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Maximizing the Number of Spatial Nulls with Minimum Sensors

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ABSTRACT

In this paper, we attempt to unify two array processing frameworks viz, Acoustic Vector Sensor (AVS) and two level nested array to enhance the Degrees of Freedom (DoF) significantly beyond the limit that is attained by a Uniform Linear Hydrophone Array (ULA) with specified number of sensors. The major focus is to design a line array architecture which provides high resolution unambiguous bearing estimation with increased number of spatial nulls to mitigate the multiple interferences in a deep ocean scenario. AVS can provide more information about the propagating acoustic field intensity vector by simultaneously measuring the acoustic pressure along with tri-axial particle velocity components. In this work, we have developed Nested AVS array (NAVS) ocean data model to demonstrate the performance enhancement. Conventional and MVDR spatial filters are used as the response function to evaluate the performance of the proposed architecture. Simulation results show significant improvement in performance viz, increase of DoF, and localization of more number of acoustic sources and high resolution bearing estimation with reduced side lobe level.

Keywords: Uniform line array; Acoustic vector sensor; Nested array; Spatial nulls; Conventional and MVDR beamformer

NOMENCLATURE

AVS	Acoustic Vector Sensor
DoF	Degrees of Freedom
NAVS	Nested AVS
ULA	Uniform Linear Hydrophone Array
MVDR	Minimum Variance Distortionless Response
KR	Khatri-Rao
DOA	Direction of Arrival
APS	Acoustic Pressure Sensor
NAPS	Nested Acoustic Pressure Sensor
CBF	Conventional Beamformer
SNR	Signal to Noise Ratio
ESPRIT	Estimation of Signal Parameters via Rotational Invariance Technique
MUSIC	Multiple Signal Classification
l	Minimum wavelength of Interest
$\otimes$	Kronecker Product
$\odot$	KR Product

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1. INTRODUCTION

Passive underwater coastal surveillance plays an important role in military applications to locate the underwater objects. In a conventional passive sonar architecture, the detection and localization performance is primarily confined by how many hydrophones are used or the effective length of that array. It is well known that the array gain, angular resolution, DoF, and capability to mitigate spatial interferences improves as the number of hydrophones in the array increases[1]. In general, hydrophones in the array are omnidirectional and pick up ambient noise in addition to the acoustic s ignature of the targets.

In this work, using a limited number of sensors, we attempt to design a non-uniform line array architecture which provides precise information about the target location in the presence of multiple interference signals. We propose a two-level nested array structure (two ULAs with different inter-element spacing) which is capable of enhancing the achievable DoF significantly beyond the limit set by a conventional ULA. This in turn can resolve sources more than its actual physical sensors[2]. The proposed architecture consists of six sensors with two levels of nesting - two ULAs, each consists of three sensors. Usually, a ULA consisting of N hydrophones is capable of positioning N-1 nulls in the angular spectrum. However, beyond the conventional limit N, the nested array is able to increase the DoF. The difference co-array of the actual array is realizable using the nested array which resembles to a longer virtual array. This is achieved with the use of second order statistics of the received signals. A better spatial resolution as well as suppression of jammers during beamforming can be achieved due to the enhancement of co-array order from $O(N)$ to $O(N^2)$[3].

An underwater passive source localization system traditionally uses an array of hydrophones to measure the spatial distribution of pressure. The azimuth and elevation angles of the targets are estimated by electronically steering the array in space. However, a single AVS is able to measure the acoustic signal pressure and particle velocity in three directions, aiding a complete characterization of the sound field and thus leading to a better localization performance. The notable advantage of AVS compared to the standard acoustic pressure sensors is that the directionality permits the AVS array to expand the precision of detection of target as well as localization of source, without the need of aperture expansion. Second advantage points to the left and right ambiguity issue that pressure sensor array suffers, never arises here[4]. The paper effectively combines the characteristic potential of an AVS with nested array. The resulting array model holds the advantage of enhanced DoF as well as high resolution localization performance.

The key contributions of this paper are encapsulated as: 1. Amalgamation of two array processing frameworks to design a compact line array with enhanced DoF 2. Theoretical development of two level NAVS array data model 3. Demonstration of narrow beam formation for accurate detection of targets along with increased number of spatial nulls, to enhance the interference rejection capability using a specified number of sensors.

Earlier works focused on detecting more sources than the sensors in different ways. In[5], the DoF are improved by creating an augmented covariance matrix and by the use of minimum redundancy arrays (MRAs). Still, for finite number of snapshots, the constructed augmented covariance matrix is not positive semi-definite. The augmented array method explained in[5] or that is done in reference 6, which constructs a suitable positive definite Toeplitz matrices, it is difficult to induce numerous DoFs by means of a ULA. It was later shown in reference 7 that, the AVS array aperture can be extended with the acoustic particle velocity information. Furthermore, the proposed AVS array compared to the standard acoustic pressure sensor array is able to afford unambiguous estimation of target bearing [8-9]. Recently, in reference 10 one can recognize up to $2N\text{-}1$ targets by an N element ULA with the assumption that sources are quasi stationary and with the notion of Khatri-Rao (KR) product deprived of higher-order statistics computation.

