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Range Density Function for Active Sensor Imaging

Askin Demirkol
University of Missouri-Rolla, Rolla MO 65409-0040, USA

ABSTRACT

In this paper, a generalised target density function (TDF) is studied for active imaging by sensors such as radar. This is achieved by estimating a new TDF which is called range density function (RDF). It is proposed by utilising of a generalised range-angle density function for imaging. While the RDF is developed by a new approach based on inverse Wandermonde matrix computation, it is obtained by considering a new range and scanning angle plane different from the conventional methods.

Keywords: Sensor imaging, SAR-ISAR, dense target environment, target density function, range density function, inverse Wandermonde matrix

1. INTRODUCTION

Imaging is a mapping process from a 3-D object to a 2-D image[1-7]. This transformation is obtained using signal transforms such as Fourier and Wavelet transforms[2,4-9]. As an active sensor, radar imaging is based on a multi-sensor image fusion technique, which is in the form of multiple-apertures and arrays[10-15]. This imaging is a reconstruction process which extracts the radar echo signals off the targets. Radar image formation consists of three consecutive phases such as signal acquisition, signal processing, and image processing[6-7].

Target density function (TDF) is the reflectivity of spatially, continuously distributed targets and is an important characteristic of radar imaging. The dense target environments that occur when the density is spread over a wider space, are studied by Fowle-Naparst[16,17]. TDF is known by different names such as ambiguity function, density function, object (target), object reflectivity function, doubly-spread reflectivity function, and reflection coefficient[8,10,18]. All these are the object representation functions obtained via multi-sensor data fusion systems.

If TDF is assumed to be a reflection coefficient, it is defined as the ratio of the received signal to the transmitted signal. By this definition, the reflected signals from the object space are amplitudes relevant to the intensities of the points on the target or objects. If the object's geometric plane is considered, since the integration of these amplitudes or the illuminated intensities reveal information related to the object shape, TDF will have an important role in obtaining the radar images.

There are two well known approaches on TDF. First one considers point scatterers reflected off the target scatterer centres. Integration of all point scatterers is able to obtain the whole object. This radar imaging technique is based on inverse Fourier transform (IFT) and used mostly in inverse synthetic aperture radar (ISAR) studies[2,4-7,19-20].

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Second method on TDF is a dense target environment approach by Fowle and Naparst[16-17]. This takes into consideration the existence of densities of the targets in a high dense target environment. It is based on the ambiguity functions with two variables as range and velocity[21]. Especially, the advanced function in the dense target environment by Naparst is developed in a novel way. Rather than typical radar imaging, this is an approach to measure the closeness of the targets to each other in the dense target environment. However, this provides an important contribution to the analysis of TDF related to the radar imaging.

In this study, a new TDF is theoretically developed by a new approach on a range-scanning angle plane different from the early approaches. This technique is developed based on inverse Wandermonde matrix.

2. PRELIMINARIES OF DENSITY FUNCTIONS

In this study, the background of the target density functions consists of SAR-ISAR reflectivity functions, ambiguity functions, and Naparst's target density functions.

2.1 SAR-ISAR Reflectivity Functions

Synthetic aperture radar (SAR) is a well known radar imaging technique used for earth surface imaging. A SAR image is a high-resolution map of surface target areas and terrain in the range and the azimuth dimension. Coherent SAR imaging is an alternative approach to remote sensing that provides contribution to the imaging over visible/infrared sensing technology[6-7,22-25].

