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Transfer Alignment Technique for Shipboard Missile Strapdown Inertial Navigation System using an Adaptive Kalman Filter

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ABSTRACT

Missile Guidance system needs accurate estimates from Inertial Navigation System (INS) for guiding the vehicle towards the target. In this paper a target point, specified before launch, in a battlefield scenario is considered for a landmark using missile Strap Down Inertial Navigation System (SDINS) aided by Master INS (MINS) placed on a moving platform. Azimuth information of the missile is one of the most critical navigation states for estimation on the moving platform before launching the missile for precise impact.

An Adaptive Kalman Filter (AKF) based on the error state model is formulated. The 7-state AKF with 4-measurement forms the core, where the filter gain of the innovation sequence (measurements) is evaluated. This approach of adaptively computing the gain is tested in a laboratory, on a van and in a ship trial, culminating in a successful guided missile launch. Mean and the covariance of the measurement residuals were used in a unique way to compute adaptive gain after the accumulation of initial samples.

A Master INS (with advanced Gyros) whose accuracy is much higher than the accuracy of the missile's SDINS is used for velocity matching algorithm before the launch with execution of an S-maneuver for generation of accelerations towards observing the states more appropriately. Estimated error states were used in a feedback mode to get near the true orientation of the Missile's slave INS. Error quaternions are used for this purpose in the feedback and the gains were selected using offline matrix Riccati equation solution in a discrete domain as used in the modern control system. The results were very encouraging with less than 5 arc minutes of error in azimuth.

Keywords: Missile guidance system; Inertial navigation system; Strap-down navigation algorithm; Adaptive kalman filter; Moving launch platforms; Velocity matching algorithm; S-maneuver; Implicit guidance scheme; Critical error probability

NOMENCLATURE

$\tilde{X}(t)$	State estimate at time t
$F(t)$	State transition matrix at time t
B	Forcing function
$U(t)$	Input at time t
$W(t)$	White zero mean guassian state noise
$\phi(t)$	State transition matrix at time t
$\tilde{Z}_k$	Observation vector at instant 'K'
H_k	Observation matrix at instant 'K'
V_k	White zero mean Gaussian observation noise
α,β,γ	Euler angle rates
V_N, V_W	North, West velocities
$\lambda, \wedge$	Latitude, Longitude
ϕ,θ,Ψ	Euler angles
γ_k	Innovation sequence
S_γ	Error covariance matrix
σ^2_{MIMU}	Variance of IMU
σ^2_X	State covariance
$\hat{R}$	Residue
Δq_i	Element of quaternion

1. INTRODUCTION

Modern missile guidance system needs accurate navigation for precise impact. Strap Down Inertial Navigation System (SDINS) is used for the typical fire and forget missile launched from a moving platform towards a land target. The principal aim is to reduce the azimuth error as well as other Navigation errors for this Strapdown Inertial Guidance System (SDIGS) where an implicit guidance Scheme based on perturbation technique is used[1]. Finding the azimuth angle on a moving platform is a major technical challenge. Alignment of SDINS is a pre-requisite for self-contained navigation and guidance for a mission-critical application. A master INS having

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highly accurate and stabilized gyros $(drift < 0.001^0 / hr)$, accelerometers ($bias < 50\mu g$) aided by gyrocompass is used for finding the azimuth.

The problem is to find out the relative azimuth of the missile's SDINS[2] with respect to the Master INS on a moving platform, which will be used to find out the near true orientation of the missile's SDINS. In addition, in a moving platform (like Ship) the pitch and roll rates are high and need a stabilization system to reduce undue oscillation of the launch platform.

Transferring the missile azimuth available on the ground to the SDINS of a missile is normally performed by optical methods using theodolites and available survey information of the launch point.

Presently similar survey information in Sea is available by the help of superior class Master INS, but the finding relative azimuth can be attempted by various methods. Optical method of finding relative azimuth being one of them.

A computer-based recursive Adaptive Kalman Filter (AKF) scheme is used for finding the missile SDINS azimuth in an elegant way in this paper. However, due to the poor observability of azimuth and the spread of noise in missile SDINS (with higher drift), an S-maneuver of the ship is required at a low frequency. The maneuver requirement is within the capability of the (less than $\pm 0.15\, m/s^2$ at $0.005\, Hz$ and maybe still less when the missile SDINS is better. SDINS without GPS using an optical-based alignment scheme is tried for surface-launched missiles.

An error propagation system involving the misalignment between Master and Slave INS is formulated for the velocity-matching algorithm using AKF based Transfer Alignment (TA). This results in lesser azimuth error leading to improved Navigation accuracy for the ease of precision Guidance. The result of the scheme led to lesser Circular Error Probability (CEP) for precise impact[3].

In this paper, the System configuration and Kalman Filter formulation for the TA scheme is presented and explained.

Typical simulation plans and results of trials are highlighted subsequently with appropriate references in section 5 of this paper.

