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Exact Solutions of an Incompressible Fluid Flow of Second Order by Forced Oscillations on the Porous Boundary

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ABSTRACT

Exact solution of an incompressible fluid of second-order type by causing forced oscillations in the liquid of finite depth bounded by a porous bottom has been obtained. The results presented are in terms of nondimensional elastico-viscosity parameter (β) which depends on the non-Newtonian coefficient and the frequency of excitation (σ) of the external disturbance while considering the porosity (K) of the medium. The flow parameters are found to be identical with that of Newtonian case as $\beta \to 0$ and $K \to \infty$. It is seen that the effect of β and the porosity of the bounding surface has significant effect on the velocity parameter. Further, the nature of the paths of the fluid particles have also been studied with reference to β and the porosity of the bounding surface.

Keywords: Elastico-viscous fluid, second-order fluid, retarded history, porous media, fluid flow

NOMENCLATURE

$g(s)$	Given history
$g_\psi(s)$	Retarded history
α	Retardation factor
S	Stress tensor
P	Indeterminate hydrostatic pressure
U_i	Velocity component in the i^{th} direction
A_i	Acceleration component in the i^{th} coordinate
T	Time parameter
ϕ_1	Coefficient of viscosity
ϕ_2	Coefficient of elastico-viscosity
ϕ_3	Coefficient of cross-viscosity
u_i	Nondimensional velocity component along the i^{th} coordinate
k	Permeability of material in the dimensional form
ρ	Density of the fluid
β	Nondimensional elastico-viscosity parameter
P	Nondimensionalised indeterminate hydrostatic pressure
a_i	Nondimensionalised acceleration component in the i^{th} direction
L	Characteristic length
K	Nondimensionalised porosity factor
υ_c	Nondimensionalised cross-viscosity parameter

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σ Nondimensional frequency of excitation

F Nondimensional external force applied

ϕ Phase parameter

A Cross-sectional area of the filter bed

μ Coefficient of viscosity

q Flux of the fluid

V Velocity vector

η Unit vector along the gravitational force

G Gravitational force

1. INTRODUCTION

Viscous fluid flow over wavy wall had attracted the attention of relatively few researchers although the analysis of such flows finds application in different areas, such as transpiration cooling of reentry vehicles and rocket boosters, cross-hatching on ablative surfaces, and film vaporisation in combustion chambers, etc. Especially, where the heat and mass transfer takes place in the chemical processing industry, the problem by considering the permeability of the bounding surface in the reactors assumes greater significance.

In view of several industrial and technological importance, the problem of the exact solutions of two-dimensional flows of a second-order incompressible fluid has been examined by Pattabhi Ramacharyulu[1] considering rigid boundaries. Later, a linear analysis of the compressible boundary layer flow over a wall was presented by Lekoudis,[2] *et al*. Subsequently, Shankar and Sinha[3] studied the problem of Rayleigh for a wavy wall, while Lessen and Gangwani[4] examined the effect of small amplitude wall waviness on the stability of the laminar boundary layer. Further, the problem of free convective heat transfer in a viscous incompressible fluid confined between vertical wavy wall and a particle flat wall was examined by Vajravelu and Shastri[5], and thereafter, by Das and Ahmed[6]. Later, Patidar and Purohit[7] studied the free convective flow of a viscous incompressible fluid in porous medium between two long vertical wavy walls. Subsequently, Rajeev, Taneja, and Jain[8] had examined the problem of MHD flow with slip effects and temperature-dependent heat source in a viscous incompressible fluid confined between a long vertical wall and a parallel flat plate.

In all the above investigations, the fluid under consideration was viscous incompressible fluid and one of the bounding surfaces had a wavy character. Recently, Ramana Murthy and Kulkarni[9] examined the problem of elastico-viscous fluid of second-order type by causing disturbances in the liquid which was initially at rest and the bounding surface was subjected to sinusoidal oscillations. The results are presented in terms of nondimensional elastico-viscosity parameter (β) which depends on the non-Newtonian coefficient and the frequency of excitation (σ) of the external disturbance while considering the porosity (K) of the medium.

The aim of the present analysis is to examine the nature of the fluid flow and also to trace the paths of the fluid particles and examine the effects of various parameters by considering an additional property namely elastico viscosity and also by creating forced oscillations in the fluid of finite depth bounded by porous bottom. The free-surface condition on the top was taken into account.

2. MATHEMATICAL FORMULATION

In the sense of Noll[10], a simple material is a substance for which stress can be determined with entire knowledge of the history of the strain. This is called simple fluid, if it has property that at all local states, with the same mass density, are intrinsically equal in response, with all observable differences in response being due to definite differences in the history. For any given history $g(s)$, a retarded history $g_\psi(s)$ can be defined as:

$$g_\psi(s) = g(\psi s) : 0 \le s \le \infty,\ 0 \le \psi \le 1 \tag{1}$$

ψ being retardation factor. Assuming that the stress is more sensitive to recent deformation than to the deformations at distant past, it has been established by Coleman and Noll[11] that the theory of simple fluids yields the theory of perfect fluids as $\psi \to 0$ and that of Newtonian fluids as a correction (up to the order of ψ) to the theory of the perfect fluids. Neglecting all the terms of the order higher than two in ψ, the incompressible elastico-viscous fluid of second-order type have been considered whose constitutive relation is governed by:

$$S = -PI + \phi_1 E^{(1)} + \phi_2 E^{(2)} + \phi_3 E^{(1)^2} \tag{2}$$

where $E^1_{ij} = U_{i,j} + U_{j,i}$ (3)

and $E^2_{ij} = A_{i,j} + A_{j,i} + 2U_{m,i}U_{m,j}$ (4)