For SAR receiving mode, if the target is composed of continuum-point targets (scatterers), by the superposition principle, the echo (reflected signal) $x(t)$, from such a target at x,y,z points is

$$x(t)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\rho(x,y,z)\,f\left(t-\frac{2R(x,y,z)}{c}\right)dxdydz \tag{1}$$

Here, f is the transmitted signal function, ρ is reflectivity function, R is the range, and c is the speed of light. As stated in Eqn (1), the returned

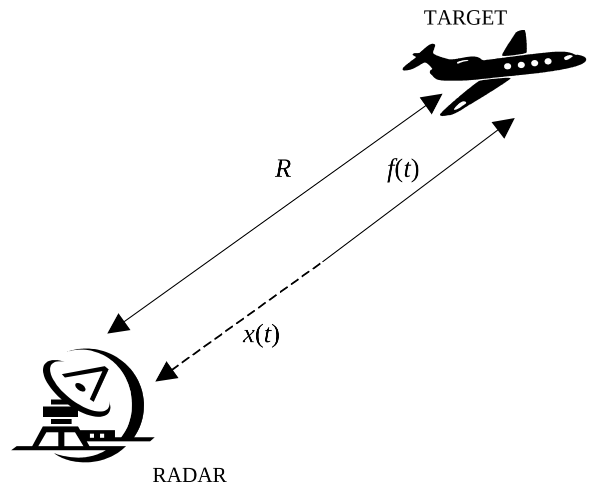

Figure 1. Radar-target detection.

signal $x(t)$ is a delayed and time-scaled version of the transmitted signal, $f(t)$.

As a TDF definition, reflection coefficient is used in inverse Fourier transform (IFT) and ISAR image formation. This technique defines the reflection coefficient by the superposition method, which involves integration of all point scatterers. Summation of the point scatterers represents the whole object[2,4-7,19-20,22-25] as shown in Eqn (1). (x, y, z) determines the integration of the point scatterers at the object or target. SAR systems are designed by moving a real aperture or antenna through a series of positions along the flight track. This corresponds to multi-aperture SAR imaging[26].

As for ISAR systems, imaging is based on similar principles to SAR imaging. However, these have different configurations. In SAR imaging, the radar is flying in space and the object is stationary, while in ISAR imaging, the object is moving and the radar is stationary. Target motion is the essence of the difference between SAR and ISAR[2,6-7,27-28].

ISAR is considered as an IFT of a 3-D object on a 2-D image. If Eqn (1) is expressed in two dimensions, after demodulation and some pre-filtering processes, the measured ISAR signal becomes:

$$x(t)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\rho(x,y)\,e^{-j2\pi f_0\frac{2R_p(t)}{c}}dxdy \tag{2}$$

for $2R_p(t)/c \le t \le T_{PRI}$

Here, T_{PRI} is pulse interval repetition, c is the speed of light, f_0 is carrier frequency, and $Rp(t)$ is the range from the radar to the point scatterer, given as:

$$R_p(t) = R(t) + x\cos[\theta(t) - \alpha] - y\sin[\theta(t) - \alpha] \quad (3)$$

In Eqn (3), α is the azimuth angle and $\theta(t)$ is the rotation angle. If IFT is applied to Eqn (2), the image (x,y) is obtained as a 2-D form of 3-D object[2,6-7].

$$\rho(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{X(f_x, f_y)}{e^{j2\pi(x\,f_x - y\,f_y)\frac{2R_p(t)}{c}}} df_x df_y \quad (4)$$

where $f_x = \dfrac{2f_0}{c}\cos\theta(t)$, $f_y = \dfrac{2f_0}{c}\sin\theta(t)$

2.2. Ambiguity Functions

The radar ambiguity function can be used as a criterion of goodness for modulation waveforms to discriminate the range and range rate of point targets[21,29-33]. Radar waveforms can usually be described as a narrowband modulation of a single radio frequency carrier. For the narrowband radar signal of short duration, the Doppler effect is simply to shift the frequency of the carrier signal by an amount proportional to both the scatterer's radial velocity wrt the radar and the signal carrier frequency. For other radar signals, however, the representation of the Doppler effect as a simple carrier frequency shift is no longer valid. When either the modulation bandwidth or coherent signal duration is sufficiently large, the Doppler effect on the modulation envelope also becomes significant, and must be properly accounted for in radar design. Such is increasingly the case as radar waveforms and signal processing increase in sophistication, with goals of achieving higher range and velocity resolution, and better clutter rejection.