2. LITERATURE SURVEY

Kalman filter as a navigation workhorse for estimation of the dynamic state vector is well documented in the literature. Small-angle dynamics mechanization for Kalman filter implementation is given by Bar Itzhack[4-6]. Transfer alignment for moving vehicles was conducted by Chaudhuri et. al. in their papers[7-9]. PK Nandi et. al. conducted detailed simulation studies for transfer alignment on oscillating moving base vide their papers[10-11].

However, both Chaudhuri and Nandi have not shown Kalman filter mechanization with LQG loop for variance estimation of the observation noise in adaptive Kalman filter mechanization.

This paper discusses the complete mechanization of Kalman filter structures for estimation of velocity-based attitude misalignment information including the necessity of S-manoeuvre in the lateral axis for the estimation of azimuth angle and down gyro bias. The velocity information is supplied by master and slave together and fused with the adaptive Kalman filter.

Paper[12] describes the basic principles of navigation. It is explained why attitude information is critical to the start of inertial navigation. Paper[13] explores robust strategies for in-motion inertial navigation explaining various statistically robust methods.

DARPA, US is relying on MEMS technology for positioning of small vehicles, for which attitude information will be necessary[14]. The need for ubiquitous inertial navigation is given in Stovall[15].

The errors of the slave system are not directly coupled with navigation outputs. For example, it is not possible to calculate gyroscope drift errors by just using the velocity differences between the two systems. This is because drift errors generate attitude errors and these attitude errors generate velocity errors. In order to estimate those indirect errors, an estimation algorithm that can process the observed differences for a sufficiently long time is necessary. The most significant factor that affects the design of such an estimation algorithm is the selection of vector types as given below that are compared. The choice of vector type specifies the overall structure of any estimation algorithm. Therefore, transfer alignment algorithms are classified according to the type of vectors compared to generating an estimate.

According to this criterion, transfer alignment algorithms can be classified as follows[16]:

- Acceleration / Rotation Rate Matching
- Velocity Matching
- Integrated Velocity Matching
- Attitude Matching
- Velocity and Attitude Matching
- Position Matching

In acceleration/rotation rate matching the samples were available at a high reception rate and were very noisy as the computation of the samples does not involve the integration process.

Hence the position/velocity matching algorithm is tried which gave a good sample with minimum noise in the measurements as they involve an integration process while calculating the samples. Also, the velocity matching algorithm does not assume rigid body assumption and can take care of lever arm and flexure moments, whilst the rotation/acceleration methods assume perfect rigid body for their working. The difference in velocity is used as input to the AKF for estimating the angular misalignments between the Master and Slave INS. The samples were available at a moderate timestamp which was suited very much for the computational requirements of the AKF.

The other methods like Integrated Velocity Matching Attitude Matching, Velocity, and Attitude Matching, Position Matching involve the further integration in the calculation of samples, which in turn introduces lag in the computations of AKF, due to which the filter became unstable and sometimes divergent[16].

3. SYSTEM CONFIGURATION FOR TA SCHEME

In case of a launch from a moving base like a ship, it is essential to align the INS in the missile (Slave unit) with that mounted on the Ship (Master unit). The Master INS is much more accurate, stable, and precisely calibrated over a period of time. Appropriate software is in-built which is aided by external instruments (Velocity log equipment for relative water velocity measurement and GPS systems). TA is the process of aligning Slave INS to the Master INS through velocity matching using natural or deliberately induced excitation (i.e., S-manoeuvre of a Ship) in the vertical direction before the vertical launching of the missile. The pitch and roll stabilization system provided in the Ship reduces the missile yaw and pitch rates before launch as shown in Fig. 1.

The S-manoeuvre of the Ship excites the sensors of both Master and Slave INS for the generation of data to be used in aligning the slave INS of SDINS.

plane is considered with sufficient observability under maneuver for the azimuth channel and the bias states.

The entire duration of the maneuver is reduced to less than 10 minutes with minimum effort of maneuvrings. The earth rotation rate is integrated into the maneuver by the gyro output during the maneuver. In the specified 10 minutes of manoeuvre, the earth's rotation rate is 2.5 degrees. In order to estimate misalignment, the differential output of slave INS and master INS is used as an input to AKF based TA algorithm.