The coefficients ϕ_1 ϕ_2, and ϕ_3 are material constants. The constitutive relation for general Rivlin-Ericksen[12] fluid also reduces to Eqn (2) when the squares and higher orders of E^2 are neglected, with the coefficients being constants. Also, the non-Newtonian models considered by Reiner[13] could be obtained from Eqn (2) when $\phi_2 = 0$ and naming ϕ_3 as the coefficient of cross–viscosity. With reference to the Rivlin-Ericksen fluids, ϕ_2 may be called as the coefficient of elastico-viscosity.

It has been reported that a solution of poly-iso-butylene in cetane behaves as a second-order fluid. In many of the chemical processing industries, slurry adheres to the reactor vessels and gets consolidated. As a result, the chemical compounds within the reactor vessel percolates through the boundaries, causing loss of production and consuming more reaction time. In view of such technological and industrial importance wherein the heat and mass transfer takes place in the chemical industry, the problem of permeability of the bounding surfaces in the reactors attracts the attention of several investigators.

The aim is to study a class of exact solutions for the flow of incompressible fluid of second–order type the porosity factor of the bounding surfaces and comparing the results with those in the Newtonian case. The disturbance due to forced oscillations of a liquid of finite depth bounded by a porous bottom has been studied. The results are expressed in terms of a nondimensional porosity parameter K, which depends on the non-Newtonian coefficient ϕ_2 and the frequency of excitation σ. It is noticed that the flow properties are identical with those in the Newtonian case (K = ∞).

If $V\ (U_1, U_2, U_3)$ is the velocity component and $F\ (F_x, F_y, F_z)$ are the body forces acting on the system, then the equation of motion in X, Y and Z directions are given by

$$\rho \frac{DU_1}{DT} = \rho F_X + \frac{\partial S_{XX}}{\partial X} + \frac{\partial S_{XY}}{\partial Y} + \frac{\partial S_{XZ}}{\partial Z} \tag{5}$$

$$\rho \frac{DU_2}{DT} = \rho F_Y + \frac{\partial S_{YX}}{\partial X} + \frac{\partial S_{YY}}{\partial Y} + \frac{\partial S_{YZ}}{\partial Z} \tag{6}$$

$$\rho \frac{DU_3}{DT} = \rho F_Z + \frac{\partial S_{ZX}}{\partial X} + \frac{\partial S_{ZY}}{\partial Y} + \frac{\partial S_{ZZ}}{\partial Z} \tag{7}$$

where $\dfrac{D}{DT} = \dfrac{\partial V}{\partial T} + V.\nabla V$

If the bounding surface is porous, the rate of percolation of the fluid is directly proportional to the cross–sectional area of the filter bed and the total force, say the sum of the pressure gradient and the gravity force. In the sense of Darcy

$$q = CA(\frac{P_1 - P_2}{H_1 - H_2} + \rho G) \tag{8}$$

where $C = \dfrac{k}{\mu}$ in which k is the permeability of the material. Since this law is empirical, therefore to generalise this result, we must have the relation for variable thickness of the porous material. A straight forward generalisation of the Eqn (8) yields

$$V = -\frac{k}{\mu}[\nabla P + \rho G \eta] \tag{9}$$

where η is the unit vector along the gravitational force (G) taken in the –ve direction. If any other external forces are acting on the system, instead of G, then one has

$$V = -\frac{k}{\mu}[\nabla P - \rho F] \tag{10}$$

In the absence of external forces, $V = -\dfrac{k}{\mu}\nabla P$, which gives $\nabla P = -\dfrac{\mu}{k}V$

Therefore, the net resulting equation (in the dimensional form) of motions in the X, Y, and Z directions are

$$\rho\frac{DU_1}{DT}=\rho F_X+\frac{\partial S_{XX}}{\partial X}+\frac{\partial S_{XY}}{\partial Y}+\frac{\partial S_{XZ}}{\partial Z}-\frac{\mu}{k}U_1 \quad (11)$$

$$\rho\frac{DU_2}{DT}=\rho F_Y+\frac{\partial S_{YX}}{\partial X}+\frac{\partial S_{YY}}{\partial Y}+\frac{\partial S_{YZ}}{\partial Z}-\frac{\mu}{k}U_2 \quad (12)$$

$$\rho\frac{DU_3}{DT}=\rho F_Z+\frac{\partial S_{ZX}}{\partial X}+\frac{\partial S_{ZY}}{\partial Y}+\frac{\partial S_{ZZ}}{\partial Z}-\frac{\mu}{k}U_3 \quad (13)$$

Introducing the following nondimensional variables as

$$U_i=\frac{\phi_1 u_i}{\rho L}\quad T=\frac{\rho L^2 t}{\phi_1}\quad \phi_2=\rho L^2\beta\quad P=\frac{\phi_1^2 p}{\rho L^2}$$

$$\frac{X_i}{L}=x_i\quad \frac{Y_i}{L}=y_i\quad \phi_3=\rho L^2\upsilon_c\quad A_i=\frac{\phi_1^2 a_i}{\rho^2 L^3}$$

$$S_{i,j}=\frac{\phi_1^2 s_{i,j}}{\rho L^2}\quad E_{i,j}^{(1)}=\frac{\phi_1 e_{i,j}^{(1)}}{\rho L^2}\quad E_{i,j}^{(2)}=\frac{\phi_1^2 e_{i,j}^{(2)}}{\rho^2 L^4}\quad k=\frac{L^2 K}{\rho}$$

$$F_i=\frac{\phi_1^2}{\rho^2 L^3}f_i\quad \mu=\frac{\phi_1}{\rho}\quad V=\frac{\phi_1 v}{\rho L}$$

where T is the (dimensional) time variable.

