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Fuzzy Logic-based Terminal Guidance with Impact Angle Control

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ABSTRACT

This paper presents a new formulation of terminal guidance law which controls the impact attitude angle while minimising the miss distance. The formulation is based on the fuzzy logic-control approach. Unlike many prevalent designs, the proposed guidance law does not require linearisation of missile-target engagement model. Numerical simulation results demonstrate that the proposed guidance law offers satisfactory performance, fulfilling its design goals.

Keywords: Terminal guidance, fuzzy logic control, impact angle control, reentry guidance

NOMENCLATURE

a_c Commanded lateral acceleration

a_m Achieved lateral acceleration

A Reference area

c_d Drag force coefficient

D Drag force

g Gravitational acceleration

h_m Altitude

m Mass

R Relative range from missile to target

t Time

v_m Missile velocity

x_m Downrange

γ_m Flight path angle

γ_{md} Desired impact flight path angle

ρ Atmospheric air density

θ Line-of-sight angle

τ Autopilot time constant

1. INTRODUCTION

The vast majority of guidance laws have one objective, i.e., to reduce the distance between the missile and the target to zero. This is not always sufficient. In some cases, the direction from which the missile approaches the target and impacts on it is also important. For example, the effectiveness of many warhead systems is a strong function of the miss distance and the relative-approach angle between the target and the missile. When a missile employs a directional warhead, the guidance system is expected to cause impact on the target at a certain body-attitude angle to achieve the maximum damage. Also, impenetrable nature of heavy armour may be susceptible to missile attacks at a relative high angle of impact wrt the horizon. Thus, impact angle control is required to modulate the trajectories of antiship and antitank missiles to achieve the

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maximum damage. In certain reentry vehicles, the body-attitude angle at impact is required to be within permissible limit wrt local vertical while achieving acceptable miss distance. For such vehicles, vertical impact on the targets also improves the miss distance due to inertial navigation error in height channel. It is noted that the general guidance schemes developed to produce zero miss distance may not achieve a proper body-attitude angle at impact, and thus such requirement cannot be satisfied by guidance laws designed to minimise the miss distance only.

The guidance laws reported in the literature suffer from certain drawbacks such as use of linearised model of missile-target engagement scenario and neglecting the aerodynamic effects such as drag in its formulation. A biased PNG law[1] is proposed to achieve a desired body-attitude angle of impact as well as zero miss distance. Similarly, a numerical solution[2] of a time-optimal impact angle control problem for the vertical plane engagements is presented. In both of these formulations, missile velocity is assumed to be constant. An optimal active homing-guidance law[3] for impact angle control is proposed, wherein the guidance law is a control energy-minimising solution with constraints to reduce the miss distance and impact angle control. In this formulation, while the missile velocity is varying, it is assumed to reduce exponentially from its initial value. Also, the estimation of time-to-go is required for implementation of the guidance law. The guidance laws, based on linear control strategies such as LQR and H^{∞}, to control the missile body attitude wrt the target at the terminal point are presented[4]. Similarly, a generalised formulation[5] of energy minimisation optimal guidance laws using the linearised model for constant speed missiles is proposed to achieve the desired impact angle as well as zero miss distance. Such guidance laws are not expected to perform satisfactorily whenever the assumption of linearisation is violated. The recently developed circular navigation guidance[6] is based on the principal of following a circular arc to the target, resulting in impact angle control. In this formulation too, the aerodynamic effect such as drag are neglected, and thus the missile velocity is assumed constant.

In the design of a terminal guidance system for certain reentry vehicles, the body-attitude angle at impact is required to be within a permissible range, while meeting the specification on miss distance. To this end, a terminal guidance for a reentry vehicle is presented[7], wherein the problem is formulated as a linear quadratic optimal-control problem with a constraint on body-attitude angle at impact. The problem was further investigated by considering the angle of attack and autopilot dynamics. These works are based on linearised models of pursuit kinematics. To implement these guidance laws, time-to-go estimation is also necessary. Three reentry guidance laws[9], namely proportional navigation, cross-product law, and tangent cubic law have been presented. While the first two cannot achieve the desired impact angle, the tangent cubic guidance law can be used to achieve the desired impact angle. In the tangent cubic law, as cubic curve is fitted between the current vehicle position and its desired final position with a tangent constraint at each end, the demanded lateral acceleration levels are usually high and may become prohibitively large form implementation point of view. On similar line of tangent cubic law, a characteristic curve approach for deriving explicit reentry guidance equations for reentry vehicles is developed[10] showing how terminal trajectory constraints on flight path angles can be obtained.