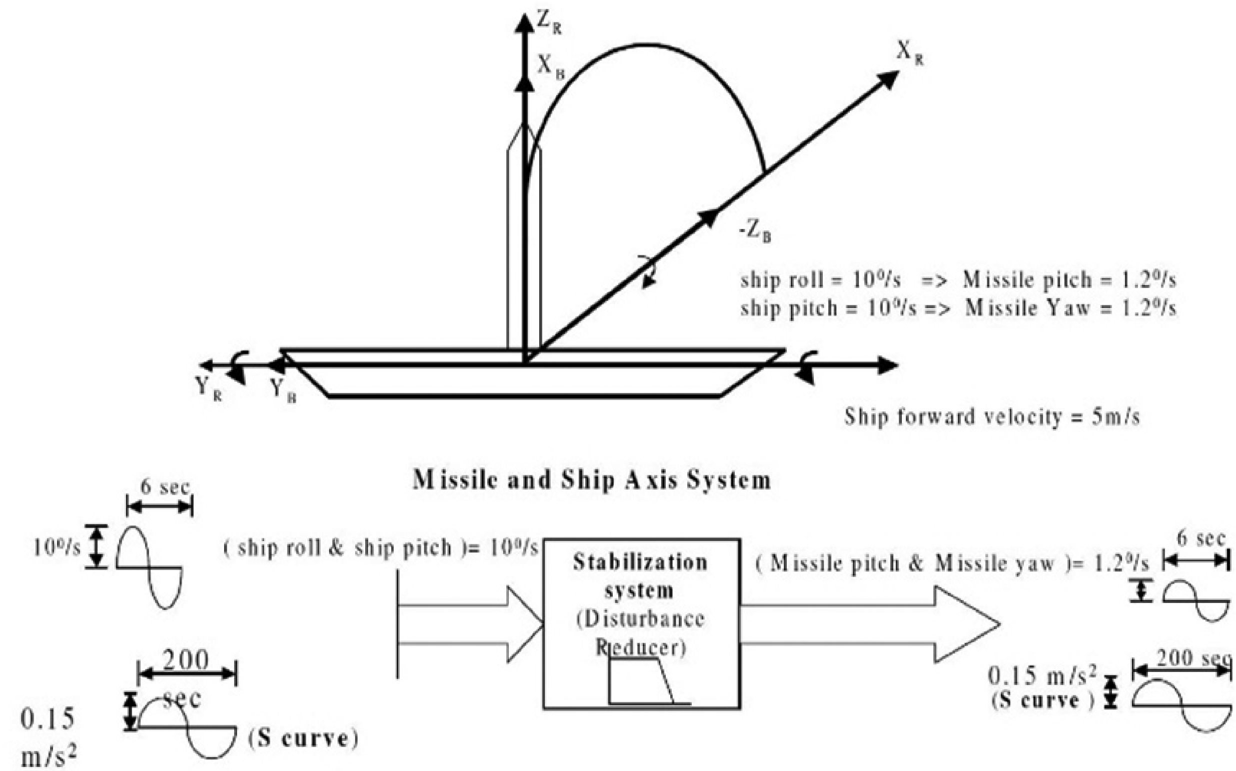

Figure 1. Excitation to master and slave imu by ship maneuvering.

There is a launcher stabilizer on the ship which isolates ship flexure and reduces the effect of the ship's roll-on missile's pitch and ship's pitch on the missile's yaw from $10^{\circ}/\sec$ to $1.2^{\circ}/\sec$. The missile's horizontal axes are nearly perpendicular to the ship's horizontal axes.

The co-located master INS (on stabilizer) and slave INS (inside missile) on the stiff stabilization system $(\approx 1Hz)$ allow very little lever arm motion during the S-manoeuvre performed by the ship in the azimuth. The physical motion excites the system states as shown in fig. 2. The master INS and slave INS as depicted in Fig. 2 give velocity and position outputs (i.e., $\tilde{V}_n, \tilde{\lambda}$).

The method of TA involves error modelling of both the systems and comparing two systems' outputs with the help of AKF. A conventional state estimator in the form of KF requiring exact process noise and measurement statistics is not suitable for this type of real-time system due to the requirement of a higher-order model and longer convergence time.

An error propagation system involving the misalignment between Master and Slave INS is formulated involving the 3-misalignment angles, 2-velocity errors, and 2-position errors. Error in north and west velocity in the horizontal

The estimator should be capable of identifying and learning the noise statistics online from the innovation sequence. An explicit adaptive optimal estimator (in the form of AKF) is used[7], which makes direct use of online identification of noise covariance. The estimated error states are used in a feedback mode to get near the true orientation of the Slave INS.

The standard Matlab based routines were used before selecting the final gains[17]. It was also proved that online computations of gains are not essential for this scheme[3]. The major part of the algorithm was introduced in an additional PC without disturbing the existing missile launch configuration.

This total engineering was done not to disturb the existing validated software and hardware configuration. Minimum additional software was introduced in the existing On Board embedded software.

4. FORMULATION OF KALMAN FILTER ALGORITHM

The KF used for TA has 7 states and uses 4 differential measurement outputs as shown in Fig. 2. The dynamic system associated with the Navigation process is fundamentally in continuous state space. Sensor signal drives the navigation equations, which are non-linear[16-17]. In order to apply linear filtering techniques linearization of these equations about a reference trajectory (S-manoeuvre) has been applied.

The states of the linear error model are North-West-Vertical (NWV) and have been already described. For very small misalignment angles vertical axis can be decoupled from the error model. However, height is updated from SDINS at a much faster rate via On-Board Computer (OBC).

The vector differential equation giving process Error dynamics may be described below in a stochastic form as given in Eqn. (1).

Reference outputs are expected to be aided by external instruments like LOG and GPS inputs from the ship. The detailed states of output and input along with feedback are shown in Fig.2. The process and measurement noises are assumed to be stationary and independent of each other, which are white Gaussian noises with a normal probability distribution.

where ϕ_k is the state transition matrix[1-2] based on the NWV frame.

The structure of the ϕ matrix need not be exactly known in the AKF formulation. Error in modelling of the ϕ_k is taken care of by the adaptive modelling of the Q matrix, as given in equation 4 below.

