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Reliability, Availability, and Maintainability of Intermittently-used Repairable Systems

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ABSTRACT

A composite measure comprising linear combination of reliability, availability, and maintainability (CMRAM) is proposed for an intermittently-used complex repairable system, namely a transport helicopter. The failure data are fitted in Weibull distribution for overlapping intervals of operating time to capture the effect of recent maintenance and other actions. Evaluation of CMRAM is helpful in formulating future operational and maintenance strategies.

Keywords: Repairable system, reliability, Weibull distribution, CMRAM, transport helicopter, simulation model, system reliability, system maintainability, Monte-Carlo simulation

1. INTRODUCTION

A repairable system can be characterised in terms of its reliability, availability, and maintainability (RAM) which is affected by such factors as locational arrangement of machines, their respective workload, operational severity, spare-parts availability, number and capability of operational personnel as well as of maintenance personnel, planning and control, safety measures, environmental severity, and infrastructural facilities. The RAM parameters can be quantified over discrete intervals of operating time. In the past, many researchers have tried to study these. Kurien[1] developed a discrete event simulation model based on the Monte-Carlo simulation technique as enumerated by Shanon[2], to study the reliability and availability of an aircraft in a training facility. Marseguerra[3], *et al*. have used genetic algorithm and Monte-Carlo simulation for optimising condition-based maintenance. Chisman[4] has suggested the use of discrete-event simulation modelling for the study of reliability and maintainability of large-scale systems. Jones and Hayes[5] have proposed a methodology for collection of field data and its analysis for assessing the current reliability of a given product. Weckman[6], *et al*. have proposed Weibull process for modelling complex repairable systems. Scarf[7] has emphasised the development of a maintenance management information system for generating data for mathematical modelling of practical systems. In the opinion of Pidgeon and Leary[8], complex system failures highlight organisational factors in the generation of accidents and disasters across a wide variety of settings.

Reliability-based optimisation of operation and maintenance of a repairable system presents a multi-objective scenario where one reliability metric gets pitched against others, requiring trade-offs to find optimal combinations of reliability, availability, and maintainability. A single composite metric could provide a convenient measure for evaluating and optimising a given system and its maintenance policies and can also help in judging whether the system

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has improved or deteriorated over time. In the event of deterioration, corrective steps to arrest the degradation can be taken. If improvement occurs, relevant operational and maintenance factors may be continued or even reinforced.

2. PROPOSED MODEL

Physically, a repairable system is modelled as an arrangement of a number of interacting sub-systems (Fig. 1). The important input parameters to the model are time between failures, repair times, and intermittent periodic maintenance activities. The parameters are measured over discrete intervals of time, which may be overlapping. The operation of model generates reliability, availability, and maintainability characteristics. These are combined by assigning appropriate weights to each, and a single composite measure of system reliability, availability, and maintainability (CMRAM) is developed.

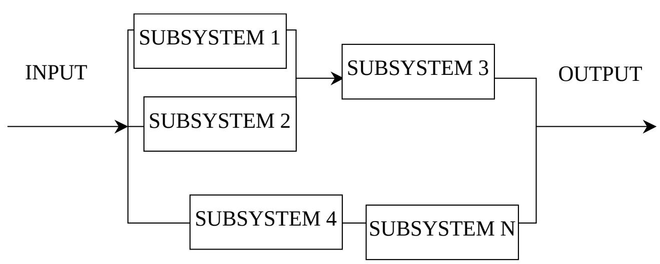

Figure 1. Model of a repairable system.

The proposed procedure to evaluate CMRAM against cumulative operating time to characterise the behaviour of the system is shown in block diagram (Fig. 2). The optimum value of CMRAM would help in deciding the best possible values of the input parameters.

3. FIELD DATA PROCESSING

For repairable systems, important field data consisting of operating time, time to failure, time to repair, schedule and time spent for different preventive and condition-based maintenance actions, and other maintenance activities necessitated by safety regulations and emergency procedures, are recorded in logbooks. These act as seed values to generate additional data (if required) by Monte-Carlo discrete-event simulation technique. The actual and generated data are processed to calculate reliability, availability, and maintainability characteristics.