The objective of this paper is to present a formulation of an explicit terminal guidance law in reentry which controls the impact attitude angle while driving the miss distance to zero. By explicit, it is meant that the guidance equations would include only position and velocity components relative to the target, without resorting to a stored nominal trajectory. The formulation is based on the fuzzy logic approach and does not involve linearisation. The missile velocity varies under drag effects and thus the assumption of constant missile velocity which is usually employed in many impact angle control formulations is avoided. With these objectives, the next section briefly reviews the fuzzy logic control approach while formulation of the guidance law based on this approach is presented in Section 3. Numerical simulations are carried out to demonstrate the effectiveness of this formulation and the results are presented in Section 4, while Section 5 concludes this work.

2. FUZZY LOGIC CONTROL

It is well known that fuzzy systems have the ability to make use of knowledge expressed in the linguistic rules without completely resorting to the precise plant models[11]. In control applications, fuzzy logic approaches using, if-then, rules can solve complex and practical problems. In recent years, researchers have also attempted to apply it on missile guidance designs[12-15]. Although, many applications of fuzzy logic theory on missile guidance and control have appeared with growing interest, no application to the terminal guidance with impact angle control problem has been attempted. This paper employs this approach to formulate a terminal guidance with impact angle control for a reentry vehicle.

The basic configuration of the fuzzy logic system is shown in Fig. 1. It is composed of four functional blocks. A rule base contains a number of fuzzy, if-then, rules and database defines the membership functions of the fuzzy sets used in the fuzzy rules. The decision-making unit performs the inference operations on the rules. A fuzzification interface transforms the crisp input into degrees of match with linguistic value. A defuzzification interface transforms the fuzzy results of the inference into a crisp output.

The fuzzy rule base consists of a collection of fuzzy, if-then, rules expressed as the form if a is A then b is B, where a and b denote the linguistic variables, A and B represent the linguistic values that are characterised by the membership functions. The input and output variables of a fuzzy system are called linguistic variables as they take linguistic values (e.g. large, small, very large, very small, etc.). The linguistic sets are described by their membership functions.

The rule-based representation of a fuzzy logic controller does not include any dynamics, and the computational structure of a fuzzy logic-control consisting of fuzzification, inference, and defuzzification is highly nonlinear. These make a fuzzy logic-control a natural nonlinear static transfer element like a static controller. A fuzzy logic system is a kind of gain-scheduling systems which can cover a wide range of operating regions. This specific feature is appropriate for the guidance design since the missile-target engagement system is a highly nonlinear model.

3. FORMULATION OF GUIDANCE LAW

In this Section, the fuzzy logic approach is employed to formulate the terminal guidance law for a point mass reentry vehicle moving in a plane. The problem being considered is to minimise the miss distance and have the missile-approach target as close as possible to some specified impact angle. To obtain the equations of motion, one considers a coordinate frame, the origin of which is fixed at the projection of the nominal reentry position of a vehicle on ground with X-axis pointing downrange and Z-axis along the local vertical as shown in Fig. 2. The equations governing the motion of the point mass reentry vehicle can be derived as