$\tilde{W}_k$ comprises of the variances in accelerometer and gyroscope noises. $\tilde{W}_k$ is given as $\begin{bmatrix} \sigma^2_{gyro}(3,1) \\ \sigma^2_{acc}(3,1) \end{bmatrix}$, with $\sigma^2_{gyro}\, and\, \sigma^2_{acc}$ being the random noise variances of gyroscope and accelerometer respectively.

G is the coupling matrix between process noise and the state of the system. The G matrix is given by

$$\begin{bmatrix} I \\ C_b^n(1:2,1:2) \\ zeros(2,3) \end{bmatrix}$$

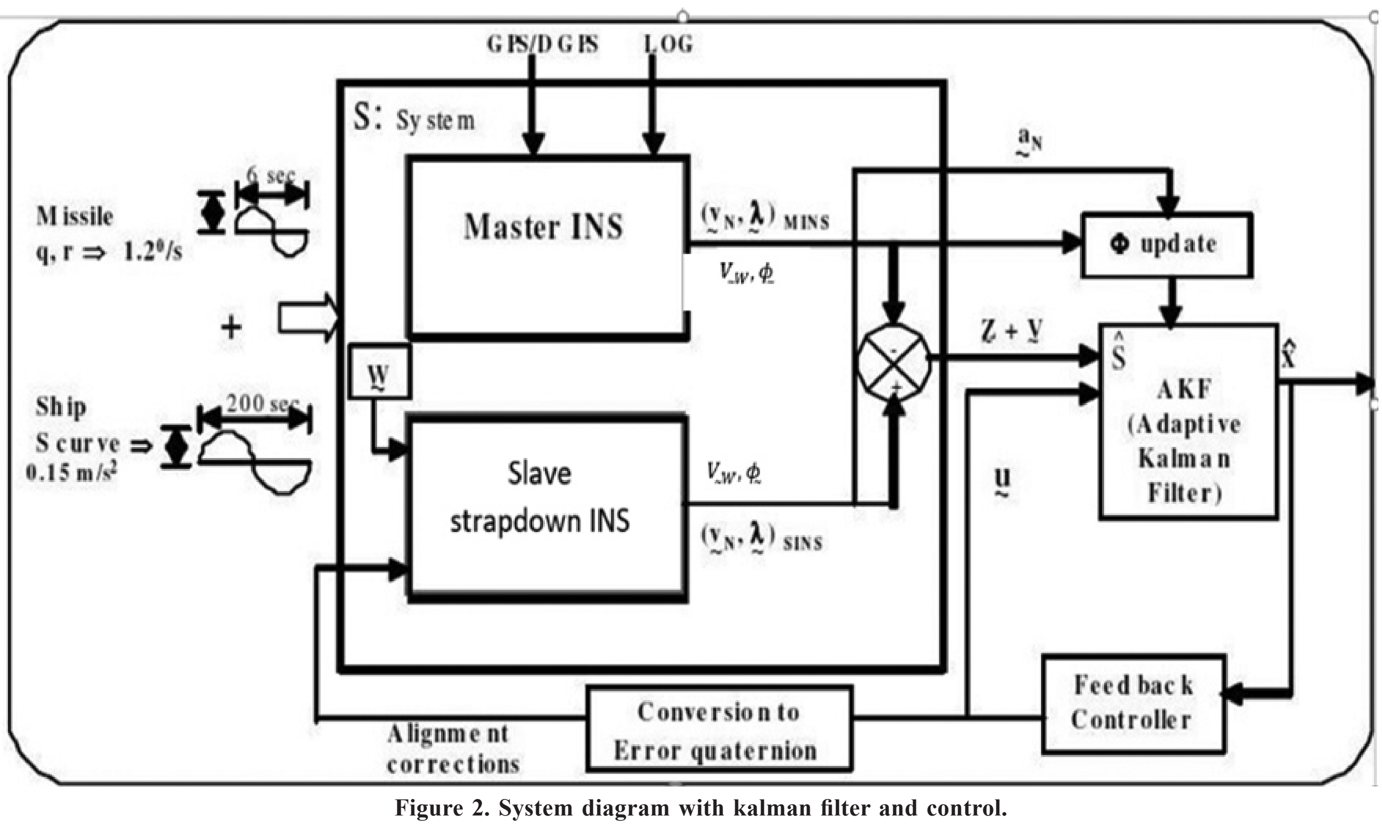

Figure 2. System diagram with kalman filter and control.

$$\tilde{X}(t) = F(t) * \tilde{X}(t) + B * \tilde{U}(t) + G\tilde{W}(t) \tag{1}$$