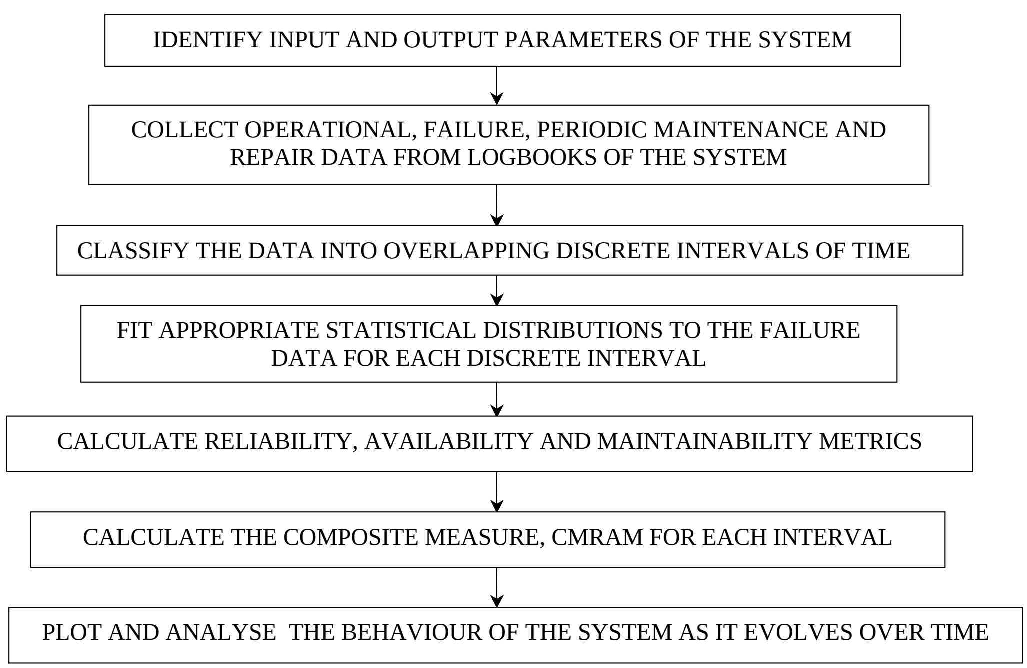

Figure 2. Procedure to analyse the proposed discrete event model.

3.1 Reliability

Reliability is defined as the probability that a component or a system will perform its required function satisfactorily for a given period of time when used under stated operating conditions. In other words, it represents the probability of non-failure over time[9]. The equipment is considered as an appropriate combination of sub-systems. Any one of such sub-system may fail individually or more than one may fail simultaneously to make the system inoperable. Each such incidence can be considered as a system failure. The total operating time can be divided into a number of overlapping discrete intervals of time (Fig. 3).

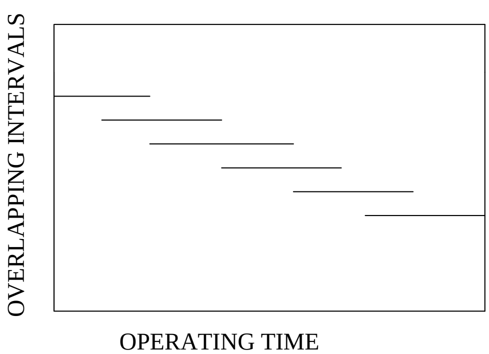

Figure 3. Overlapping discrete intervals of operating time.

The actual or generated time (t) at which failures occur, are arranged in an ascending order in each interval and so-called cumulative failure function, $F(t)$ may be calculated using the following expression[9].

$$F(t) = (i - 0.3)/(n + 0.4) \qquad (1)$$

where n is the total number of failures in an interval and i is its sequence number.

The two-parameter Weibull distribution[9], having the following expression, is capable of representing most observed failure patterns.

$$R(t) = 1 - F(t) = \exp\,[-\,(t/\theta)^{\beta}] \qquad (2)$$

The values of β and θ, called the shape factor and scale factor respectively may be calculated with the help of the computed cumulative failure function $F(t)$, as given below[2].

$$\beta = [n\Sigma xy - \Sigma x\Sigma y]/[n\Sigma x^2 - (\Sigma x)^2] \qquad (3)$$

$$? = \exp\left[-\frac{\left(\sum y - ß\sum x\right)/n}{ß}\right] \qquad (4)$$

where $x = \log_e t$ and $y = \log_e[\log_e\{1/(1\text{-}F(t))\}]$

If a system is typically used continuously for time t, then the operational characteristic time, $t_{oc} = t$. Its operational reliability $R(t_{oc})$, given by the following expression, is a valuable characteristic of system performance.

$$R(t_{oc}) = \exp[-(t_{oc}/\theta)^{\beta}] \qquad (5)$$

3.2 Inherent Availability

Availability is defined as the probability that a component or a system is performing its required function at a given point of time when operated and maintained in a prescribed manner[9]. In case of inherent availability (A_{inh}), the downtime due to corrective maintenance (repair) is only considered. Other causes of downtime, e.g., ready time, waiting time, preventive-maintenance time, condition-based maintenance time, logistics, etc are not included.

$$A_{inh} = \frac{T_{up}}{T_{up} + T_{dn}} = \frac{1}{1 + T_{dn}\,/\,T_{up}} \qquad (6)$$

where T_{up} is the total uptime (operating time) and T_{dn} is the total downtime (repair time) during the given discrete interval.

3.3 Maintainability

Maintainability, $M(t)$ is defined as the probability that a failed component or system will be restored or repaired to a specified condition within a stated period of time when maintenance is performed in accordance with the prescribed procedures[9]. If repair rate is μ, which is reciprocal of mean time to repair (MTTR), then it is given[10] by the following expression.

$$M(t) = 1 - e^{-\mu t} \qquad (7)$$

A characteristic maintenance time, t_{cm}, can be defined as the time by which over 90 per cent of the maintenance jobs on the system can be completed. MTTR is the average value of all repair times in any discrete interval. Maintainability at time, $t = t_{cm}$ may be called characteristic maintainability $M(t_{cm})$ and expressed as

$$M(t_{cm}) = 1 - e^{-\mu\, t_{cm}} = 1 - e^{-\left(t_{cm}/\mathrm{MTTR}\right)} \qquad (8)$$

4. PROPOSED COMPOSITE MEASURE

The operational reliability $R(t_{oc})$, inherent availability A_{inh}, and characteristic maintainability $M(t_{cm})$, can be combined into a single composite measure (CMRAM) taking relative weightages of all the three characteristics.

$$\mathrm{CMRAM} = w_r \times R(t_{oc}) + w_a \times A_{inh} + w_m \times M(t_{cm}) \qquad (9)$$

$$w_r + w_a + w_m = 1 \qquad (10)$$

where w_r, w_a, w_m are the relative weightages of $R(t_{oc})$, A_{inh}, $M(t_{cm})$, respectively.