The nondimensional form of Eqn (10) will now be

$$v=-K(\nabla p-f) \quad (14)$$

In the absence of external forces $v=-K\nabla p$ which yields

$$\nabla p=-\frac{v}{K} \quad (15)$$

A class of plane flows given by the velocity components has been considered

$$u_1=u(y,t)\ \text{ and } u_2=0 \quad (16)$$

in the directions of rectangular Cartesian coordinates x and y. The velocity field given by Eqn (16) identically satisfies the incompressibility condition. The stress can now be obtained in a nondimensional form as

$$s_{xx}=-p+\upsilon_c\left(\frac{\partial u}{\partial y}\right)^2 \quad (17)$$

$$s_{yy}=-p+(\upsilon_c+2\beta)\left(\frac{\partial u}{\partial y}\right)^2 \quad (18)$$

$$s_{xy}=\frac{\partial u}{\partial y}+\beta\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial t}\right) \quad (19)$$

In view of the above, the equation of motion in the x-direction is given by

$$\frac{\partial u}{\partial t}=-\frac{\partial p}{\partial x}+\frac{\partial^2 u}{\partial y^2}+\beta\frac{\partial}{\partial t}\left(\frac{\partial^2 u}{\partial y^2}\right)-\frac{1}{K}u+f_x \quad (20)$$

where f_x is the external force acting along the x-direction.

The equation of motion in the y-direction in the absence of any external forces is given by

$$0=-\frac{\partial p}{\partial y}+(2\beta+\upsilon_c)\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)^2 \quad (21)$$

The pressure gradient in Eqn (20) can only be a function of time for this flow.

Using Eqn (20) if

$$-\frac{\partial p}{\partial x}=\xi(t),\quad \int_{p_0(t)}^{p}\partial p=-\xi(t)\int_0^x\partial x$$

$$p=p_0(t)-\xi(t)x \quad (22)$$

where $p_0(t)$ is the initial pressure.

From Eqn (21)

$$\frac{\partial p}{\partial y}=(2\beta+\upsilon_c)\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)^2$$

which on integration wrt y yields

$$p = (2\beta + \upsilon_c)(\frac{\partial u}{\partial y})^2 + \text{a function of } x \text{ say } \lambda(x) \quad (23)$$

Using Eqn (22) and Eqn (23), one has

$$\lambda(x) = p_0(t) - \xi(t) - (2\beta + \upsilon_c)(\frac{\partial u}{\partial y})^2 \quad (24)$$

Using Eqn (24) in Eqn (23), one has

$$p = p_0(t) - \xi(t)x \quad (25)$$

Considering $\xi(t)$ = 0, and using Eqn (25) in Eqn (20), the flow characterised by the velocity is given by

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + \beta\frac{\partial}{\partial t}(\frac{\partial^2 u}{\partial y^2}) - \frac{1}{K}u + f_x \quad (26)$$

where K is the nondimensional porosity constant and f_x is the external force (nondimensional) acting along the *x*-direction. It may be noted that the presence of β changes the order of differential from two to three.

3. FORCED OSCILLATIONS OF A LIQUID OF FINITE DEPTH BOUNDED BY A RIGID BOTTOM

Let the fluid of the depth L_h bounded by the rigid bottom $y = 0$ be influenced by the (nondimensional) external force $Fe^{i\sigma\tau}$ in the *x*-direction. In such a situation, Eqn (26) will now get modified as

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + \beta\frac{\partial}{\partial t}(\frac{\partial^2 u}{\partial y^2}) - \frac{1}{K}u + Fe^{i\sigma t} \quad (27)$$

with the no-slip condition at the bottom

$$u(0,t) = 0 \quad (28)$$

and the free surface condition on the top

$$s_{xy} = 0, \text{ at } y = h \quad (29)$$

Assuming the trial solution as

$$u(y,t) = Ff(y)e^{i\sigma t} \quad (30)$$

$$f''(y) - p^2 f(y) = -\frac{1}{(1+i\beta\sigma)} \quad (31)$$

where $p^2 = \dfrac{i\sigma + \frac{1}{K}}{1+i\beta\sigma} = \dfrac{(\beta\sigma^2 + \frac{1}{K}) + i(\sigma - \frac{\beta\sigma}{K})}{(1+\beta^2\sigma^2)}$ (32)

When expressed in polar form

$$p = r(\cos(\frac{\pi}{4} - \frac{\varepsilon}{2}) + i\sin(\frac{\pi}{4} - \frac{\varepsilon}{2})) \quad (33)$$

where $r = \dfrac{[(\beta\sigma^2 + \frac{1}{K})^2 + (\sigma - \frac{\beta\sigma}{K})]^{1/4}}{\sqrt{(1+\beta^2\sigma^2)}}$,

$$\varepsilon = \tan^{-1}(Q) \text{ and } Q = \frac{\frac{1}{K} + \beta\sigma^2}{\sigma - \frac{\beta\sigma}{K}}$$