The sensor signals in slave IMU consist of substantial random error and may exceed the deterministic error covariance used in the formulation. A discrete form stochastic difference equation with a zero mean White Gaussian sequence W (k) is used,

$$\tilde{X}_{k+1} = \phi_k * \tilde{X}_k + B_k * U(k) + G\tilde{W}_k \tag{2}$$

The observation model consists of a linear equation of the error state as,

$$\tilde{Z}_k = H_k * \tilde{X}_k + \tilde{V}_k \tag{3}$$

Where $H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$

In Figure 2, the Kalman Filter is used with discrete form stochastic difference equation with a zero mean White Gaussian sequence $\tilde{W}_k$

$$\tilde{Z}_k = H_k * \tilde{X}_k + \tilde{V}_k \tag{3}$$

Where, $\tilde{X}_k$ is the state vector $\tilde{X}_k = \begin{bmatrix} \alpha \\ \beta \\ \gamma \\ \Delta v_N \\ \Delta v_W \\ \Delta\lambda \\ \Delta\wedge \end{bmatrix}$

$\tilde{W}_k$ is the Plant noise from SINS gyros

$\tilde{Z}_k$ is the measurement vector

$$= \begin{bmatrix} v_N \\ v_W \\ \lambda \\ \wedge \end{bmatrix}_M - \begin{bmatrix} v_N \\ v_W \\ \lambda \\ \wedge \end{bmatrix}_S = \begin{bmatrix} \Delta v_{N\wedge} \\ \Delta v_{W\wedge} \\ \Delta\lambda_\wedge \\ \Delta\wedge_\wedge \end{bmatrix}$$

$\tilde{V}_k$ is the Measurement noise from SINS accelerometers.

B_k is the control input selection Matrix

$\tilde{U}_k$ is the control input vector

$$\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} \Delta\phi \\ \Delta\theta \\ \Delta\psi \end{bmatrix}$$

The state transition matrix ϕ_k of the system[16] comprises of seven error states, i.e. 3-angular errors, 2-velocity errors, 2-position errors in the form of a column vector $[\Delta\phi, \Delta\theta, \Delta\psi, \Delta V_N, \Delta Vw, \Delta\lambda, \Delta\wedge]^T$.

The measurements are the errors in velocities and position between master INS and slave INS. There are 4-measurements consisting of 2-velocity errors, 2-position errors given by a column matrix as $[\Delta V_N, \Delta Vw, \Delta l, \Delta\wedge]^T$. The state transition matrix propagating the state of the system between instants of time. Since this is only an estimator without state vector augmentation, the B_k matrix is considered null matrix. w_k is white gaussian zero mean noise considered to be arising due to modelling error with $E\left[w_k w_k^T\right] = Q$, the process noise covariance. The process noise covariance matrix has the same dimensions of the state vector. z_k is the measurement of difference between the velocities of $\Delta v_n, \Delta v_w$ being the north and west velocity errors $\Delta\lambda$, $\Delta\wedge$ being the latitude and longitude errors.

The stochastic disturbance vectors $\tilde{W}_k$ and $\tilde{V}_k$ are treated as independent non-stationary Gaussian noise sequences with following properties:

$$E\left[\tilde{W}_i\right] = q_i,\ E\left[\left(W_i - q_i\right)\left(W_j - q_j\right)^T\right] = Q_i\delta_{ij} \tag{4}$$

$$E\left[\tilde{V}_i\right] = r_i,\ E\left[\left(V_i - r_i\right)\left(V_j - r_j\right)^T\right] = R_i\delta_{ij} \tag{5}$$

where,

$$\delta_{ij} = 0 \ \ if \ \ i \neq j$$
$$= 1 \ if \ \ i = j$$

q_i, r_i are true means, Q_i, R_i are true moments about the mean of the states and the observation sequences.

The state and output equations used for the INS error model in the TA scheme are given in the above two equations (2) and (3). The DCM computation, velocity update, and position updates are carried out in the On Board Computer (OBC) for getting SDINS system normal output$(\tilde{V}_N, \tilde{\lambda})$s. It is advisable not to use SDINS accelerometer noise in the process noise term for writing the state propagation equation [i.e., ΔV_n and ΔVw]. Further, handling measurement noises $\tilde{V}_k$ in KF formation may include Slave IMU (SIMU) accelerometer latitude, longitude noises.

These will help in considering the MINS as a reference and the complete AKF based TA's accuracy will be dictated by master performance only.

Slave INS (missile IMU) will be different from true value due to the presence of slave INS Gyro and Accelerometer noises and biases. Further, they get aligned to the proper NWV frame from the misaligned N W V frame based on error quaternion [□□q], which is fed back from the KF estimator (ref. Fig. 2). Quaternions are extracted from small angles due to ease of propagation of the quaternion with available rates. The feedback controller trims the feedback and the feedback applied is not instantaneous as is the case when a feedback controller is unavailable. The availability of feedback controller reduces the transient fluctuations of the filter. Propagation of these states based on the slave INS different from the true value is based on the state transition matrix (ϕ)[16] which gets updated based on the master INS $(\tilde{V}_N, \tilde{\lambda})$ and slave INS $(\tilde{a}_{NWV})$.

The estimated states i.e. angular error, velocity error, position errors $[\alpha, \beta, \gamma, \Delta V_N, \Delta V_W, \Delta\lambda, \Delta\wedge]^T$ are fed back from the KF at the end of each Measurement Update (MUP) and is used to turn the misaligned frame of the slave to true NWV frame INS during TA as shown in the Fig. 2.