The respective weightages are decided on the basis of the relative importance of the three metrics in the context of system requirements and costs. Their relative frequency distributions, experts' opinion, and Bayes' theorem are used to calculate numerical values of weightages. Once the weightages are decided, the CMRAM can be calculated for each discrete interval of operating time representing the system's performance.

5. CASE STUDY

A single helicopter has been studied for its 880 flight hour in terms of time to failure, repair time, and periodic and condition-based maintenance time. The failure and repair data, picked up from logbooks, were arranged in chronological order and divided into 11 overlapping intervals, namely 0-150 h, 75-225 h, 150-300 h, 225-375 h, 300-450 h and-so on. The helicopter was divided into five sub-systems, viz., air frame, engine, electricals, instruments, and radio and radar. The data pertain to all the five sub-systems.

The effect of periodic maintenance on failures is shown in Fig. 4. Out of 34 periodic maintenances, 17 were after every 25 h, 8 after every 50 h, 4 after every 100 h, and 5 after every 200 h of operation. It was noticed that many failures occur immediately after completion of 25 h, 50 h, or 100 h maintenance. Figure 5 and Table 1 show failures for each sub-system. The total number of sub-system failures in a given interval may not add up to system failures because multiple sub-systems can fail simultaneously. Out of a total of 66 system failures, the sub-systems failures were of air frame (31), engine (22), instruments (19), electricals (13), and radio and radar (8). Pie diagram (Fig. 6) revealed that over 80 per cent of the failures belong to components and assemblies of airframe, engine, and instruments, and the main causes were excessive vibration, oil/fuel/valve leakages, mismatching of engine-control parameters, and instrument malfunctioning.

5.1 Operational Reliability

In failure data, for the total system as well as for the sub-systems, Weibull distribution was fitted and values of θ, β and correlation coefficient were obtained (Table 2). The values of correlation coefficient varied from 0.805 to 0.985, indicating a good fit[2] and justifies the use of Weibull distribution. The value of β varied from 0.364 to 0.659, showing early failures in all the sub-systems.

The operational characteristic time, t_{oc} was decided by looking into the flight operations. Helicopter sortie generally consists of about 1 h of flight. Hence, a value of t_{oc} = 1 h was chosen. Operational reliability, $R(1)$ was calculated for all the 11 intervals using Eqn. (5). The graph for the same [Fig. 7 (a)] shows that between 150 h to 450 h of cumulative flight, the operational reliability improved from 0.6621 to 0.9123, and thereafter, there was a drop in its values up to 0.6753 till 528 h of cumulative flight time. It again increased to 0.7829 at 601 h, and finally attained a value of 0.6818. The pattern of changes in the values of $R(1)$ indicate the variable nature of the failure processes. The median value of $R(1)$ was found to be 0.6818.

5.2 Inherent Availability

Inherent availability, A_{inh}, was calculated using operating time and repair time data of all the 11