Also the conditions satisfied by $f(y)$ are

$$f(0) = 0, \quad (34)$$

This yields the solution

$$f(y) = \frac{1}{p^2(1+i\beta\sigma)}\left[1 - \frac{\cosh p(h-y)}{\cosh ph}\right] \quad (35)$$

In view of Eqn (30)

$$u(y,t) = RP\frac{Fe^{i\sigma t}}{p^2(1+i\beta\sigma)}\left[1 - \frac{\cosh p(h-y)}{\cosh ph}\right] \quad (36)$$

hence,

$$u(y,t) = \frac{F[A-B]}{[\frac{1}{K^2} + \sigma^2][\cosh(2ah) + \cos(2bh)]} \quad (37)$$

where

$$A = \begin{pmatrix} (\frac{\cos\sigma t}{K} + \sigma\sin\sigma t)\{\cosh 2ah + \cos 2bh - \\ \cosh a(2h-y)\cos by - (\cosh ay)\cos b(2h-y)\} \end{pmatrix} \quad (38)$$

$$B = \left(\begin{array}{l} (\frac{\sin \sigma t}{K} - \sigma \cos \sigma t)\{\sinh a(2h-y)\sin by \\ + \sinh ay \sin b(2h-y)\} \end{array} \right) \tag{39}$$

On, $y = h$, the velocity is

$$u(h,t) = \frac{F[A_1 - B_1]}{[\frac{1}{K^2} + \sigma^2][\cosh(2ah) + \cos(2bh)]} \tag{40}$$

where

$$A_1 = \left(\begin{array}{l} (\frac{\cos \sigma t}{K} + \sigma \sin \sigma t)\{\cosh 2ah + \\ \cos 2bh - 2\cosh ah \cos bh\} \end{array} \right) \tag{41}$$

$$B_1 = \left((\frac{\sin \sigma t}{K} - \sigma \cos \sigma t)\{2\sinh ah \sin bh\} \right) \tag{42}$$

The paths of particles may be obtained by integrating Eqn (36) wrt t

$$x = RP \frac{Fe^{i\sigma t}}{i\sigma p^2 (1 + i\beta\sigma)} \left[1 - \frac{\cosh p(h-y)}{\cosh ph} \right] \tag{43}$$

The constant of integration may be conveniently taken to be zero for particles starting from the same point (taken as origin) on the bottom.

For the case of large h,

$$u = \frac{F}{\frac{1}{K^2} + \sigma^2} \left[\begin{array}{l} (\frac{\cos \sigma t}{K} + \sigma \sin \sigma t) + \\ e^{-yr\cos(\frac{\pi}{4} - \frac{\varepsilon}{2})} . \sin\left\{ yr \sin(\frac{\pi}{4} - \frac{\varepsilon}{2}) - \sigma t \right\} \end{array} \right] \tag{44}$$

which for $y >> \delta$, reduces to

$$u \approx \frac{F}{\frac{1}{K^2} + \sigma^2} \left(\frac{\cos \sigma t}{K} + \sigma \sin \sigma t \right) \tag{45}$$

where $\delta = \frac{1}{r}$

The paths of the particles in this case are given by

$$x = \frac{F}{\frac{1}{K^2} + \sigma^2} \left[\begin{array}{l} (\frac{\sin \sigma t}{K\sigma} - \sin \sigma t) + e^{-yr\cos(\frac{\pi}{4} - \frac{\varepsilon}{2})} . \\ \cos\left\{ yr \cos(\frac{\pi}{4} - \frac{\varepsilon}{2}) - \sigma t \right\} \end{array} \right] \tag{46}$$

which for $y >> \delta$,

$$x \approx \frac{F}{\frac{1}{K^2} + \sigma^2} \left(\frac{\sin \sigma t}{K\sigma} - \cos \sigma t \right) \tag{47}$$

$$x^* = \frac{RPx\left(\frac{1}{K^2} + \sigma^2 \right)}{F\left(\frac{\sin \sigma t}{K\sigma} - \cos \sigma t \right)} \tag{48}$$

$$y^* = \frac{y(1 + \beta^2\sigma^2)^{\frac{1}{4}}}{\sqrt{2}} [\frac{(\frac{1}{K} + \beta\sigma^2)^2 + (\sigma - \frac{\beta\sigma}{K})^2}{(1 + \beta^2\sigma^2)^2}]^{\frac{1}{4}} \tag{49}$$

Phase parameter in this case is given by

$$\phi = \frac{\varepsilon}{\sigma} = \frac{\tan^{-1}(Q)}{\sigma} \tag{50}$$

4. DISCUSSIONS AND CONCLUSIONS

As $K \to \infty$, the results obtained for the velocity field, paths of the particles are in agreement with those of Pattabhi Ramacharyulu[1]. In the absence of external forces, the results coincide with that of Ramana Murthy and Kulkarni[9]. The case of Newtonian fluid can be realised as $\beta \to \infty$.

Figure 1 illustrates the effect of elastico-viscosity on the velocity profiles. It is observed that as the elastico-viscosity increases, there is a decreasing trend in the velocity of the fluid particles. And at times, it is also noticed that there is a back flow at the boundary layer. This is in agreement with the real-life situation due to the fact that the intramolecular forces are strong as elastico-viscosity increases, which results in the decrease of fluid velocity. Further, it is seen that as the porosity of the plate increases, the velocity profiles are found to be more significantly distributed.