The recursive discrete KF, which runs indefinitely, leads to a divergence problem. The basic sources may be modelling error, round-off error, and Observability problems. Round-off error and Observability problems can be taken care of by appropriate states, measurement, and computational states. Since no exact mathematical model is possible in the case of moving base INS alignment problem, therefore adaptive KF gives better performance by estimating the measurement noise under maneuver which is capable of tuning itself to an ambient real-life situation, thus giving a better accuracy in Euler angle estimation.

In AKF exact knowledge of (ϕ, H, Q, R) matrices are not essential. In the real-world application of KF, one of the quantities available in judging filter performance are the residuals and their predicted statistics. Normal adaptive filter models provide feedback from residuals.

4.1 Adaptation of Innovation Sequence

Mehra[20] considered several techniques to estimate error covariance matrices. This was followed further in the next decade by several researchers in this field. Meyers and Tapley's[21] approach for Adaptive Limited Memory Filter (ALMF) for implementation of innovation correlation is followed in this case [11]. A variant of this algorithm is implemented for the present AKF, where the innovation sequence (measurement vector) is adapted to minimize the observability problem and faster convergence of the filter.

Empirical estimators are derived in a 'batch' form under the assumption of constant values for $\hat{q}_i \hat{Q}_i \hat{r}_i \hat{R}_i$ over N successive stages of noise samples. The measurement residuals can be expressed as:

$$\gamma_k = \tilde{Z}_k - H_k \tilde{X}_k^- = H_k \left[\tilde{X}_k - \tilde{X}_k^- \right] + \tilde{V}_k \qquad (6)$$

An Unbiased estimator for γ, first-order moment of measurement noise is taken as the sample mean:

$$\hat{\gamma}_k = \frac{1}{N} \sum_{k=1}^{N} \gamma_k \qquad (7)$$

An unbiased estimator for R is obtained by, first constructing an estimator for S_γ, the covariance of measurement residuals as:

$$\hat{S}_\gamma = \frac{1}{N} \sum_{k=1}^{N} [\left(\gamma_k - \hat{\gamma}_k\right)\left(\gamma_k - \hat{\gamma}_k\right)^T] \qquad (8)$$

The expected value of this quantity can be shown as:

$$E\left[\hat{S}_{\tilde{a}}\right] = \frac{1}{N} \sum_{k=1}^{N} [H_k P_k^- H_k^T] + \hat{R} \qquad (9)$$

The expected value of covariance, as given in Bar Shalom[24] is

$$\left(\tilde{Z} - H_k \tilde{X}_K\right) = \sigma_X^2 - \sigma_{MIMU}^2$$

$$\tilde{R} = E\left[\hat{S}_{\tilde{a}}\right] - \frac{1}{N} \sum_{k=1}^{N} [H_k P_k^- H_k^T]$$

$$\hat{R} = \hat{S}_{\tilde{a}} - \frac{1}{N} \sum_{k=1}^{N} [H_k P_k^- H_k^T]$$

$$\hat{R} = \frac{1}{N} \sum_{k=1}^{N} [\left(\gamma_K - \hat{\gamma}_K\right)\left(\gamma_K - \hat{\gamma}_K\right)^T] - \frac{1}{N} \sum_{k=1}^{N} [H_k P_k^- H_k^T] \qquad (10)$$

These estimates are based on N samples of $\gamma_k, k = 1....N$, which is assumed to be statistically independent. The Adaptive Kalman Filter (AKF) uses 30 samples in a moving window and uses Joseph form for Kalman Filter gain computation [6]. Initially, for the first 30 samples collection, non-adaptive KF is used. After getting the 30 samples, AKF is used with 30 samples moving window for adaption.

The state transition matrix is also updated during the time update cycle. The 30 samples moving window was chosen after studying 10 and 50 samples moving window. With 10 samples the filter was diverging because of fewer samples and with 50 samples the convergence was taking a lot of time for gathering samples and processing the same. It was found from using the chi-square distribution model, that an ensemble of more innovation sequences is advantageous. However, more time will be required to switch over from non–adaptive to adaptive KF during S–maneuver. This is not a desirable situation since it is shown that non–adaptive KF gives rise to larger errors[7][11]. It was suggested[11] to modify the measurement update of the state below where $\hat{\gamma}_k$ is the additional term

$$\tilde{X}_k^+ = \tilde{X}_k^- + K_k \left[\gamma_k - \hat{\gamma}_k \right] \qquad (11)$$

Where $K_k = PH^T (HPH^T + R)^{-1}$. Refer Bar-Shalom[17].

The above modification will remove the undue bias and the same was tried in the 7-states AKF for TA trials. After deploying the adaptive estimator, the error feedback gains are selected using Linear Quadratic Gaussian Regulator and offline Matrix Riccati equation solution in the discrete domain [8,10].

Final gains are selected using standard Matlab routines computed offline[3,10].

The control vector which is the output of the LQG regulator, $\tilde{u}_k = -K \tilde{X}_k$ as described in Fig. 2. The control vector is feedback to the slave INS after the LQG regulator for attenuating the transient behaviour of the Kalman filter estimator of Euler angles.