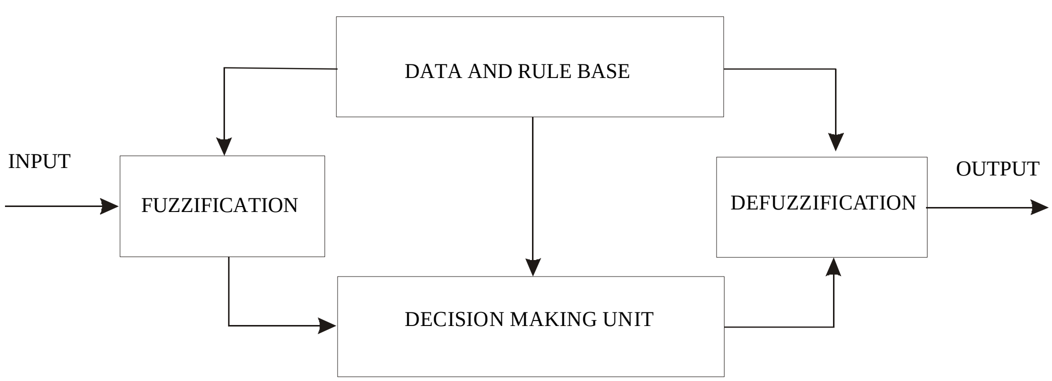


Figure 1. General structure of a fuzzy control system.

$$
\begin{aligned}
\dot{v}_m &= -\frac{D}{m} - g\sin\gamma_m \\
\dot{\gamma}_m &= \frac{a_m}{v_m} - \frac{g\cos\gamma_m}{v_m} \\
\dot{h}_m &= v_m\sin\gamma_m \\
\dot{x}_m &= v_m\cos\gamma_m \\
\dot{a}_m &= -\frac{a_m}{\tau} + \frac{a_c}{\tau} \\
\dot{R} &= -v_m\cos(\gamma_m - \theta) \\
\dot{\theta} &= \frac{1}{R}[-v_m\sin(\gamma_m - \theta)]
\end{aligned}
$$

where v_m, γ_m, h_m, x_m and a_m are the velocity, flight-path angle, altitude, downrange and acceleration of the missile respectively. The quantities R and θ are the relative range from missile to target i.e. Line-of-sight (LOS) and LOS angle, respectively. The quantities D and m are the drag and the mass of the vehicle, g is the gravitational acceleration, and a_c is the commanded lateral acceleration in vertical plane. The autopilot is modelled as first- order dynamics having a time constant of τ. In the present formulation of guidance law, aerodynamically the vehicle is being modelled as a sphere. Defining an error angle (λ) as

$$\lambda = \gamma_m - \gamma_{md} \quad (2)$$

where γ_{md} is the desired impact angle, the objective is to minimise this quantity as the vehicle approaches towards its target. The input variables of fuzzy logic-based guidance law, also called the linguistic variables are chosen as R, $\dot{?}$, λ, and v_m, while the output variable is the commanded lateral acceleration (a_c). The fuzzy logic-based guidance law has been designed using Mamdani implication and centroid defuzzification[11]. To simplify the computation in the actual operation, triangular membership functions are suggested, as it has been found that using complex forms of membership functions such as bell-shaped functions, cannot bring any advantage over the triangular ones. The values assigned to the membership functions used here have been arrived at through numerical simulations.

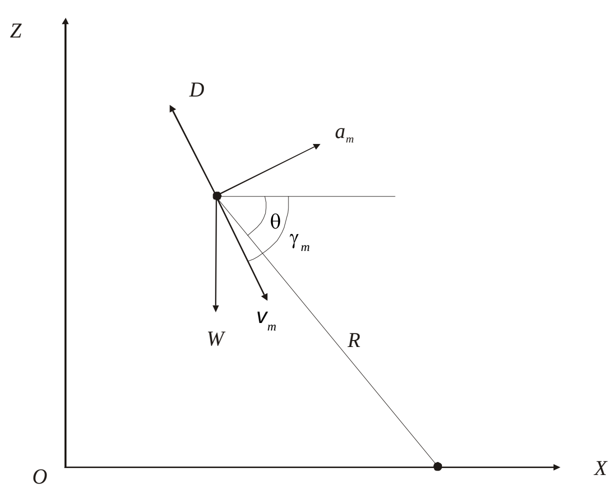


Figure 2. Coordinate systems.