The radar ambiguity function describes the response of a particular range-velocity resolution cell of a radar to a point target, as the target range and velocity vary. Radar performance in terms of capability to resolve target and clutter scatterers in range and velocity dimensions can be assessed by direct examination of the ambiguity function surface in the range-velocity ambiguity plane.

Recognising the importance of ambiguity functions as a signal design criterion in mathematical radar theory is generally credited to Woodward[21]. The ambiguity function $A(x,y)$ of a signal $u(t)$ is given by

$$A(x,y) = \int_{-\infty}^{\infty} u(t - \frac{x}{2})\,\hat{u}(t + \frac{x}{2})\,e^{-j2\pi y t} dt \quad (5)$$

where $\hat{u}$ is the complex conjugate of u. As seen, the ambiguity function $A(x,y)$ is a 2-D correlation function varying wrt x and y variables. If $A(x,y)$ is considered for the radar, the quantities x and y are, respectively, the time delay in the echo due to the range of the object, and the Doppler, or frequency shift in the echo, due to the object's velocity.

As Woodward[21] observed, the ambiguity function $A(x,y)$ of a transmitted signal $u(t)$ measures the uncertainty with which the returning echo distinguishes, simultaneously, both ranges and velocities of a target system. In all cases of the ambiguity function $A(x,y)$, the number of objects and their various ranges and velocities obviously are not completely determined by the pattern, and the ambiguity in the determination is described by the structure of $A(x,y)$. Hence, it is natural to call $A(x,y)$ the ambiguity function for the waveform $u(t)$.

2.3 Target Density Function

First density term related to the TDF is called as dense and density by Fowle[16], *et al*. In their paper, in general, two types of target density environment including multiple target complex are considered: (i) case of high density of similar targets, and (ii) case of single target in a dissimilar clutter background.

In Fowle's work, a particular aspect of the radar resolution problem was studied. Whereas many discussions of radar resolution are concerned with the highest possible degree of separation of two very nearby targets, usually in one dimension.

Fowle was concerned with the problem of the detection and resolution in two dimensions(range and range rate) of a large number of targets in a fixed part of the target space. He claims that the high target density is more characteristic of this resolution problem than is the close proximity of target pairs.

While Fowle had focused on the problem of the detection and resolution in two dimensions of a large number of targets in a fixed part of the target space, he was inspired of ambiguity functions.

After Fowle, dense target environment term was used by Naparst's paper[17]. In addition to dense target environment definition, Naparst also described a TDF as a new concept in the same work. Contrary to Fowle's low target density environment, Naparst method was on multiple and high-density target environment. His new approach was based on ambiguity and cross-ambiguity functions. In this work, the dense target environment was defined as the closeness of a lot of targets at a distance, when their velocities are so close to each other. While Naparst developed the new technique with the high target density environment, he was inspired by Fowle's study, which defines that the high target density is more characteristic of this resolution problem than is the close proximity of target pairs.

As per the definition by Naparst, density of targets at distance x and velocity y is $D(x,y)$. In this case, the echo or the reflected signal from targets is

$$e(t) = \int_0^{\infty} \int_{-\infty}^{\infty} D(x,y)\sqrt{y}\; s(y(t-x))dxdy \qquad (6)$$

In this approach, it is assumed that all targets are illuminated equally. As stated, the target density function is a function of the range and velocity variables similar to the ambiguity functions.

Reconstruction of the TDF in Naparst algorithm by the ambiguity and orthonorrmal functions in Hilbert[17] space is finalised as:

$$D(x,y) = \sum_{n,m=0}^{\infty} < e_n, s_m > A_{nm}(x,y) \qquad (7)$$

where s_m are signals sent out and e_n are their echoes. The cross-ambiguity function of the signals sent out (s_1, s_2,...) is

$$A_{nm}(x,y) = \int_{-\infty}^{\infty} s_n(y(t-x))\,\overline{s}_m(t)dt$$

3. IMAGING BY A NEW TARGET DENSITY FUNCTION

In this paper, an active sensor imaging has been studied by an alternative TDF, which is based on the range-scanning angle. New target density function, $g(R,\beta)$ is composed of two variables, which are the range R, and the scanning angle β. Definition of $g(R,\beta)$ is given as follows.