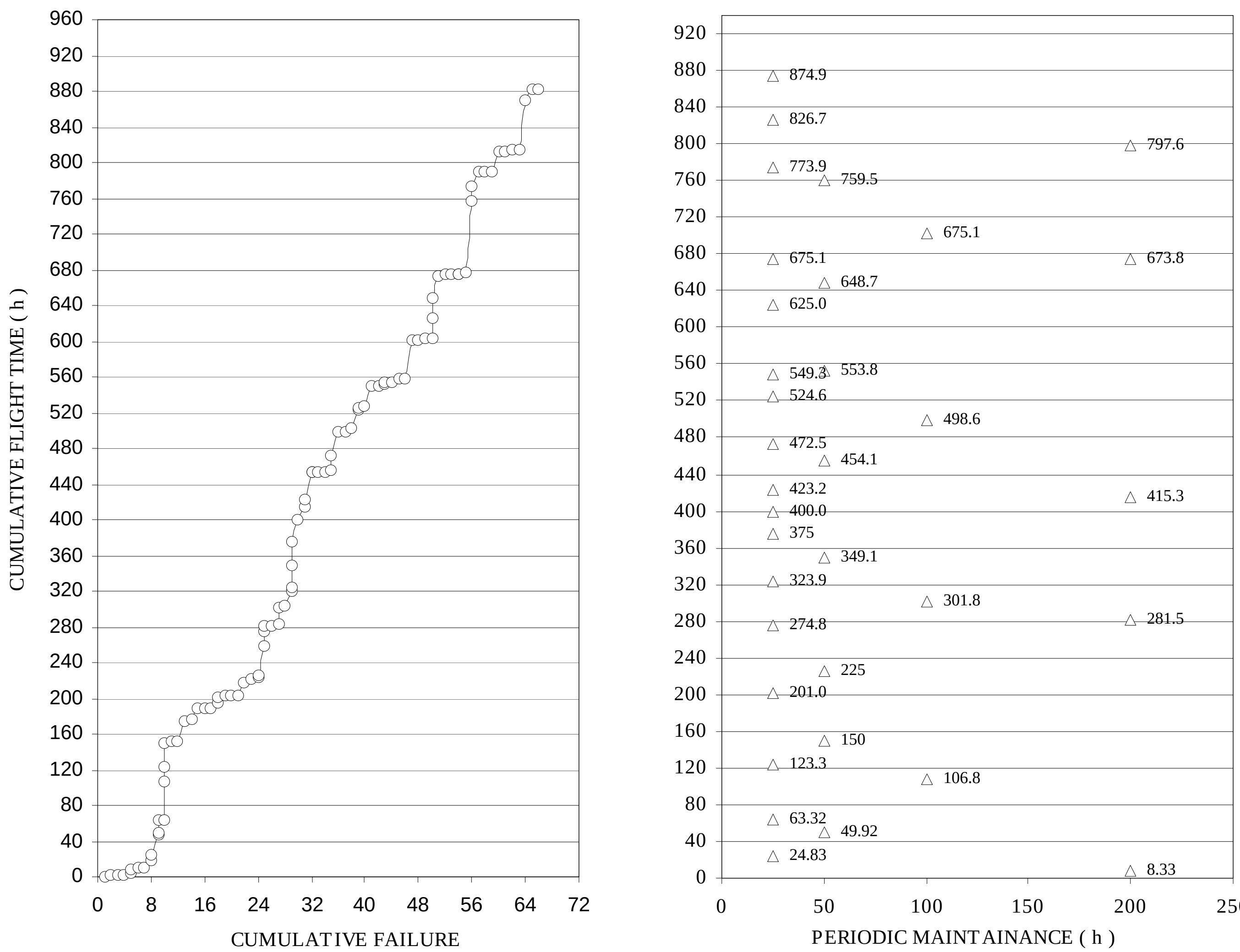

Figure 4. Cumulative flight time versus cumulative failures and periodic maintenance.

intervals [Table 3 and Fig. 7(b)]. It can be observed that A_{inh} improved from 0.2639 to 0.9223 during cumulative flight time from 150 h to 415 h, indicating a large increase of 249.5 per cent. Thereafter, the A_{inh} value kept decreasing and reached 0.3423 at 882 h. The median value of inherent availability was found to be 0.3423.

5.3 Characteristic Maintainability

For all the 11 flight time intervals, values of MTTR, μ, M(5), M(7), M(10) were calculated (Table 4). The characteristic maintainability at 5 h of operation, M(5), was found to vary from 0.1800 to 0.8702 and never reached a value of 0.9. M(7) values varied from 0.2426 to 0.9426. However, M(10) values, in 5 intervals out of 11, were in the vicinity of 0.90. In remaining 6 intervals, the values varied between 0.3277 to 0.7658. The characteristic maintenance time was better fulfilled by M(10). The graph of the same is shown in Fig. 7(b).

5.4 Composite Measure of Reliability, Availability, and Maintainability

The relative weightages, w_r for operational reliability, w_a for inherent availability, and w_m for characteristic maintainability were adjudged and fixed as 0.5, 0.3 and 0.2, respectively as per the procedure in Section 4. CMRAM was calculated using Eqn (9). The plot for the same is shown in Fig. 7(c). To determine the system behaviour, five-point moving averages (MA) of CMRAM, i.e., CMRAM (MA) were plotted [Fig. 7(d)]. The CMRAM

Table 1. Cumulative failures of the helicopter and its subsystems

Cumulative flight time (h)	Cumulative failures					
	Total system	Air frame	Engine	Electricals	Instruments	Radio and radar
50.0	9	5	2	3	6	2
100.0	10	5	3	3	6	2
150.0	10	5	3	3	6	2
200.0	18	7	6	5	6	2
250.0	24	9	6	6	9	3
300.0	27	10	6	7	10	4
350.0	29	11	6	7	10	5
400.0	29	11	6	7	10	5
450.0	30	11	6	7	11	5
500.0	36	14	7	7	14	7
550.0	40	17	8	7	15	7
600.0	45	20	11	8	15	7
650.0	50	23	12	9	15	8
700.0	55	27	13	10	17	8
750.0	55	27	13	10	17	8
800.0	59	31	17	11	18	8
850.0	63	31	21	11	19	8
882.8	66	31	22	13	19	8

Table 2. Computed Weibull parameters, the corresponding correlation coefficient, and operational reliability during different time intervals

Time interval number	Cumulative flight time (h)	Flight time (h)	θ	β	Correlation coefficient	Operational reliability R(1)
1	151.62	151.62	4.35	0.602	0.846	0.6621
2	224.50	161.18	4.36	0.497	0.976	0.6181
3	303.02	151.40	7.90	0.492	0.973	0.6966
4	415.37	190.87	37.39	0.659	0.933	0.9123
5	454.93	151.91	14.72	0.364	0.982	0.6869
6	528.18	112.81	7.71	0.458	0.985	0.6753
7	600.85	145.92	9.30	0.631	0.971	0.7829
8	674.03	145.85	6.50	0.558	0.901	0.7031
9	759.90	156.05	6.26	0.403	0.805	0.6206
10	814.57	140.54	5.74	0.444	0.808	0.6311
11	882.80	125.90	7.77	0.468	0.872	0.6818