Previous works on AVS were carried out for DOA estimation such as Capon based method[11], Estimation of Signal Parameters via Rotational Invariance (ESPRIT)[12-14], self-initiating multiple signal classification (MUSIC) method[15], Quaternion method[16], successive MUSIC method[17], tri-linear decomposition-based method[18] and propagator-based method[19]. Even though these methods can provide good bearing estimation performance, the array geometries utilized are the same as ordinary ULA. Nested array concept has been applied in AVS array[20], which estimates signal parameters via tensor modelling. Though the pressure and vector sensors in AVS are collocated, resulting advantages have not been fully exploited here.

In this paper, a totally passive set-up with enhanced DoF based on the idea of difference co-array is explored. Some previous studies[21-22] dealt with specific array geometries with co-array concept. The proposed scheme is compared to the earlier methods as well as the more recent one based on KR product.

The outline of the paper is as follows. A brief description on AVS array and nested array is given in sections 2 and 3 respectively. The corresponding acoustic field models are described in section 4. The conventional and MVDR beamforming schemes are illustrated in section 5, followed by the methodology adopted in section 6. Section 7 discusses the simulation results and Section 8 concludes the findings.

2. ACOUSTIC VECTOR SENSOR ARRAY

Underwater acoustic systems mostly use an array of scalar pressure sensors in order to estimate the geometrical coordinates of the sources by spatially sampling the propagating pressure field. However, the fundamental energy transfer in the medium is governed by the vector wave equation, which itself suggests the use of an AVS, to effectively measure the propagating signal vector. An AVS comprises of a hydrophone and tri-axial accelerometer by which the hydrophone detects the acoustic pressure and the latter detects the particle velocity in Cartesian coordinates x, y and z directions. An Acoustic Pressure Sensor (APS) measures only the pressure values, hence only a part of information can be obtained. Whereas an AVS is capable of measuring three Cartesian components of particle velocity, in addition to the pressure field variables. This aids the complete characterization of acoustic field which in turn improves the localization performance. To measure the tri-axial particle velocity, AVS must be designed to be neutrally buoyant in the water. Though AVS array processing techniques utilize lesser number of sensors, the hardware and the channel number are similar to the APS array. This happens because the sensor number drop in the AVS array is balanced by the number of measurements per sensor increment.

The pressure sensor which is omnidirectional responds equally in all directions, while the vector sensor gathers data about the sound source direction. Put another way, “all directions are ambiguous to the pressure sensor, but no directions are ambiguous to the vector-sensor”[23]. This means that a single vector sensor is inherently directional. Pressure-sensor is capable of resolving signals and rejecting noise by exploiting the time or phase delay computations. Since the sensor components in vector sensors are co-located, it can offer extra phase information, also the gain information is gathered by directional components. Thus, the vector sensor array is said to be more effective, when its processing is capable of exploiting both phase and gain quantities.

Consider a plane wave parameterized by azimuth $\varphi \in [0, 2\pi]$ and $\psi \in [-\pi/2, \pi/2]$ be the elevation which impinges on a vector sensor array with N sensors. The unit vector $\boldsymbol{u}$, points from the origin to the source, i.e.,

$$\boldsymbol{u} = [\cos j \ \cos y \ , \sin j \ \sin y \ , \sin y \ ]^T \tag{1}$$

The acoustic pressure and particle velocity is related through Euler's conservation of momentum equation, i.e.,

$$\frac{\partial \boldsymbol{v}}{\partial t} + v^T \nabla \boldsymbol{v} = \frac{-\nabla P}{r} \tag{2}$$

where r is density, $\boldsymbol{v}$ is fluid velocity, and P is pressure. The term $\boldsymbol{v}^T \nabla \boldsymbol{v}$ (convective acceleration term) is neglected and this equation is linearized for acoustic propagation [24]. Pressure P relate to time and position $\boldsymbol{x}$ through sound speed c, with a plane wave consideration:

$$P(\boldsymbol{x}, t) = f\left(\frac{u^T \boldsymbol{x}}{c} + t\right) \tag{3}$$

$$\nabla P = \frac{\boldsymbol{u}}{c} \frac{\partial P}{\partial t} \tag{4}$$

Substituting equation 4 in equation 2 gives,

$$\boldsymbol{v} = \frac{-\boldsymbol{u}}{r\,c} P \tag{5}$$

The velocity vector $\boldsymbol{v}$ can be measured due to the alignment of accelerometers with the coordinate axes. Thus, the AVS array performance analysis simplifies to a great extend because of the linear relationship between pressure and particle velocity components.