Target density function is the limit of the ratio of the amplitude of the signal reflected from an infinitesimally neighborhood about the point (R,β) to the amplitude of the incoming signal. By this definition, the new TDF $g(R,\beta)$ is

$$g(R,\beta) = \lim_{d(\Omega)\to 0} \frac{A_r}{A_t} \qquad (8)$$

where $d(\Omega)$ is the diameter of the disc about the point $(R,\beta)\in\Omega$, A_r and $\$A_t$ are the amplitudes of the reflected and the transmitted signals, respectively.

In this definition, the TDF is relevant to the reflectivity of spatially, continuously distributed targets. This approach is different from the conventional TDF definitions stated early. Instead of ambiguity functions based on range-velocity variables, the imaging is taken by a new TDF with the range and scanning angle.

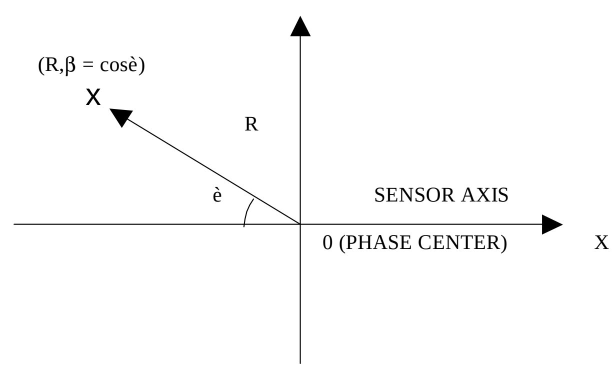

Figure 2. Radar imaging plane.

Let one consider the target plane shown in Fig. 2, where β is cos θ and R is the range from the target to the radar.

As seen in Fig. 2, the TDF is a function of the spatial coordinates (R,β) in the upper semi-plane. Now a new target density function is defined by utilizing Fig. 2. This function is the range density function (RDF).

3.1 Imaging by Range Target Density Function

In a radar system, if R is the range from the sensor in a fixed direction β, which is direction cosine of the line joining the point and the phase centre, as a new TDF, the new RDF is defined as follows:

Range density function, $g(R)$ is the reflectivity of the point at range R.

By this definition, $g(R)$ represents the image along the range or the distance to the sensor. Let one formulate this definition. The direction density function $g(R,\beta)$ at a fixed direction β is

$$g(R) \equiv g(R)_\beta \equiv g(R,\beta) \tag{9}$$

Let one write the TDF wrt N targets on a semi-upper plane in Fig. 2 as

$$g(R) = \sum_{i=1}^{N} g(R)\,\delta(R-R_i) \tag{10}$$

where (R-R_i) is the Delta Dirac function.

$$g(R) = g(R)(\sum_{i=1}^{N} \delta(R-R_i)) \tag{11}$$

Let $P(t)$ be the signal transmitted in the fixed direction β, which can be done by a directed single antenna or a phased array using beamforming[34-35].

$$P(t) = e^{j\omega_c t} \tag{12}$$

Then, the reflectivity of one point at $g(R)$ is

$$y(t) = P(t-2R/c)g(R) \tag{13}$$

where $y(\omega_c,t)$ is the output of the sensor located at the centre (the feature space), and c is the speed of light. Let one generalise Eqn (13) for the whole radar-target semi upper plane by superpositioning principle considering all point scatterers related to the range angle.