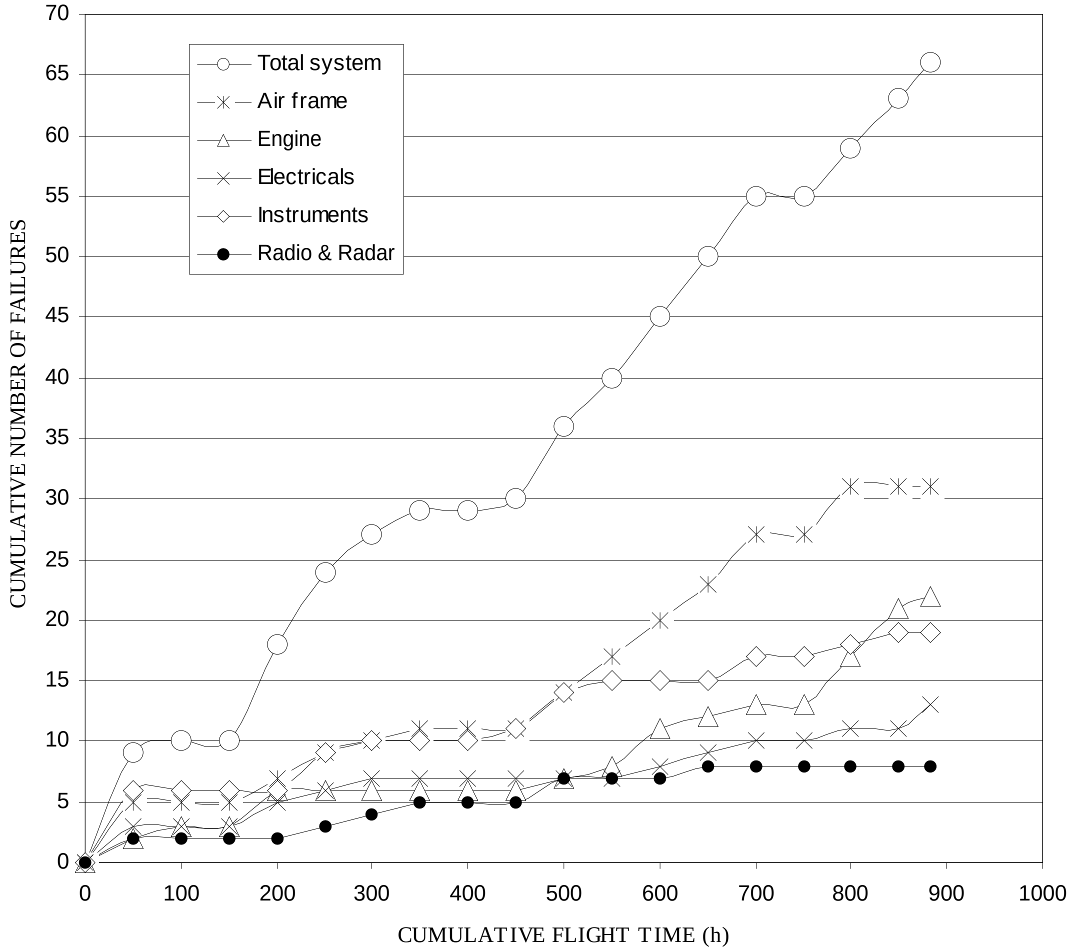

Figure 5. Cumulative failures of the subsystems of the the total system and the helicopter.

steadily improves from 0.5238 to 0.9295 between 150 h to 415 h, thereafter, there is a general decline, till it reaches a value of 0.5121 at 880 h. The CMRAM (MA) confirms a general decline from 0.7077 at 450 h to 0.5364 at of 880 h. The decline can be attributed partly to the failures occurring immediately after the periodic maintenances performed at 25 h, 50 h, and 100 h (Fig. 4). Therefore, one can infer an inadequacy of the above-mentioned maintenance actions, which need to be probed and improved upon.

The CMRAM indicates quantitatively the behaviour of the system as it evolves over time and provides a forewarning for initiating corrective actions. These actions can comprise changes in intensity of workload, number and expertise of operating personnel, maintenance and supporting staff, severity of operations, spare-parts availability, flight decisions based on environmental severity, new safety measures, and improved infrastructural facilities. Additionally, the factors that lead to degradation need to be identified and neutralised.

5.5 Parametric Study

The effect of different parameters, viz., ratio of downtime to uptime (T_{dn}/T_{up}), θ, β, and μ on CMRAM were studied by varying one parameter while keeping all other parameters fixed at their mid values (Fig. 8). The T_{dn}/T_{up} ratio was varied from 0.01 to 7.00 while keeping the values of θ = 7.71, β = 0.49, and μ = 0.14. It was found that when the ratio of T_{dn}/T_{up} is changed from 0.01 to 2.0, there is a steep reduction (24.7 %) in the value of CMRAM from 0.7969 to 0.5999. For the remaining range of T_{dn}/T_{up} from 2.0 to 7.0, the CMRAM values dropped by only 10.4 per