3. TWO LEVEL NESTED ARRAY

Nested array when compared to other arbitrary sensor arrays has higher DoF as well as a structure with closed-form expression.[25, 27-28] Basically, two ULAs are concatenated together to form a two-level nested array with an inner and outer ULA. Inner one has N_1 elements and having d_1 spacing whereas the outer ULA has N_2 elements and d_2 spacing which is equal to $(N_1+1)d_1$ [3]. The structure of two-level NAVS array is depicted in Fig.1, used for simulating the array data vector to demonstrate the increasing the number of spatial nulls with minimum number of sensors. Then the sensor locations become;

$$S_{inner} = \{ad_1, a = 1, \ldots\ldots, N_1\} \text{ and } S_{outer} = \{bd_2, b = 1, \ldots\ldots, N_2\}$$

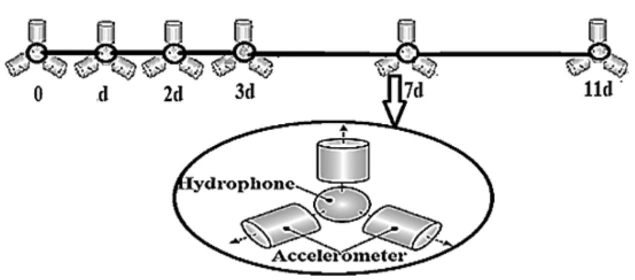


Figure 1. A two-level NAVS array with three sensors per level.

. Thus, a difference co-array is obtained which is positioned according to the following expression with $2N_2(N_1+1)-1$ elements,

$$S_{dca} = \{bd_1, b = -M, \ldots\ldots, M, \ M = N_2(N_1+1)-1\} \tag{6}$$

3.1. The Difference Co-array

An array of N sensors is considered having the position vector of say, i^{th} sensor as $\boldsymbol{x}_i$. Now the set D is defined as,

$$D = \{\boldsymbol{x}_i - \boldsymbol{x}_j\} \tag{7}$$

Where $i, j = 1, 2, \ldots\ldots, N$ .Let D_u denotes the distinct elements from D, then the given array has a difference co-array having sensors positioned at locations given by D_u. The signal received by the array is having a covariance matrix having cross-correlation terms with distinct values which is directly decided by the number of elements in the difference co-array[26].

4. NAVS ARRAY DATA MODEL

This section presents the mathematical formulation of a measurement model which accurately reflects the acoustic field received by a two-level NAVS array. The acoustic field originates from spatially distributed sources and thus the energy travels through the medium supported by multi-mode acoustic wave propagation. The received array data vector is generated by modelling the signals radiated from the acoustic sources, the medium characteristics and the ambient noise.

According to[25], NAVS array holds more relevance than a co-prime array because of its closed-form expression of structure and enhanced DoF. The mathematical manipulation of AVS can be better understood in this section by explaining array model concepts. This is followed by a systematic presentation of the development of the mathematical expression for generating two-level nested array data vector, with assumptions made for the sensor array, signal sources and ambient noise. The foremost assumptions[11, 29] include;

- The hydrophone and three accelerometers are located in space at the same point and the alike state is detected. This assumption is valid only if the sensors are packaged very close to each other compared to the minimum wavelength of operation.
- The output of every accelerometer gives a proportional relation between the cosine of the angle between the axis and the source. This ensures each accelerometer maximally responds to particle acceleration along its axis.
- Sound waves follow direct-path propagation only, or ocean is an unbounded medium.
- The signal is modelled as a slowly varying time signal occupying small bandwidth $\Delta\Omega$ such that the variation in the envelope can be neglected during the time taken by the plane-wave to pass the array. This meets the requirement of narrowband assumption in array processing and is expressed as $\Delta\Omega . t_{max} \odot$, where t_{max} is the maximum travel time across the array

- The unit vector to the target from each of the sensor is similar irrespective of the location of elements. This is because at each sensor and across its array, the sound waves remain planar.

4.1. AVS Array Data Model

The physical dimension of every AVS within the hybrid array is as small as possible to measure acoustic vector field ρ, v_x, v_y, v_z at some extent in space. The measurement at each AVS is taken to be a 4-dimensional vector, the acoustic pressure P and therefore the three velocity components v_x, v_y, v_z. But v_z is neglected since the vertical component of the shallow ocean ambient noise is much higher. The particle velocity measurements are scaled by the factor $\sqrt{2}\rho c$ to render the measurements dimensionally uniform. Consider an AVS array with *N* sensors and *J* sources with θ_j because the azimuth angle that the j^{th} source makes with the array axis. The law Conservation of momentum governs the acoustic pressure P and particle velocity v relation at time t and at a point $r = (x, y, z)$, i.e.,

$$\rho \frac{\partial \boldsymbol{v}(r,t)}{\partial t} + \nabla P(r,t) = 0 \tag{8}$$

