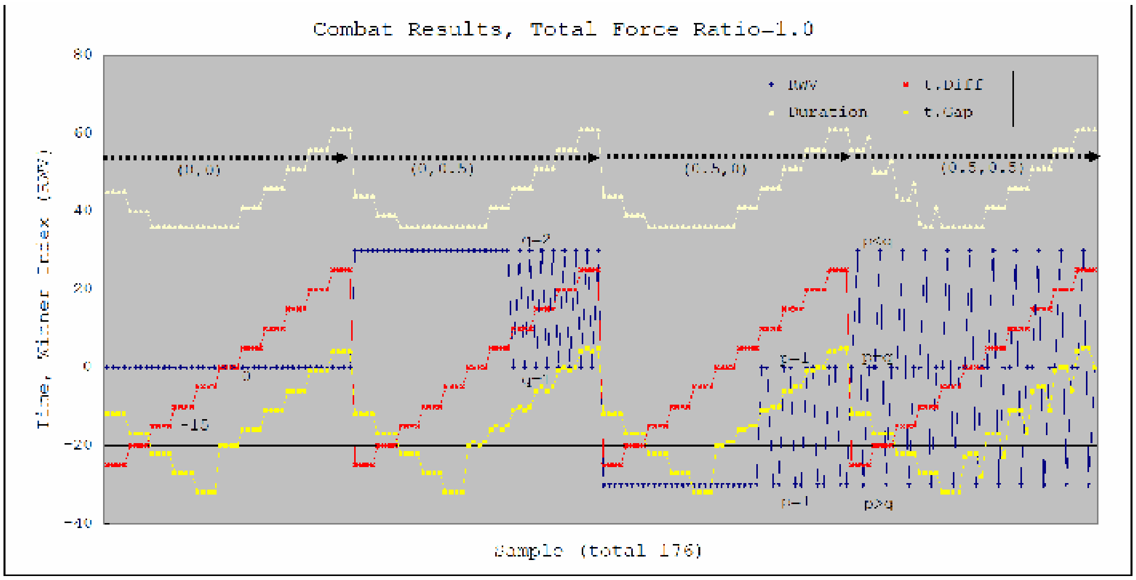


Figure 7. Combat results.

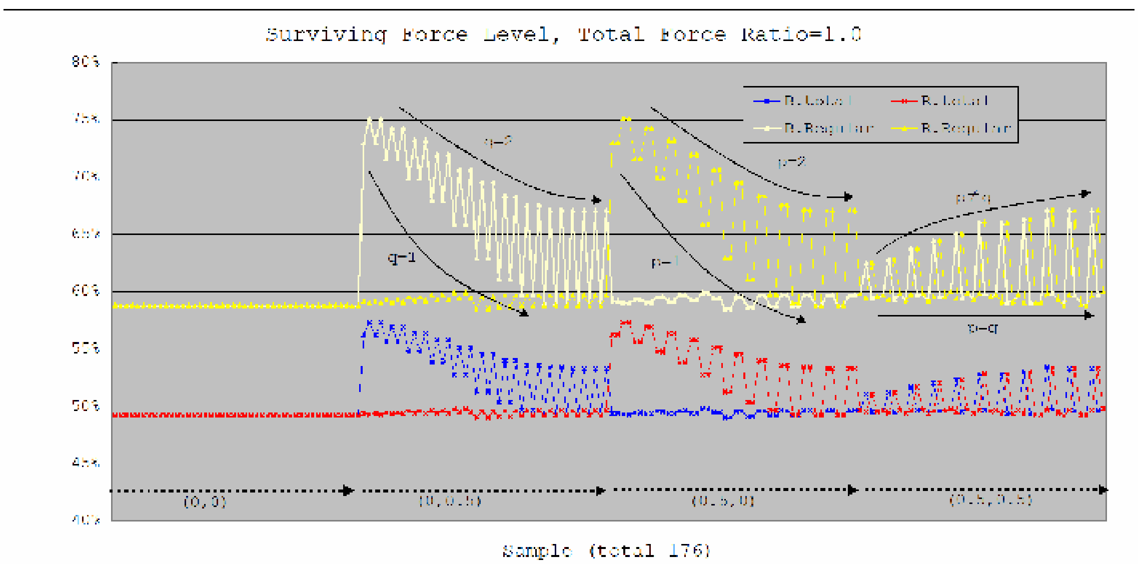


Figure 8. Surviving force level.