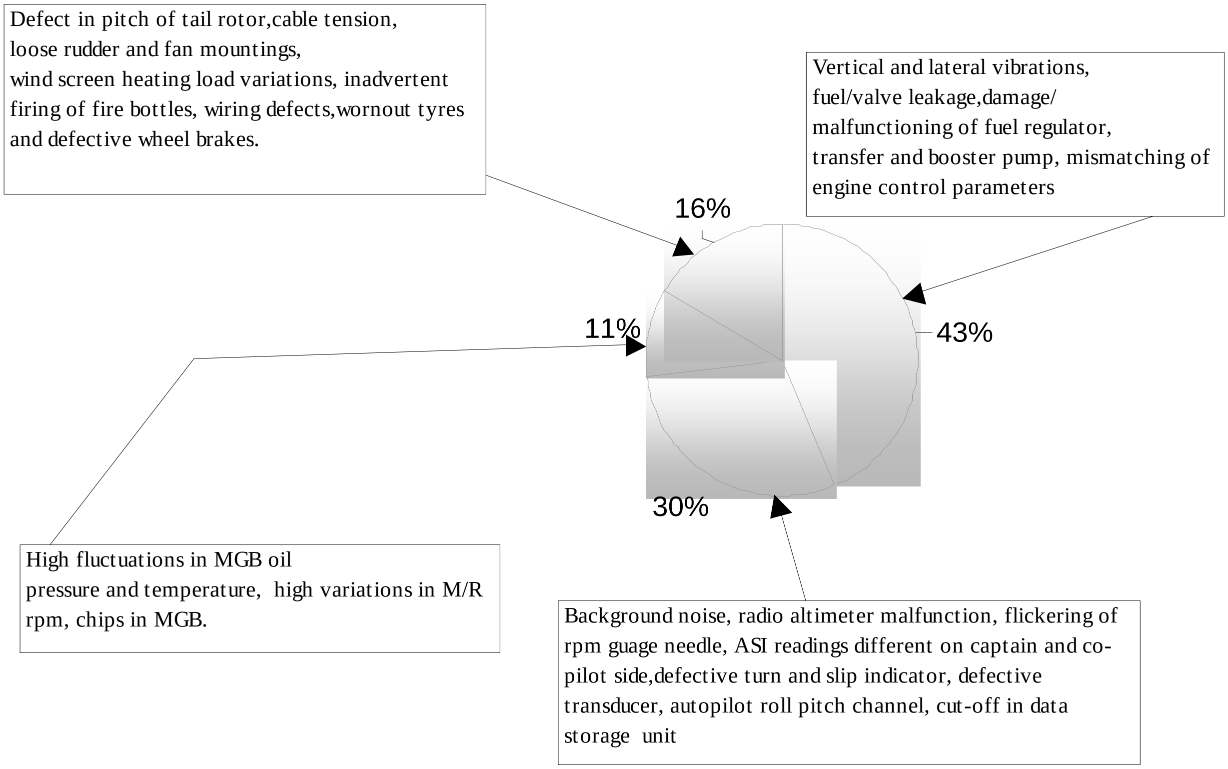

Figure 6. Pie-diagram representation of failures of the helicopter subsystems.

Table 3. Computation of inherent availability during different time intervals

Time interval number	Cumulative flight time (h)	Flight time (h)	Repair time (h)	A_{inh}
1	151.62	151.62	422.91	0.2639
2	224.50	161.18	104.50	0.6067
3	303.02	151.40	112.67	0.5733
4	415.37	190.87	16.09	0.9223
5	454.93	151.91	80.00	0.6550
6	528.18	112.81	100.00	0.5301
7	600.85	145.92	344.09	0.2978
8	674.03	145.85	501.33	0.2254
9	759.90	156.05	340.41	0.3143
10	814.57	140.54	369.67	0.2755
11	882.80	125.90	241.92	0.3423

cent from 0.5999 to 0.5374 [Fig. 8 (a)]. To maintain high values of CMRAM, therefore, the ratio of T_{dn}/T_{up} should be kept below 2.0.

The scale parameter, θ is the time by which 63.2 per cent of failures occur[9]. Figure 8(b) shows improvement in values of CMRAM by 20.2 per cent (from 0.5574 to 0.6702) when θ was increased from 4 to 30 and 2.6 per cent, i.e., from 0.6702 to 0.6879 for θ increased from 30 to 50. This suggests maintaining θ values above 30. As for the effect of β, the values of CMRAM improved by 35.5 per cent (from 0.5131 to 0.6951) for β varying from 0.2 to 1.0. In the range of β from 1.0 to 2.0, the values of CMRAM improve by only 7.5 per cent, i.e., from 0.6951 to 0.7475 [Fig. 8 (c)]. This indicates that β values should be maintained near 1.0, i.e., in constant failure zone. The repair rate, μ, presents a similar scenario. For a range of μ from 0.04 to 0.3, the CMRAM values increased by 24.1 per cent (i.e., from 0.5153 to 0.6394). For further increase of μ between 0.3 to 0.45, the values of CMRAM improved by only 1.2 per cent (i.e., from 0.6394 to 0.6472). Therefore, μ should not go below a value of 0.3.