At a time sample t, the signal vector received at the array can be given by,

$$\boldsymbol{x}(t) = \boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{n}(t) \in C^{3N\times 1} \tag{9}$$

where,

$$\boldsymbol{A} = \left[\boldsymbol{a}(\theta_1)\boldsymbol{a}(\theta_2)\ldots\ldots\boldsymbol{a}(\theta_J)\right] \tag{10}$$

Here **A** is the array manifold matrix with size $3N \times J$ and $\boldsymbol{a}(\theta j)$ represents the steering vector of the source direction θ_j also, $\boldsymbol{a}(\theta j) = \boldsymbol{c}(\theta j) \otimes \boldsymbol{d}(\theta j)$, here $\otimes$ symbolizes the Kronecker product.

$$\boldsymbol{c}(\theta j) = \left[1 e^{-jkd\cos\theta_j} \ldots\ldots e^{-j(N-1)kd\cos\theta_j}\right]^T \tag{11}$$

where $k = 2\pi/\lambda$ and,

$$\boldsymbol{d}(\theta j) = \left[1, \sqrt{2}\rho c\cos(\theta_j), \sqrt{2}\rho c\sin(\theta_j)\right] \tag{12}$$

The source signal vector is given by,

$$\boldsymbol{s}(t) = \left[s_1(t), s_2(t), \ldots\ldots s_J(t)\right]^T \tag{13}$$

and the array noise vector is given as,

$$\boldsymbol{n}(t) = \left[n_1(t), n_2(t), \ldots\ldots n_{3N}(t)\right]^T \tag{14}$$

where $n_1(t), \ldots, n_{3N}(t)$ has variance σ^2 and they are independent identically distributed circular complex random variables. The data vector is taken as $\boldsymbol{y}(t)$ and its correlation matrix can be,

$$\boldsymbol{R}_{3N} = E\left[\boldsymbol{y}(t)\boldsymbol{y}(t)^H\right] \tag{15}$$

4.2. Two Level Nested Array Model

Let us consider a non-uniform linear array with *N* elements, and a steering vector in the direction θ is taken as $\boldsymbol{a}(\theta)$ of size $N \times 1$. **A** be the array manifold matrix of size $N \times J$, the amplitude of the source signal which is slowly varying can be taken as $\boldsymbol{s}[t]$ with size $J \times 1$ and $\boldsymbol{n}(t)$ is the Gaussian noise with size $N \times 1$. Now the signal received by the array is,

$$\boldsymbol{x}[t] = \boldsymbol{A}\boldsymbol{s}[t] + \boldsymbol{n}[t] \tag{16}$$

where **A**, $\boldsymbol{s}[t]$ and $\boldsymbol{n}(t)$ is defined as,

$$\boldsymbol{A} = \left[\boldsymbol{a}(\theta_1)\boldsymbol{a}(\theta_2)\ldots\ldots\boldsymbol{a}(\theta_J)\right] \tag{17}$$

A is the array manifold matrix with size $N \times J$ and $\boldsymbol{a}(\theta j)$ represents the steering vector of the source direction θ_j, i.e.,

$$\boldsymbol{a}(\theta j) = \left[1 e^{-jkd\cos\theta_j} \ldots\ldots e^{-j(N-1)kd\cos\theta_j}\right]^T \tag{18}$$

The amplitude of target signal is given by,

$$\boldsymbol{s}(t) = \left[s_1(t), s_2(t), \ldots\ldots s_J(t)\right]^T \tag{19}$$

and the array noise vector $\boldsymbol{n}(t)$ is given as,

$$\boldsymbol{n}(t) = \left[n_1(t), n_2(t), \ldots\ldots n_N(t)\right]^T \tag{20}$$

The correlation matrix of $\boldsymbol{s}(t)$ is diagonal with the assumption that the acoustic sources are uncorrelated and linearly independent. Then,

$$\boldsymbol{R}_{xx} = E\left[\boldsymbol{x}\boldsymbol{x}^H\right] = \boldsymbol{A}\boldsymbol{R}_{ss}\boldsymbol{A}^H + \sigma_n^2\boldsymbol{I} \tag{21}$$

Now vectorize $\boldsymbol{R}_{xx}$ to get the following vector,

$$z = \overrightarrow{(\boldsymbol{R}_{xx})} = \overrightarrow{\left[\sum_{i=1}^{D}\sigma_i^2\left(\boldsymbol{a}(\theta_i)\boldsymbol{a}^H(\theta_i)\right)\right]} + \sigma_n^2\tilde{I}_n = \boldsymbol{A}^* \odot \boldsymbol{A}p + \sigma_n^2\tilde{I}_n \tag{22}$$

where $p = \left[\sigma_1^2 .. \sigma_D^2\right]^T$, and $\tilde{I}_n = \left[e_1^T \ldots e_N^T\right]^T$. Consequently, vector $\mathbf{z}$ acts as the received signal at an array with the manifold matrix as, $(\boldsymbol{A}^* \odot \boldsymbol{A})$ where $\odot$ symbolizes KR product. Since the manifold of a longer array is similar to the distinct rows of $(\boldsymbol{A}^* \odot \boldsymbol{A})$, this array can be considered indeed as the difference co-array of the proposed array[26].