The linguistic values taken by these variables are expressed by linguistic sets. The variable R is assumed to take five linguistic values defined as VS (very small), S (small), M (medium), L (large), VL (very large), while $\dot{?}$ is taking three linguistic values as N (negative), Z (zero) and P (positive). The variable λ is taking five values as NL (negative large), NS (negative small), Z (zero), PS (positive small) and PL (positive large). The velocity of missile which is another variable that takes three values, namely S (small), M (middle) and L (large), while the output variable (a_c) takes five values like λ. For these variables, the corresponding membership functions are shown in Fig. 3 and the guidance law employs the rule base as given in Table 1.

4. SIMULATION AND RESULTS

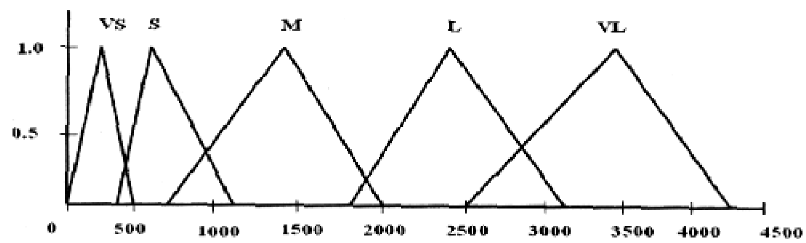
To assess the performance of the fuzzy logic-based terminal guidance law, numerical simulations have been carried out using the equations of motion of Eqn (1). The initial conditions are the same as used by Kim and Grider[7]: $v_m(0)$ = 609.6 m/s, $\gamma_m(0)$ = –35°, $h_m(0)$ = 3048 m, $x_m(0)$ = 0 m, $a_m(0)$ = 0 m/s^2, $R(0)$ = 4310.5 m, and $\theta(0)$ = –45°. The drag is calculated using

$$D = 0.5\rho v^2_m c_d A$$

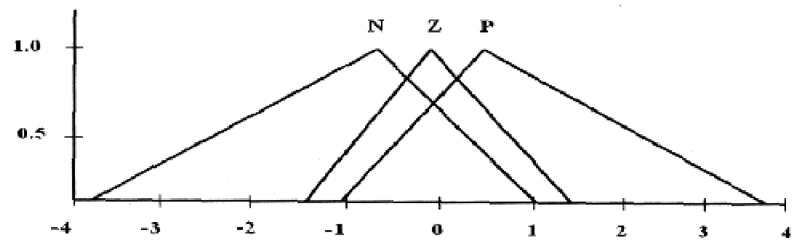
where density of air is taken as

$$\rho(h) = \rho_{sl}\exp\left(-\frac{hm}{\beta}\right)$$

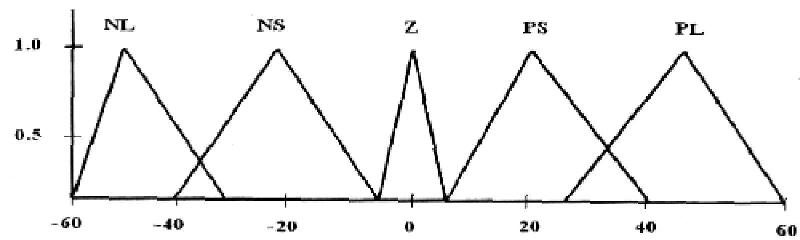
with the sea-level density, ρ_{st} = 1.12 kg/m^3 and β = 9296.0. The vehicle data for drag coefficient



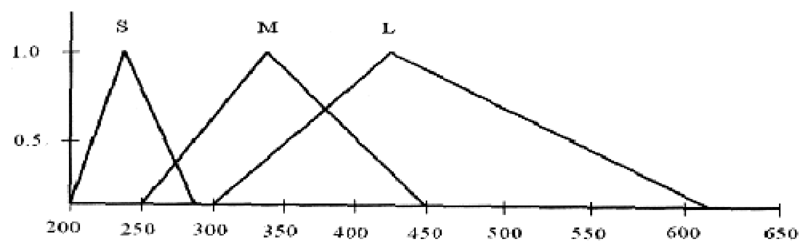
(a) Membership function of R



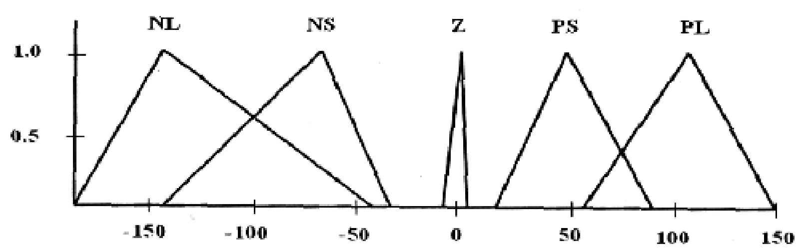
(b) Membership function of $\dot{\theta}$