If $g(R)$ is the reflectivity of the point (R) at the fixed direction β, of the interest target area, then the total reflected incoming signal to the phase center will be

$$y(\omega_c,t) = \int_{-\infty}^{\infty} g(R)P(t-2R/c)dR \tag{14}$$

$P(t)$ includes ω_c term. Substituting Eqn (11) in Eqn (14) gives:

$$y(\omega_c,t) = \int_{-\infty}^{\infty} [\sum_{i=1}^{N} g(R_i)\,\delta(R-R_i)]P(t-2R/c)dR \tag{15}$$

Then the algorithm is as follows:

$$y(\omega_c,t) = \sum_{i=1}^{N} g(R_i)P(t-\frac{2R_i}{c}) \tag{16}$$

and $$y(\omega_c,t) = \sum_{i=1}^{N} g(R_i)\,e^{j\omega_c t}\,e^{-j\omega_c\frac{2R_i}{c}} \tag{17}$$

Upon demodulation Eqn (17) via $e^{-j\omega_c t}$

$$\bar{y}(\omega_c) = \sum_{i=1}^{N} g(R_i)\,e^{-j\omega_c\frac{2R_i}{c}} \tag{18}$$

is obtained.

For $\omega_c = 0$, $$\bar{y}(0) = \sum_{i=1}^{N} g(R_i) \tag{19}$$

then

$$\begin{aligned} \bar{y}(1) &= \sum_{i=1}^{N} e^{-j\frac{2\omega_c}{c}R_i}\, g(R_i) \\ \vdots\quad & \vdots \qquad\qquad \vdots \\ \bar{y}(\omega_c) &= \sum_{i=1}^{N} e^{-j\frac{2\omega_c}{c}R_i}\, g(R_i) \end{aligned} \tag{20}$$

Let $$Z_i = e^{-j\frac{2}{c}R_i} \tag{21}$$

then $\bar{y}(\omega_c) = \sum_{i=1}^{N} Z_i^{\omega_c} g(R_i)$ (22)

where $Z_i^{\omega_c}$ term in Eqn (22) is in a Wandermonde matrix form. Thus, it makes the calculation of $g(R_i)$ easy. Then, let write Eqn (22) in matrix form as follows:

$$Y=WG \tag{23}$$

where Y is the radar output function, W is Wandermonde matrix, and G is the desired TDF. Then, the desired result is obtained as follows;

$$G=W^{-1}Y \tag{24}$$

In case of W or Z^{ω_c} matrix with a large dimension, solution of the problem becomes very complex and requires an alternative computation. To reduce it to a smaller dimension, let consider the transmitted signal $P(t)$ as a band-limited as ($\omega \le \omega_c \le k\omega$). Then,

$$\omega_c = k\omega, \quad 0 \le k \le (n-1) \tag{25}$$

Now for $\omega_c = 0$, the steps after Eqn (18) are rewritten as follows:

$$\bar{y}(0) = \sum_{i=1}^{N} g(R_i) \tag{26}$$

then

$$\begin{aligned} \bar{y}(1) &= \sum_{i=1}^{N} e^{-j\frac{2\omega}{c}R_i} g(R_i) \\ \mathrm{M} \quad & \mathrm{M} \qquad \mathrm{M} \\ \bar{y}(k) &= \sum_{i=1}^{N} e^{-j\frac{2k\omega}{c}R_i} g(R_i) \end{aligned} \tag{27}$$

Let $Z_i = e^{-j\frac{2\omega}{c}R_i}$ (28)

then, $\bar{y}(k) = \sum_{i=1}^{N} Z_i^k g(R_i)$ (29)

i.e.

$$\begin{aligned} \bar{y}(0) &= g(R_1) + \Lambda + g(R_n) \\ \bar{y}(1) &= Z_1 g(R_1) + \Lambda + Z_n g(R_n) \\ \mathrm{M} \; & \mathrm{M} \quad \mathrm{M} \qquad \mathrm{M} \qquad \mathrm{M} \\ \bar{y}(k) &= Z_1^k g(R_1) + \Lambda + Z_N^k g(R_n) \end{aligned} \tag{30}$$