5. DOA ESTIMATION USING BEAMFORMING

DOA estimation is an important function in array processing. Many such algorithms have been proposed for different scenario. This work mainly utilizes two methods, viz, Conventional and Minimum Variance Distortionless Response (MVDR).

5.1. Conventional Beamforming

The idea of scanning across the angular region of interest, thereby estimating the target signal direction which corresponds to the output power peak is known as conventional beamforming method (CBF). The power spectrum can be measured as,

$$P(\theta) = \frac{\boldsymbol{a}^H(\theta)\boldsymbol{R}_{xx}\boldsymbol{a}(\theta)}{\boldsymbol{a}^H(\theta)\boldsymbol{a}(\theta)} \tag{23}$$

Here, $\boldsymbol{a}(\theta)$ is the array response vector, calculated by incrementally varying θ across the space. Thus, the angle corresponding to the peak value of output spectrum agrees with the exact DOA. Though a robust scheme, poor angular resolution, inferior interference mitigation capability an higher side lobe levels are the significant issues associated with this algorithm[30].

5.2. MVDR Beamforming

MVDR spatial filter results in a significant improvement in resolution and mitigation of multiple strong interferences. This method involves minimizing the output power with the limitation of unity gain in the desired direction. The constraint optimization problem can be solved to obtain the weight vector as,

$$\boldsymbol{w} = \frac{\boldsymbol{R}_{xx}^{-1}\boldsymbol{a}(\theta)}{\boldsymbol{a}^H(\theta)\boldsymbol{R}_{xx}^{-1}\boldsymbol{a}(\theta)} \tag{24}$$

which gives the angular spectrum as,

$$P(\theta) = \boldsymbol{w}^H\boldsymbol{R}_{xx}\boldsymbol{w} = \frac{1}{\boldsymbol{a}^H(\theta)\boldsymbol{R}_{xx}^{-1}\boldsymbol{a}(\theta)} \tag{25}$$

Here also the angle corresponding to the output peak in the spectrum agrees with the estimate of true DOA. As compared to CBF, this beamformer needs an extra matrix inversion without compromising the resolution and interference mitigation characteristics in most cases[30].

6. METHODOLOGY

The implementation of ULA data model with six sensors is a basic level of simulation setup which is carried out in MATLAB environment. With same number of sensors, a nested APS (NAPS) array and an NAVS array both having two-levels must be constructed. DOA estimation is done for both models using two techniques. Conventional and MVDR beamforming were used for this purpose. Obtained results are to be compared in terms of DoF, number of sources able to detect, side lobe reduction, beam width, and probability of detection. The overall steps are indicated in Fig. 2.

7. RESULTS AND DISCUSSIONS

The simulation results obtained from the performance of our algorithm is presented in this section. The proposed system is compared with conventional line array. Six sensors are used for generating three array models viz., uniformly spaced hydrophone array (ULA), NAPS array and NAVS array. The upper design frequency is taken as 5 KHz and the six sensors are uniformly spaced at $\lambda/2$ to construct ULA, where λ corresponds to the minimum wavelength of interest. We use a

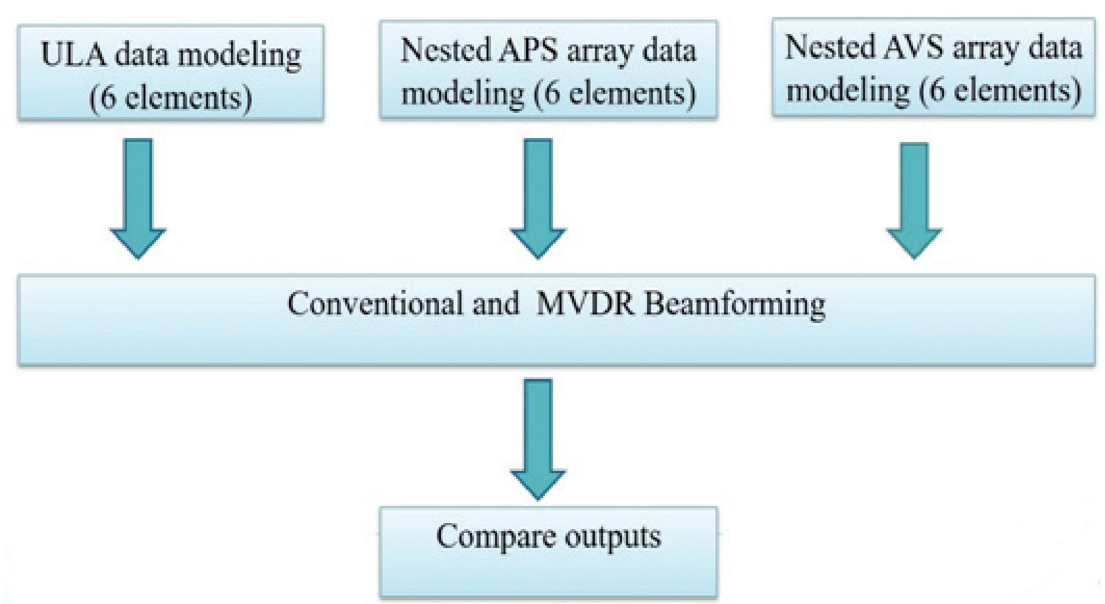


Figure 2. Methodology for the proposed system.