The effect of porosity on the nature of the velocity profiles is illustrated in Fig. 2. In each situation, it is noticed that as the porosity increases, the fluid velocity also increases. The profiles are significantly distributed and are found to be more parabolic in the case with the inclusion of elastico-viscosity term in the governing equation of motion.

Figure 3 illustrates the effect of the frequency of excitation of the bounding surface on the velocity profiles. In the case of Newtonian fluid ($\beta = 0$), it is observed that as the frequency of excitation increases, the fluid velocity also increases. The velocity profiles are found to be parabolic in nature. However, a reverse trend is observed in case of fluid whose elastico-viscosity parameter β=0.6. As the elastico-viscosity of the fluid increases, which in turn can be attributed to the strong intramolecular forces, the velocity decreases, i.e., the bulk of the fluid behaves like a rigid body.

The effect of elastico-viscosity of the fluid on the velocity profiles has been illustrated in Fig. 4 for the case of t = 0.5 and t = 1 respectively. It is observed that as the elastico-viscosity increases, the fluid velocity decreases and even a back flow is observed in certain cases. Further, over a period of time as increases, the fluid velocity increases.

Figure 5 illustrates the paths of the fluid particles wrt the variation in the elastico-viscosity of the fluid. As the elastico-viscosity increases, the trace of the fluid particles are shifted more towards left. As the porosity of the plate decreases from k = 10 to k = 0.1, it is seen from Fig. 6 that the paths of the fluid particles are more drifted to left.

The effect of the elastico-viscosity on the paths of the fluid particles for t = 0.1 and t = 1, i.e., at the very short duration are illustrated in Fig. 7. Initially, as the elastico-viscosity of the fluid increases,

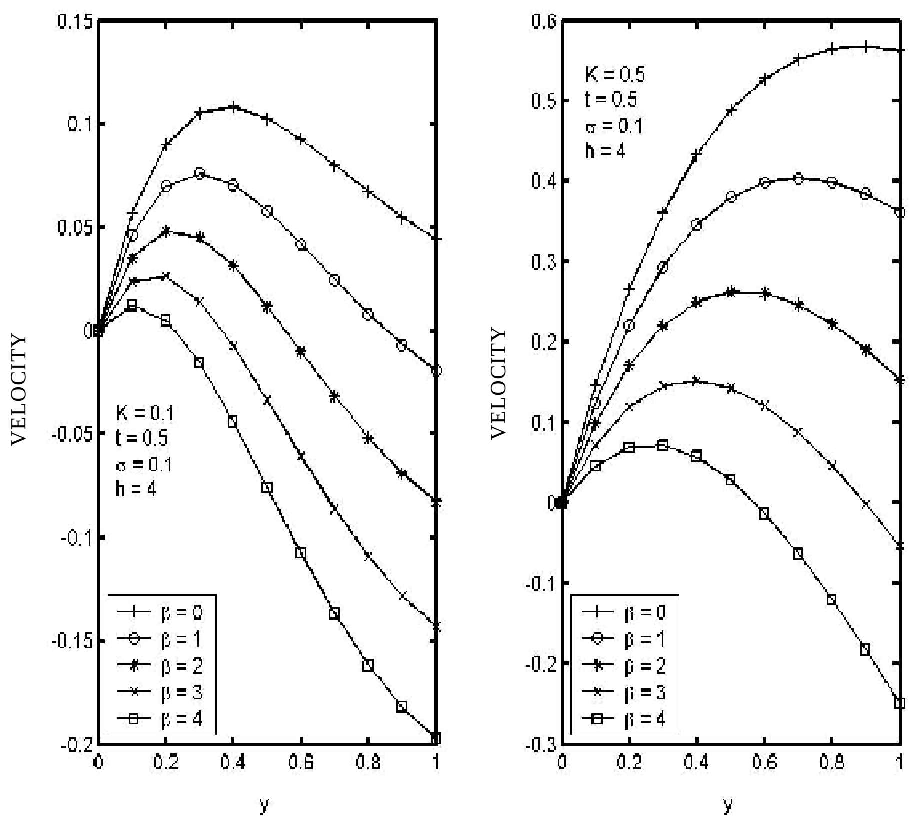

Figure 1. Effect of elastico-viscosity (β) on the velocity profiles.