(c) Membership function of ●



(d) Membership function of v_m



(e) Membership function of a_c

Figure 3. Membership function.

(c_d = 0.1), mass (m = 92 kg) and reference area (A = 0.152 m^2) corresponds to the data of generic reentry vehicle[17] and the same is used here for simulation purpose. The autopilot time constant (τ) is taken equal to 0.1 s. It is required to have an impact angle of about 70° wrt local horizon and so γ_{md} as required in Eqn (2) is set to –70° and the acceptable miss distance is taken as 100 m.

As stated in the last section, the input variables of fuzzy logic-based guidance law are R, θ, λ and v_m, while the output variable is the commanded lateral acceleration (a_c) and their membership functions are as shown in Fig. 3. The guidance law employs the rule base as given in Table 1.

Using this formulation, a number of numerical simulations have been carried out for various initial condition errors, and the trajectories were analysed. Simulation results for a representative case are given in Fig. 4. In Fig. 4 (a), the trajectory of missile is shown from where one can note that the guidance law has achieved a miss distance of 90 m.

Table 1. Fuzzy rule base

Rule	Antecedent				Consequent
	R	$\dot{\theta}$	λ	v_m	a_m
1	VL	P	PL	-	NL
2	L	P	PS	-	NS
3	M	-	PS	-	NS
4	VS	Z	Z	-	Z
5	S	P	Z	-	NS
6	VS	Z	PS	-	NS
7	-	N	NL	-	PS
8	-	-	-	L	NL
9	-	-	-	M	NS
10	-	Z	Z	-	Z
11	-	N	-	-	PS
12	-	-	-	L	NS