2. When (m, n)=(0, 0.5), the Red regular forces has lower robustness and declined combat power due to decapitation effects, but the Blue regular forces sustain their combat power, M_b=1. The Blue regular forces win in most of the cases except *t.Diff*≥10 and q=1, in such cases, the results are parity.

   From operational point of view, if the decapitation combat is initiated after the regular combat has started and the Blue did not launch any undermining operations against the Red (q=1) then the Blue may not win the war.

3. The third section, (m, n)=(0.5, 0), shows the Blue regular forces with lower robustness and affected by the decapitation effects but the Red is not. Then the Red will win in most of the cases except *t.Diff* ≥ 10 and p = 1, in such cases parity is achieved. That means the Red should conduct undermining missions to secure the success of their invasion.

4. It seems that the time difference between decapitation combat and regular combat has no influences on the war victory when (m, n)=(0.5, 0.5), the victory is totally dependant on the effect of undermining operations. When p=q parity is achieved; when (p, q)=(1, 2) the Blue wins, the Red wins when (p, q)=(2, 1).

5. It is observed that –15≤*t.Diff*≤0 will yield minimum ground combat duration of 33 time units in sections (m, n)=(0, 0), (0, 0.5) and (0.5, 0), in section of (0.5, 0.5) only *t.Diff*=–5 or 0 will give the same combat duration. This observation means that if the Red desires a quick victory then the capital decapitation operation should be initiated at the right time. To be more precisely, when (m, n)=(0.5, 0.5), for example, the Red should commit undermining operation (p=2) and initiate capital decapitation 5 time units before regular combat, or at the same time as regular combat, to win the war in minimum of 33 time units.

6. Except for victory and combat duration, casualties is also one of the most concerned factors in military operation planning. Figure 8 reveals that the pattern of surviving force level varies on (m, n) values also.

   In the section of (m, n)=(0, 0.5), the Blue surviving level declines in accordance with increasing *t.Diff* but the Red varies within limited range; in (0.5, 0) section, the situation is reversed.

   The effects of undermining operation, (p, q), is the other influencing element. In section (0, 0.5), q=2 will produce higher Blue surviving level than q=1; in section (0.5, 0), p=2 produce higher Red surviving level than p=1.

   The situation in section (0.5, 0.5) is more complex, the Blue surviving level increase along with raising *t.Diff* when (p, q)=(1, 2); the same thing happens to the Red surviving level when (p, q)=(2, 1); when p=q the surviving levels of both sides are equal and do not vary tremendously.

6.5 Example Summary

From the defensive's (Blue) point of view, the most optimistic situation, (m, n)=(0, 0.5), the defender should provoke the attacker to start capital decapitation as early as possible so that the Blue has high possibility to win the war with low casualties. But this does not seem a usual case in history.

The most pessimistic result comes from (m, n)=(0.5, 0), but the result could be parity if the Blue is able to delay the initiation of offensive capital decapitation operation concurrently to protect critical assets from hostile undermining operation or able to launch undermining operation against the attacker. It seems that the only way to prolong duration of combat is to delay the ignition of capital decapitation combat and this is also the way to increase the casualties of the Red.

In the fair case, (m, n)=(0.5, 0.5), the dominating factor is undermining operation. That is, the Blue will win if the Blue manages to make the Red suffer higher undermining effects, p<q; or the Blue might lose the war if p>q.

7. CONCLUSION

The design of the Lanchesterian decapitation warfare model is based on historical observation which suggests that both undermining factors and capital decapitation combat have effects on regular combat. Modelling hypotheses and assumptions are derived from such observation. In order to apply the proposed model, a discrete time simulation method is used to solve the complexity of the model. Through simulation various options and possibilities could be explored and the collected data will be helpful to operation planning.

In the illustrative example only equal force level is discussed, but any possible total ground force ratio and force allocation (between capital and regular battlefield) could be analysed to find the best force allocation rule.

Beside total force level and force allocation, the proposed model provides extensive coverage of decision variables and initial conditions, such as fixed-force-level or proportional-force-level breakpoints, attrition rate, time interval between combats, forces' robustness, and undermining effects. The wide coverage gives the model flexibility to analyse complex warfare involving decapitation strategy.

However, to produce precise prediction on the outcomes of war, further detailed historical study is required to identify or estimate critical model parameters such as robustness (m, n) and effects of undermining operation (p, q).

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