The navigation computer computes and propagates the attitude in quaternion format. Hence the error angles obtained as estimates from the AKF are to be converted into quaternion format and feedback to the navigation solution. The alignment corrections $(\tilde{u})$ are thus converted to the error quaternion (ref. Fig. 2) by the following approximation:

$$\left[\Delta q_0 \; \Delta q_1 \, \Delta q_2 \, \Delta q_3\right]^T = \left[\frac{u_1}{2} \; \frac{u_2}{2} \; \frac{u_3}{2} \right]^T \qquad (12)$$

The AKF based TA algorithm with the formulation mentioned in this section along with the alignment correction of the error quaternions was implemented. After a detailed design and a number of simulation runs, it was decided to go for a measurement update at an interval of 1.8s (much faster than ship roll and pitch period of oscillation) and a time propagation step size of 0.36s[2].

5. SIMULATION SETUP AND RESULTS

Various new testbeds were designed in the lab, van, and ship (using a similar fixture, dummy aerospace vehicle as well as real aerospace vehicle) before the final launch. A test fixture was used to mount both master INS and slave INS. Several ship trials were performed under various Sea states and the spread of results for azimuth error was found to be less than 5 arc minutes when proper S-manoeuvre was performed by the ship [22] and slave IMUs. Master IMU was mounted with its corresponding original plate. The fixture was used in Laboratory as well as Van trials. The same fixture was mounted during the Ship trial directly on the stabilizer for carrying out TA before putting Slave IMU in the actual missile.

This result of TA was also validated optically by measuring the relative azimuth angle between both IMUs using an alignment tool in place of Master IMU and slave IMU with its poro-prism for lab, van, and ship trials. Finally, the same optical validation method is used during ship trial. The master IMU is placed on the stabilizer with its corresponding original reference plate and slave IMU in the missile.

The test configurations for TA are as follows:

- Fixture with both IMUs in static condition
- Fixture with both IMUs in Van
- Fixture with both IMUs on Ship stabilizer
- Slave IMU in dummy missile with Master
- Slave IMU in real missile and Master IMU on the stabilizer

All the above tests were performed chronologically to have minimum problems with the last configuration which will be used during a real launch. Further, the data from the ship trial was brought into Hardware In Loop Simulation (HILS) set up to find any inherent bugs in the software or in the concept. The Slave IMU in loop 6 DOF simulation run was performed