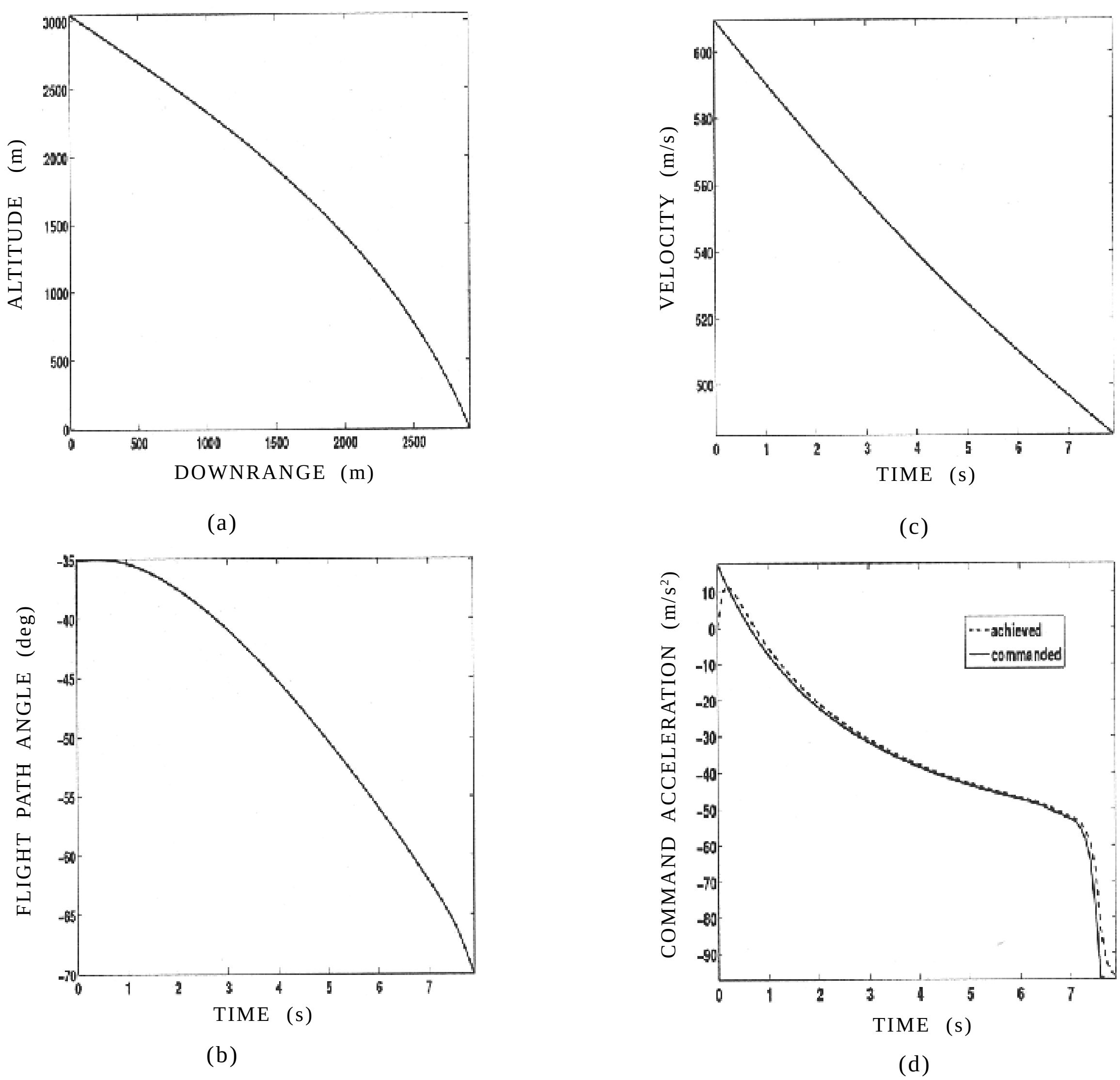


Figure 4. Performance of fuzzy logic guidance: (a) Missile trajectory, (b) missile flight-path angle history, (c) missile velocity history, and (d) commanded acceleration history.

The flight-path angle history is shown in Fig. 4 (b), from where it is obvious that the impact angle is very close to the desired value of –70°. The corresponding velocity profile of missile is shown in Fig. 4 (c). Fig. 4 (d) gives the commanded acceleration history.

During numerical simulations, it was observed that the guidance law achieves miss distance requirement while obtaining the desired impact angle for initial missile flight-path angles ranging from –20° to –40°. This range for flight-path angle is catered as it represents a practical range for a typical reentry vehicle. For any initial flight-path angle outside this range, the guidance law will need to be modified accordingly by changing the values assigned to the membership functions, and to this end, further work needs to be carried out. It was also observed that as you reduce the relative range (*R*), the objectives are met but with higher commanded lateral acceleration.

To assess and compare the performance of present formulation, guidance laws for impact angle control have also been designed using other approaches such as the tangent cubic law[9] and the predictive control approach[18-19] and numerical simulations were carried out using the same set of data and initial conditions. The details of these formulations are not included in the paper. The simulation results