Because the right side of Eqn (30) is in a typical Wandermonde matrix form, it is rewritten in a matrix form as

$$\begin{bmatrix} \bar{y}(0) \\ \bar{y}(1) \\ \bar{y}(2) \\ \mathrm{M} \\ \bar{y}(k) \end{bmatrix} = \begin{bmatrix} 1 & 1 & \Lambda & 1 \\ Z_1 & Z_2 & \Lambda & Z_N \\ Z_1^2 & Z_2^2 & \Lambda & Z_N^2 \\ \mathrm{M} & \mathrm{M} & \mathrm{M} & \mathrm{M} \\ Z_1^{n-1} & Z_2^{n-1} & \mathrm{M} & Z_N^{n-1} \end{bmatrix} \times \begin{bmatrix} g(R_1) \\ g(R_2) \\ g(R_3) \\ \mathrm{M} \\ g(R_n) \end{bmatrix} \tag{31}$$

Iff

$$Y = \begin{bmatrix} \bar{y}(0) \\ \bar{y}(1) \\ \bar{y}(2) \\ \mathrm{M} \\ \bar{y}(k) \end{bmatrix}; \; \mathrm{W} = \begin{bmatrix} 1 & 1 & \Lambda & 1 \\ Z_1 & Z_2 & \Lambda & Z_N \\ Z_1^2 & Z_2^2 & \Lambda & Z_N^2 \\ \mathrm{M} & \mathrm{M} & \mathrm{M} & \mathrm{M} \\ Z_1^{n-1} & Z_2^{n-1} & \mathrm{M} & Z_N^{n-1} \end{bmatrix};$$

$$G = \begin{bmatrix} g(R_1) \\ g(R_2) \\ g(R_3) \\ \mathrm{M} \\ g(R_n) \end{bmatrix} \tag{32}$$

where Y is the radar output function, W is Wandermonde matrix, and G is the desired TDF. Then Eqn (31) is rewritten in matrix form as

$$Y=WG \tag{33}$$

By utilising the inverse Wandermonde matrix properties, the desired target density function G, is obtained in form of inverse Wandermonde matrix by multiplied the radar output function as follows:

$$G=W^{-1}Y \tag{35}$$

which is the desired result. Thus, using a novel target density function in form of the range density function $g(R)_\beta$, the radar targets can be imaged by utilising an inverse Wandermonde matrix approach.

3.2 Comparison

The developed technique here was inspired partly by analogy to Fowle-Naparst and SAR-ISAR approaches.

- Comparing to Fowle-Naparst approach: As an advanced work of Fowle's study, Naparst has developed a TDF for a high dense target environment with multiple targets, whose velocities are close to each other. This TDF acts like a separator rather than an imaging function for the targets at the distance with a given velocity. The significant difference arises from the imaging approach. However, the contribution of especially Naparst, to the new TDF studies is quite remarkable.
- Comparing to ISAR approach: While ISAR imaging is based on the integration of the point scatterers on the target in ISAR, the proposed TDF is produced by the integration of the ranges at the fixed scanning angles.

4. SUMMARY AND CONCLUSIONS

In this paper, an alternative TDF is obtained by a new algorithm and technique different from the conventional approaches. Two main contributions of this study are:

- *New Target Density Function Plane*: New imaging TDF was presented in a novel range and scanning angle plane.
- *Novel Direction-Range Target Density Function*: A new TDF was defined to imaging by active sensors in a variable direction angle and at a range.
- I*maging by a New TDF*: A new TDF which was called direction density function, was defined to imaging by active sensors in a variable range and at fixed-direction angles.
- *Proposed TDF Algorithm*: New range density function was produced by a new technique based on inverse Wandermonde matrix

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Contributor

Dr A. Demirkol received his BS (Electrical Engineering) from the Yildiz Technical University, Istanbul, Turkey. He received his MS and PhD in 1993 and 1997 respectively. During 1998-2003, he was an Assistant Professor in the Computer Engineering Dept of Sakarya University. Presently, he is working in the Dept of Electrical and Computer Engineering, University of Missouri-Rolla. His research interests include: Radar signal processing, radar imaging, and multisensor data fusion.