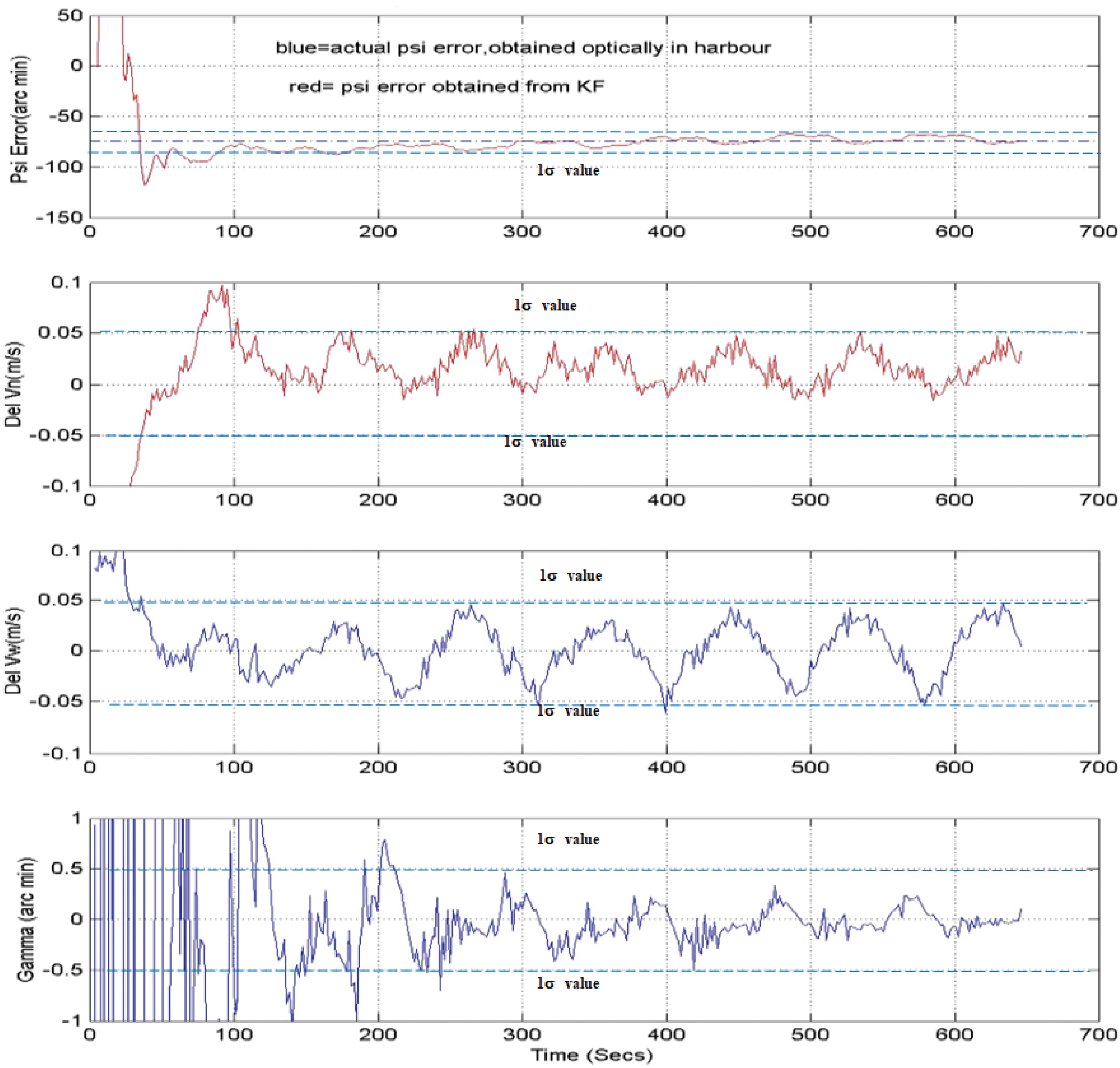

Figure 3. TA Result – the convergence of azimuth misalignment between master and slave IMUs and velocities.

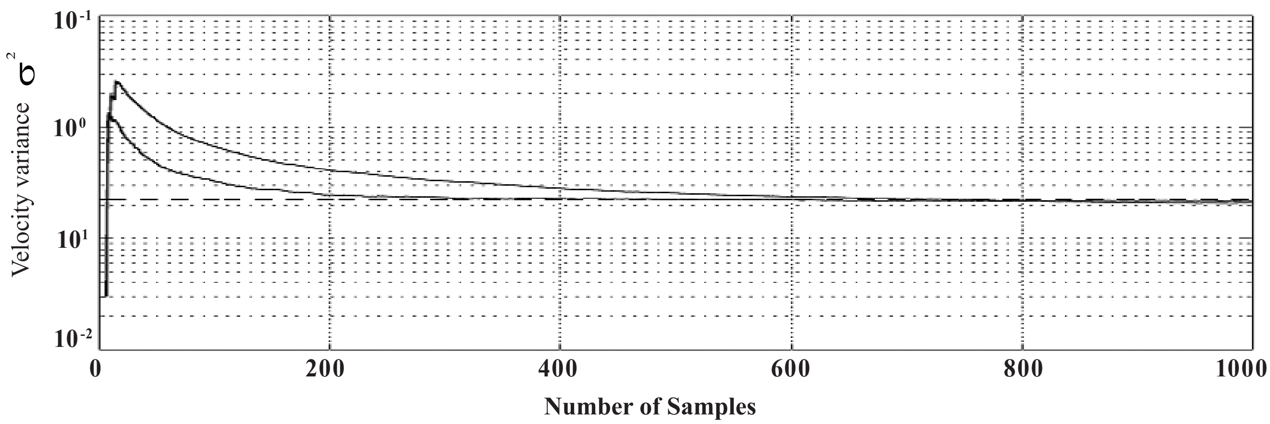

Figure 4. The adaptive measurement noise variance of diagonal elements.

with initial quaternions from Ship TA results and oscillations of the simulated Ship using Missile Motion Simulator (MMS). In addition, AKF with ϕ matrix simulating Master IMU in PC testbed was tried independently. It may be highlighted that linear acceleration cannot be given to MMS for generating S-curves for testing TA software.

Therefore, Ship trials for validating TA in realistic conditions became a necessity, where S-curve is given to tackle the unobservable azimuth problem. A typical TA trial test result is given in Fig. 3, which shows the proximity of converged misalignment between Master and Slave IMUs in azimuth.

The value was validated by optical measurements performed near the shore in the calm Sea. The convergence was achieved within 250 sec. The velocity matching in the North and West directions is near 0.01 m/s and AKF corrections are observed in γ (i.e. an error in azimuth). Due to randomness in nature and real-time data undersea wave simulation conditions, the data is oscillatory and the mean is shifted from '0' mean to 0.025m/s which is well within the acceptable band of $\pm 0.05 m/s$, where the convergence was good and the navigation results were satisfactory.

The Figure 4 shows the on-line estimated diagonal elements (Del V_N and Del V_W) of measurement noise variance, where dotted line is the true variance. Each line represents the time evolution of the diagonal elements. Fig.4 depicts the variance in the states of error velocities in North and West directions. They do not necessarily represent the error variances of the measurement and process noises (q_i, r_i).

6. CONCLUSION

The efficacy of an implementable data fusion scheme using AKF for TA is validated, which shows its usefulness for the missile guidance system. The reduced-order AKF devoid of vertical velocity error state for TA has shown that an explicit optimal estimator using online identification of noise covariance from the innovation sequence based on stable mean is capable of giving less than 5 arc minutes azimuth in moving platform.

Further, the poor observability of azimuth and spread of noise in missile SDINS demands S-manoeuvre of the Ship at a low frequency. The idea may be tried with Fuzzy logic for future developments. AKF with the reduced number of states may be attempted in the future upgrade.

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