6. CONCLUSION

Repairable systems are becoming increasingly complex in operation and maintenance. These require

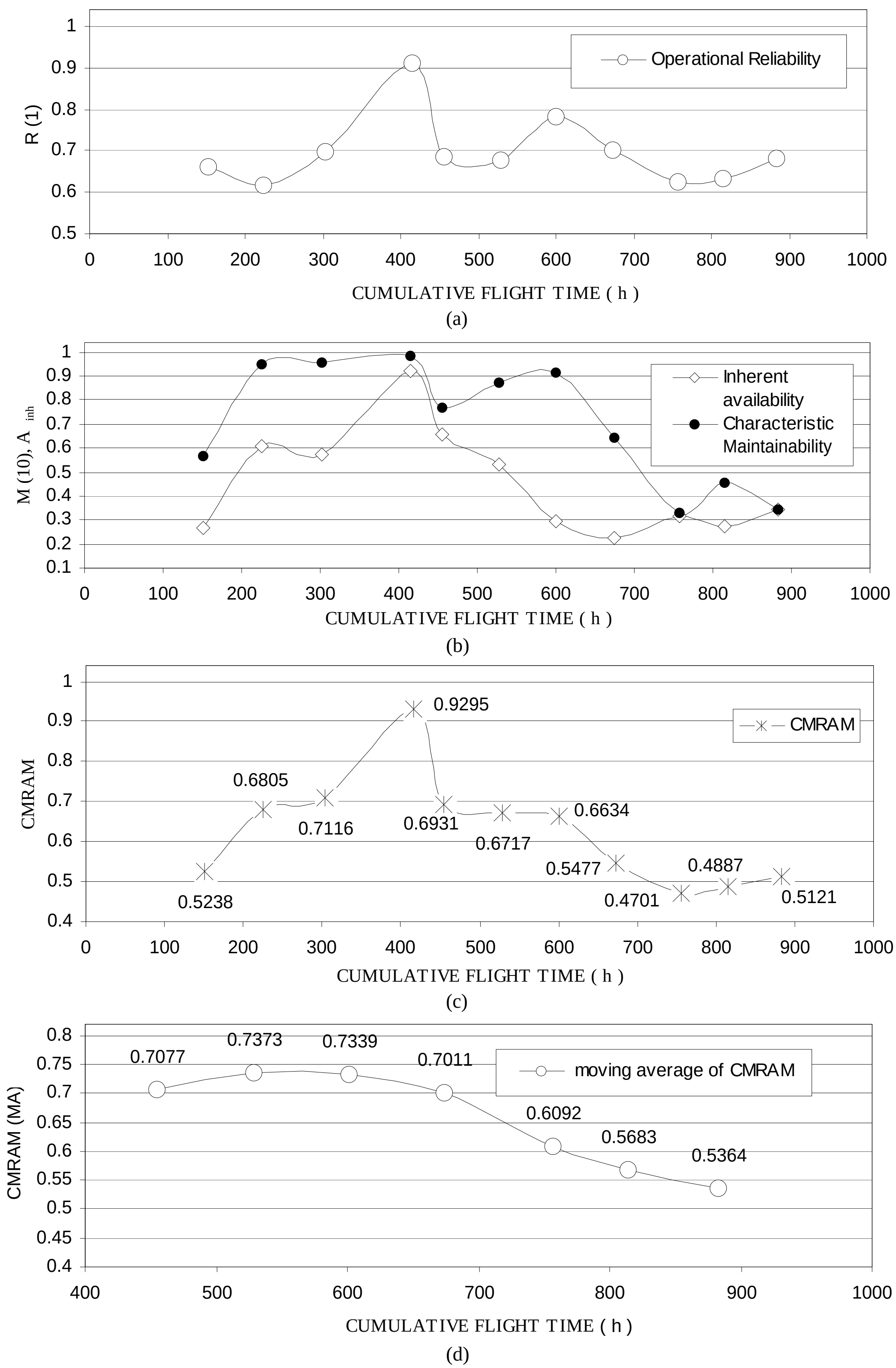

Figure 7. Variations of computed values of operational reliability, inherent availability, characteristic maintainability, and CMRAM with cumulative flight time.