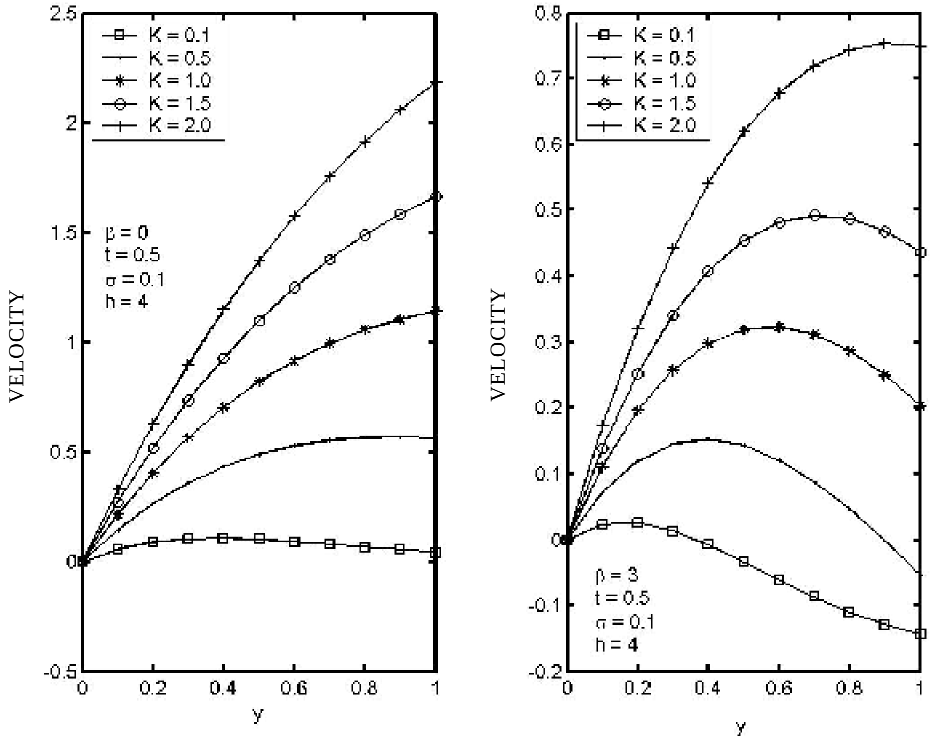

Figure 2. Effect of porosity (*K*) on the velocity profiles.

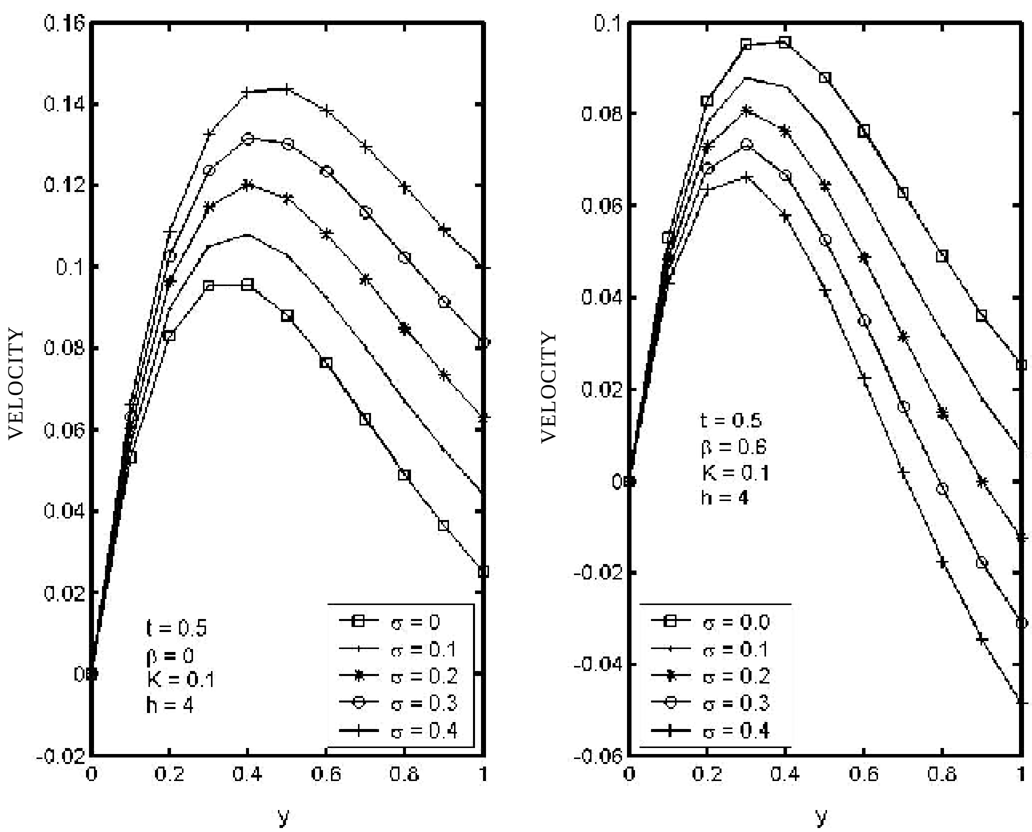

Figure 3. Effect of σ (frequency of excitation) on the velocity profiles.

the paths of the fluid particles are more parabolic and are drifted to right. However, the reverse trend is observed for t = 1.

Figure 8 illustrates the effect of the elastico-viscosity of the fluid on the phase parameter. It is seen that for a constant value the frequency of