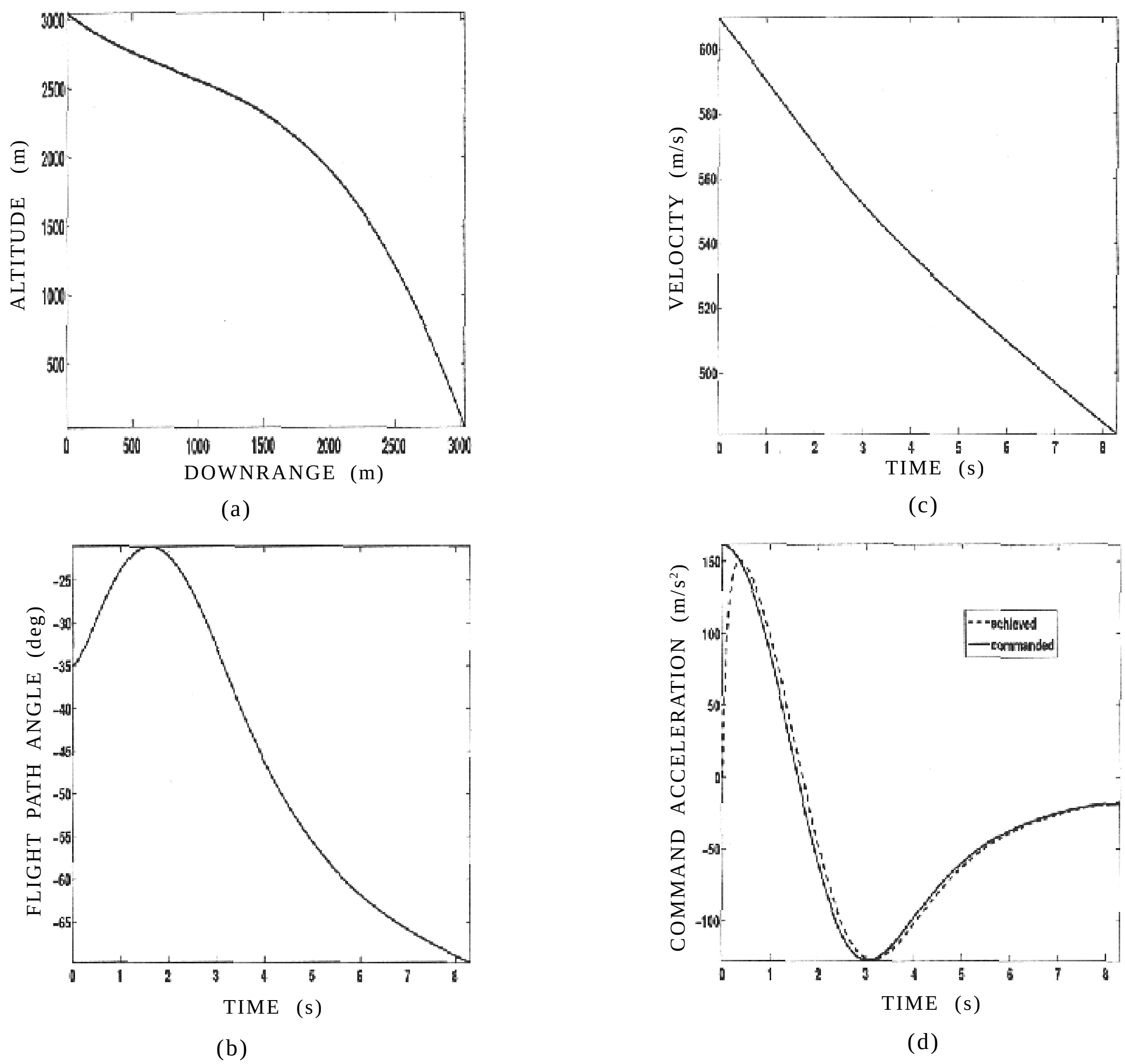


Figure 5. Performance of tangent cubic guidance: (a) Missile trajectory, (b) missile flight-path angle history, (c) missile velocity history, and (d) commanded acceleration history.