horizontal configuration of the line array where we are interested in the steering or surveillance only in the azimuth steering. All the figures are drawn with x-axis as azimuth bearing (varies from 0 to 180 degrees). The NAPS and NAVS arrays are constructed with two level inter sensor spacing as shown in the Fig.2. The target of interest is modeled as a slowly varying zero mean narrowband complex envelope centered at the upper design frequency of the array. Conventional and MVDR spatial filters are used as the response function to evaluate the performance of the three array models.

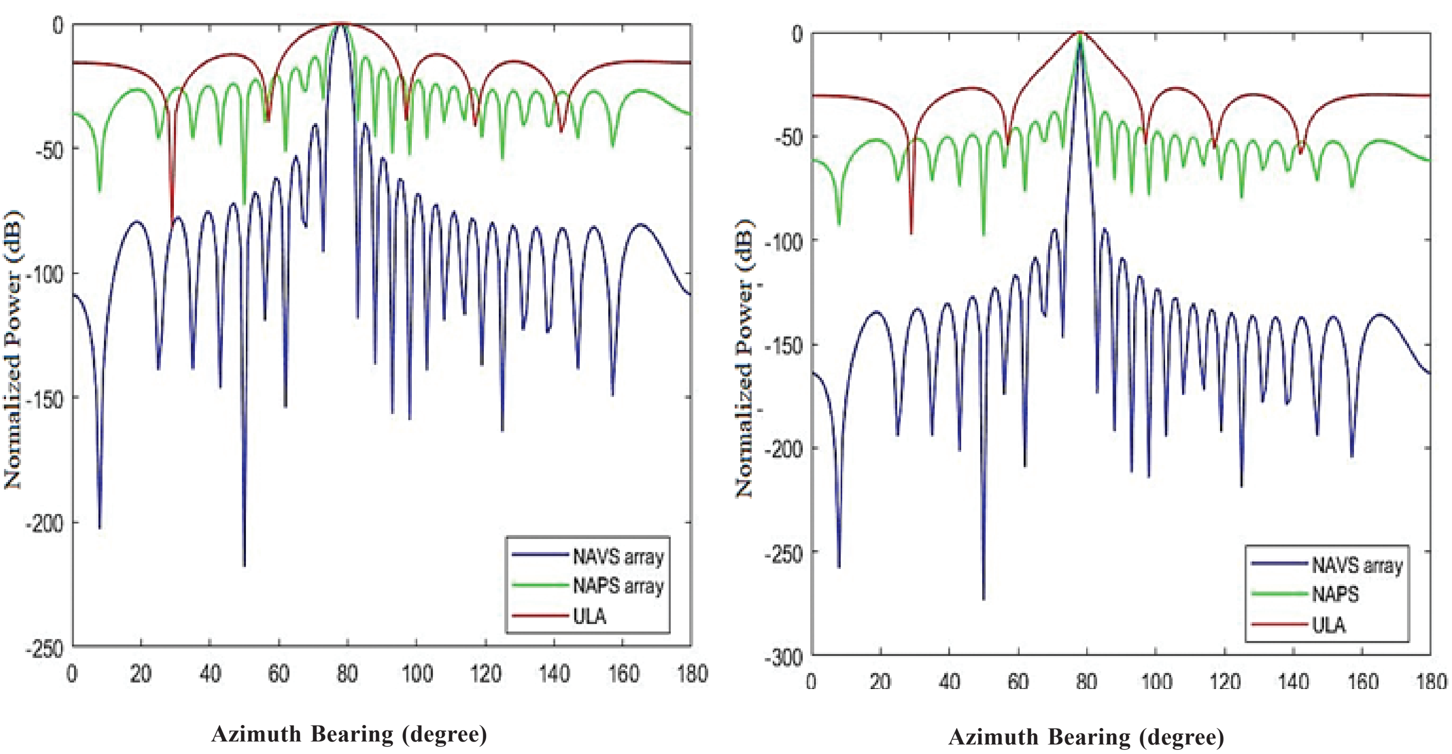
Figure 3 demonstrates the comparative performance of six sensor ULA, two-level NAPS and two-level NAVS array with a single target positioned at 78^0 azimuth bearing. The angular spectrum is plotted using both the conventional and MVDR spatial response functions. It is seen that, the angular resolution and side lobe performance of NAVS array is much better in comparison with NAPS and ULA. The higher angular spectral resolution and increased side lobe performance are primarily due to the increased length of the virtual array due to the nesting method. Also, use of AVS enhances the performance due to the particle velocity measurements in addition to the scalar pressure.