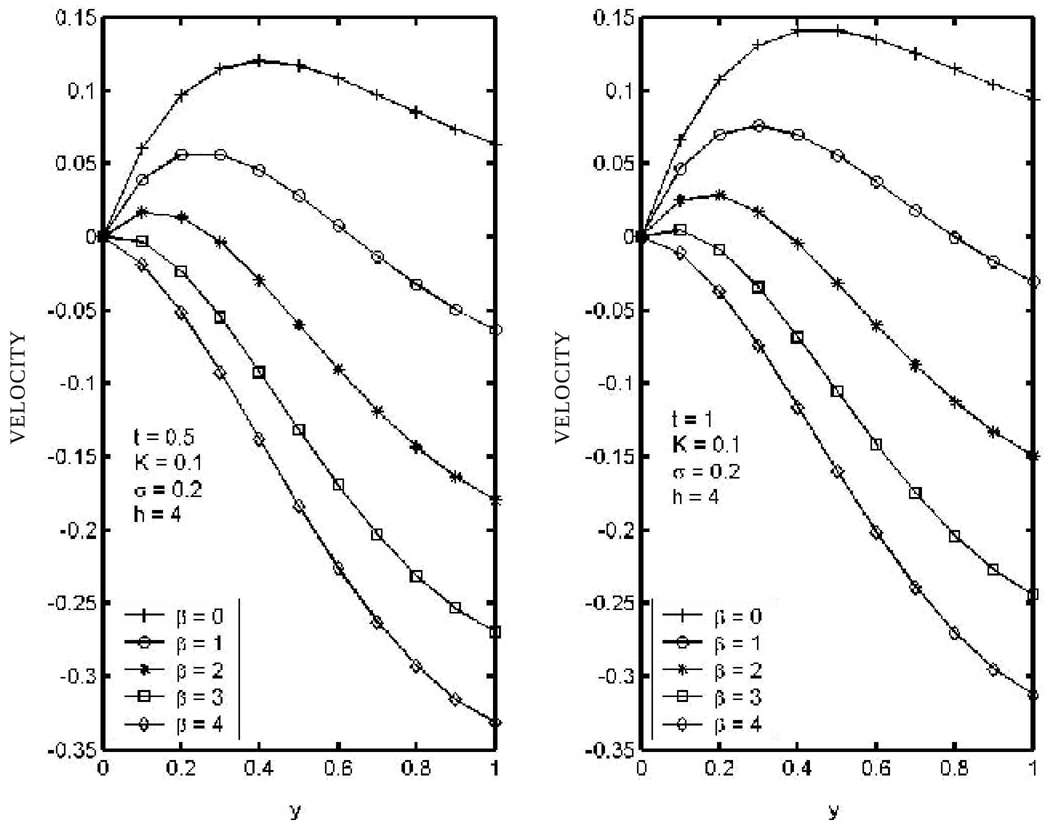

Figure 4. Effect of elastico-viscosity (β) on the velocity profiles.

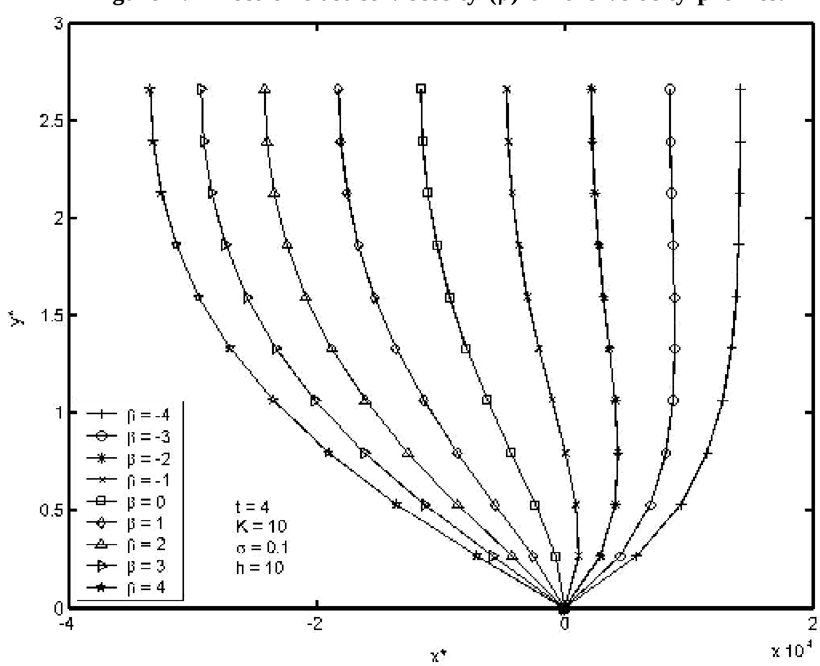

Figure 5. Effect of elastico-viscosity (β) on the paths of the fluid particles for *K*=10.

the excitation of the bounding surface, the phase parameter increases as the elastico-viscosity increases. Further, for any fluid under consideration, the phase parameter and the frequency of excitation of the bounding surfaces are inversely proportional to each other.

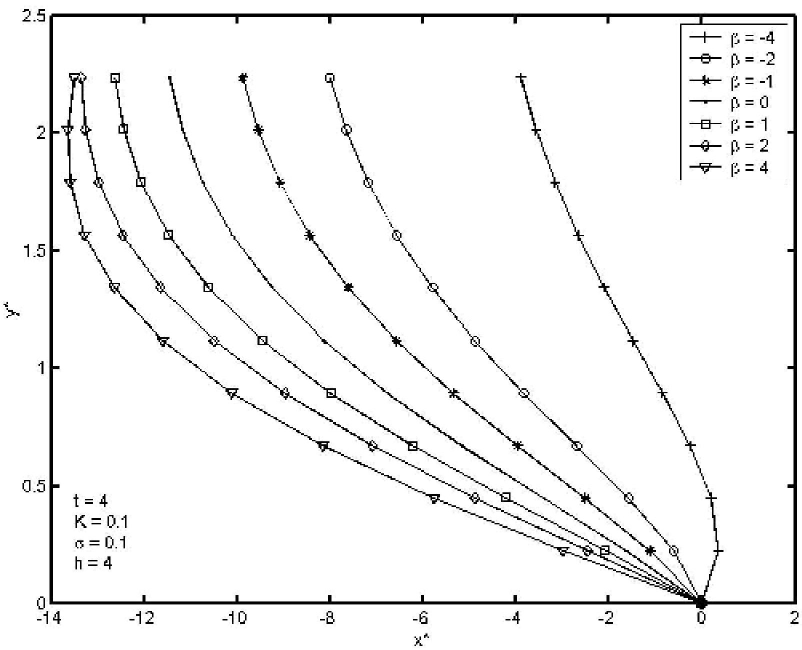

Figure 6. Effect of elastico-viscosity (β) on the paths of the fluid particles for *K*=0.1.