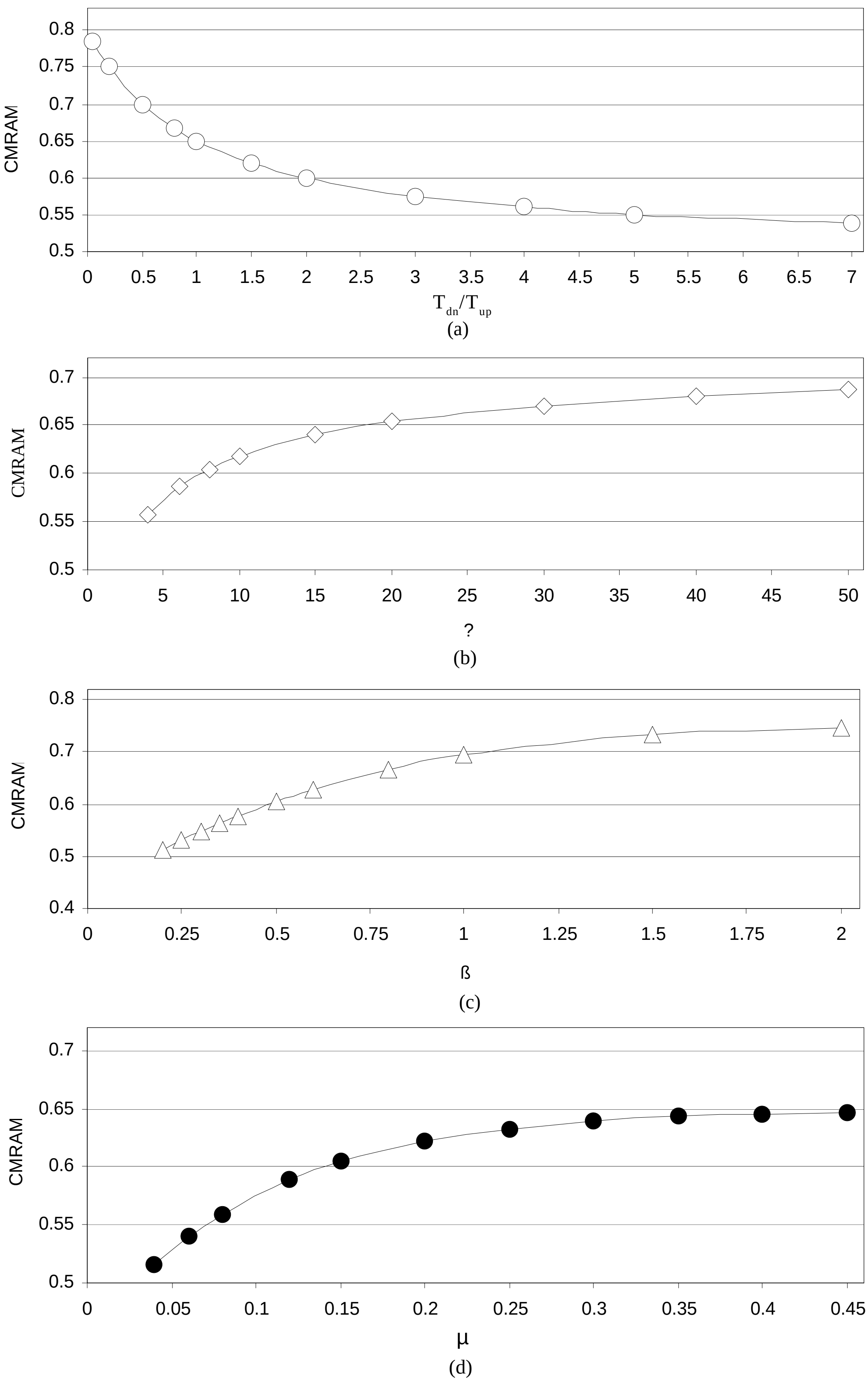

Figure 8. Effect of different parameters on the composite measure (CMRAM).

Table 4. Computation of characteristic maintainability M(*t*) for different values of *t*.

Time interval number	Cumulative flight time (h)	MTTR (h)	μ (1/ MTTR)	M(5)	M(7)	M(10)
1	151.62	11.91	0.0840	0.3428	0.4444	0.5681
2	224.50	3.40	0.2944	0.7705	0.8726	0.9473
3	303.02	3.19	0.3139	0.7919	0.8889	0.9567
4	415.37	2.45	0.4084	0.8702	0.9426	0.9832
5	454.93	6.89	0.1452	0.5161	0.6380	0.7658
6	528.18	4.80	0.2082	0.6468	0.7671	0.8753
7	600.85	4.09	0.2442	0.7051	0.8191	0.9130
8	674.03	9.72	0.1029	0.4021	0.5133	0.6426
9	759.90	25.19	0.0397	0.1800	0.2426	0.3277
10	814.57	16.61	0.0602	0.2599	0.3439	0.4523
11	882.80	9.25	0.1081	0.4175	0.5307	0.6607

periodical evaluation of their performance to optimise operation and maintenance activities. A composite measure of reliability, availability, and maintainability has been proposed to quantify the performance of the system. It is a linear weighted combination of the operational reliability, inherent availability, and characteristic maintainability metrics. Usefulness of composite measure has been illustrated by applying it to the failure and repair data of a helicopter operating unit.

REFERENCES

1. Kurien, K.C. Reliability and availability analysis of repairable system using discrete event simulation. Indian Institute of Technology Delhi, New Delhi, 1988. (PhD Thesis).

2. Shanon, R.E. *In* System simulation, the art and science. Prentice Hall, New Jersey. 1975. pp. 64, pp. 84, pp. 87.

3. Marseguerra, M.; Zio, E. & Podofillini, L. Condition-based maintanance optimisation by means of genetic algorithm and Monte-Carlo simulation. *Reliab. Engg. Syst. Safety*, 2002, **77**, 151-66.

4. Chisman, J. A. Using discrete simulation modelling to study large scale system reliability/availability. *Comput. Oper. Res.*,1998, **25**, 169-74.

5. Jones, J.A. & Hayes, J.A. Use of a field-failure database for improvement of product reliability. *Reliab. Engg. Syst. Safety*, 1997, **55**, 131-34.

6. Weckman, G.R.; Shell, R.L. & Marvel, J.H. Modelling the reliability of repairable systems in aviation industry. *Comput. Indust. Engg.*, 2001, **40**, 51-63.

7. Scarf, P. A. On the application of mathematical models in maintenance. *Eur. J. Oper. Res.*,1997, **99**, 493-06.

8. Pidgeon, N. & Leary, O.M. Man-made disasters: Why technology and organisations (sometimes) fail. *Safety Science*, 2000, **34**, 15-30.

9. Ebling, C.E. *In* An introduction to reliability and maintainability engineering. Tata McGraw Hill, New Delhi, 1997. pp.6, pp.54, pp.58, pp.59, pp.299.

10. Ertas, A. & Jones, J. C. The engineering design process, Ed. 2. John Wiley, Toronto, 1996. 359 p.

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