Figure 4(a) illustrates two target localization using MVDR at 68^0 and 78^0 and Fig. 4(b) shows a similar scenario with further closer targets with 6^0 target separation. Among the ULA, NAPS array and NAVS array, a resolution of 10^0 is observed using both NAPS and NAVS array. As the targets get closer, it is noteworthy that the resolution obtained using NAVS array (blue graph) shows peaks exactly at the simulated angles and target resolution capability is much higher than the other two array structures. This improvement may be attributed to the increased DoF of NAVS array architecture.

Figure 5 shows the probability of detection versus SNR plot of ULA, NAPS array and NAVS array.

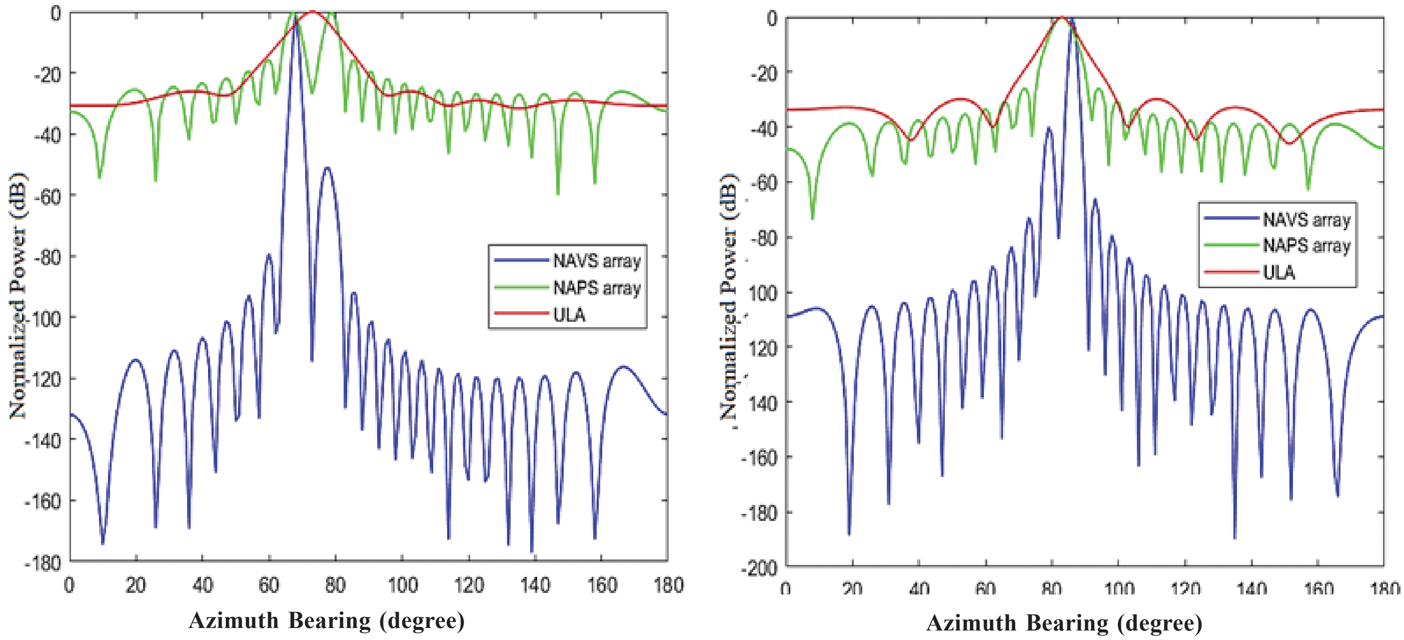
The ambient noise experienced at the array is modeled as statistically independent ands identically distributed circular complex Gaussian random variables with equal variance. We have considered 1000 Monte Carlo runs and probability of detection versus SNR is plotted. This is defined as $\frac{F_k}{F}$ where F_k is the number of times K sources are detected and F is the trial number.

Figure 6 shows six targets localized using conventional beamforming. It is now seen that the proposed NAVS array configuration is also capable of localizing six targets which is not possible by a six element ULA. It is noteworthy that with only six array elements, an NAVS array structure is able to localize more targets beyond its restrictions and is capable of locating closely placed targets. Thus a high resolution bearing estimation with reduced side lobe level is observed. This performance improvement of NAVS array can be attributed to the enhancement of DoF by nested array structure and more information gathering capability of AVS array structure. The overall performance parameters of three models handled in this work having six sensors are tabulated in Table 1.



(a) (b)

Figure 3. Angular response function using six element ULA, NAPS and NAVS array. Target is located at 78 degree (a) Conventional beam output power, and (b) MVDR beam output power.