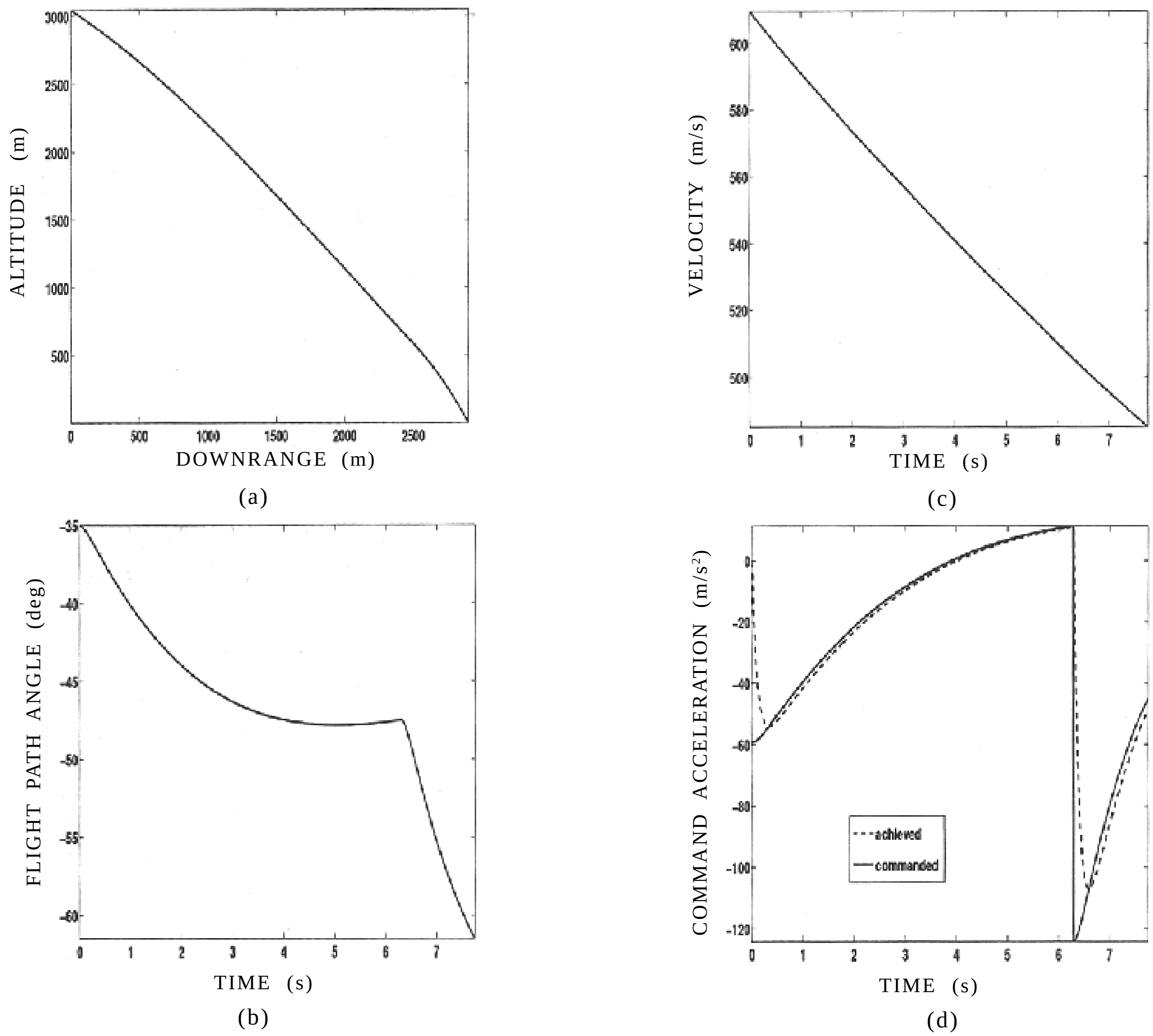


Figure 6. Performance of predictive guidance: (a) Missile trajectory, (b) missile flight-path angle history, (c) missile velocity history, and (d) commanded acceleration history.

using the tangent cubic guidance are presented in Figs 5(a) - 5(d) from where one can note that the miss distance is 25 m and but the asked lateral acceleration is much higher than that of the fuzzy logic based guidance. Figures 6(a)-6(d) present simulation results for guidance law based on the predictive control approach. The miss distance in this case is 120 m, and here too, one can note that the commanded acceleration is higher than that of the fuzzy logic-based guidance. Also, the acceleration switches its sign abruptly towards the end which is not a desirable feature.

5. CONCLUSION

In this paper, a new formulation of a terminal guidance law, based on fuzzy logic, to control the impact angle while achieving acceptable miss distance is proposed. The formulation does not require linearisation of the otherwise nonlinear missile-target engagement model. Numerical simulations have been carried out to assess the performance of this guidance law and the results are presented. Numerical simulation results show that the guidance law offers satisfactory performance.

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