(a) (b)

Figure 4. MVDR response function using six element ULA, NAPS and NAVS array. Multiple target is located at (a) 68 and 78 degree and (b) 80 and 86 degree.

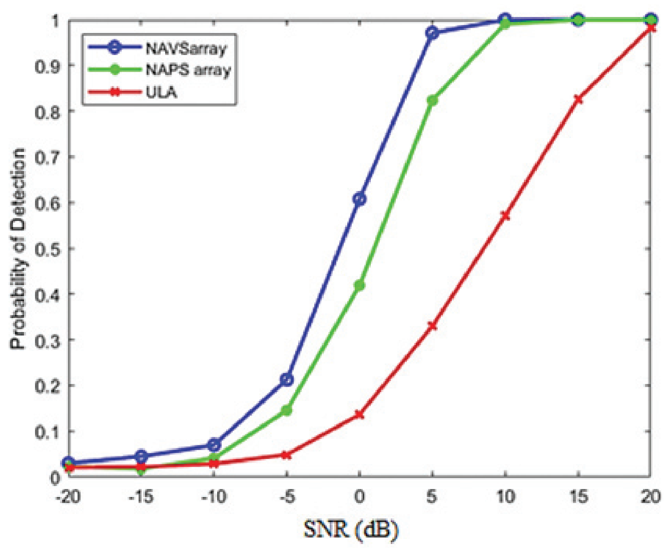


Figure 5. Probability of Detection versus SNR plot for NAVS array, NAPS array and ULA.

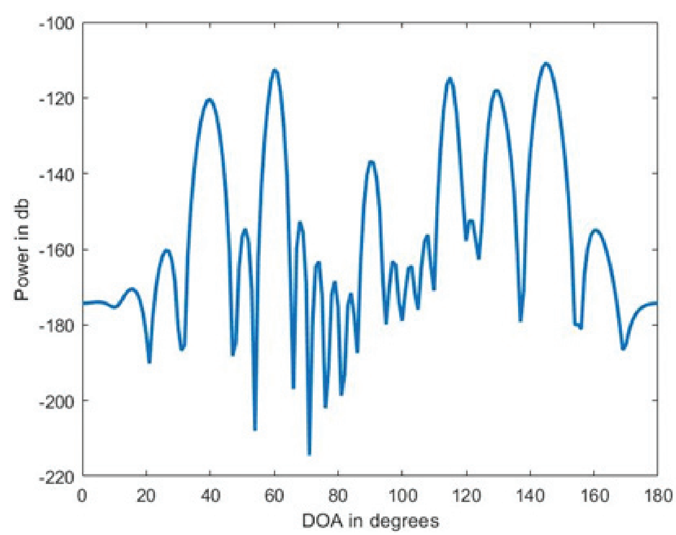


Figure 6. Six target localization using NAVS array at bearings, 40, 60, 90, 115, 130, 145.

8. CONCLUSION

NAVS array concept is systematically developed using six vector sensors with two level of nesting and its superior localization performance is presented. Ocean data model for the NAVS array is developed to analyze the performance of the proposed array architecture viz, DoF, capability to detect more number of acoustic sources, high resolution bearing estimation with reduced side lobe levels etc. Conventional or MVDR spatial filter followed by energy detector is used as the response function to check the precision of the proposed architecture with the ULAs using the equal number of sensor elements. It is to be noted that, the six sensor nested array is equivalent to 23 sensor ULA while, the NAPS and NAVS arrays attains 23 DoFs. The simulation results show that, the proposed NAVS architecture enhances the DoF of the spatial filter to sharpen the main lobe characteristics with low side lobe level, positioning more number of nulls to mitigate the multiple interferences and detection of more number of acoustic sources present in the ocean. Future work will address the generic higher order multi-level nested underwater sensors to enhance the DoF in Shallow Ocean which supports multipath wave propagation.

Table 1. Performance Analysis
(Measured first Sidelobe level and main lobe width - array steered at 78 ^{0})

Array Type		ULA	NAPS Array	NAVS Array
DoF Achieved		6	23	23
Resolution (degrees)		>10	<10	<6
1st sidelobe level at 5Khz (dB)	CBF	-13.3	-13.3	-39.8
	MVDR	-38.1	-50.2	-125.2
Main lobe width at 5Khz (degrees)	CBF	17.5	4.4	2.6
	MVDR	3.2	0.4	0.3
1st sidelobe level at 3.5Khz (dB)	CBF	-13.3	-13.3	-39.9
	MVDR	-25.5	-36.5	-89.8
Main lobe width at 3.5Khz (degrees)	CBF	25.2	6.4	3.8
	MVDR	9.6	1.4	1.0
1st sidelobe level at 3Khz (dB)	CBF	-13.3	-13.3	-39.6
	MVDR	-27.2	-37.5	-92.4
Main lobe width at 3Khz (degrees)	CBF	29.3	7.4	4.4
	MVDR	10.5	1.4	1.0

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