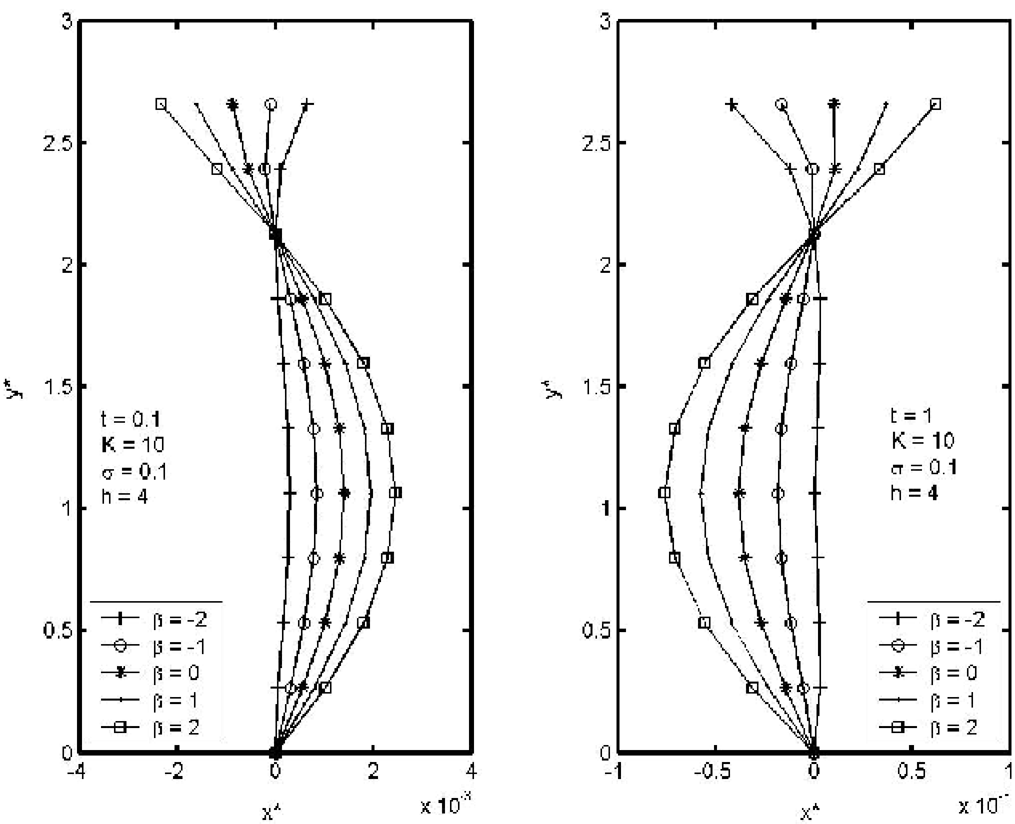

Figure 7. Effect of elastico-viscosity (β) on the paths of the fluid particles.

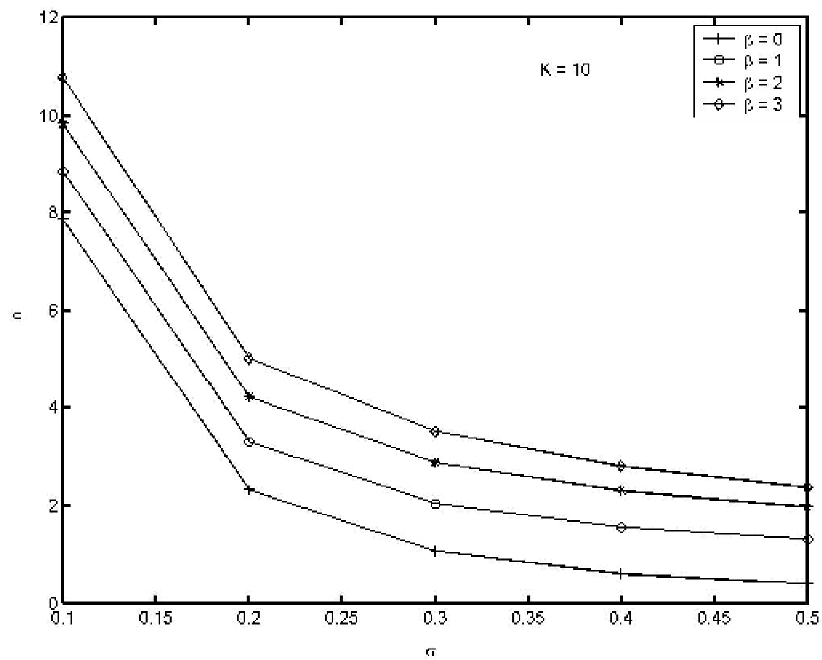

Figure 8. Effect of elastico-viscosity (β) on the phase parameter.

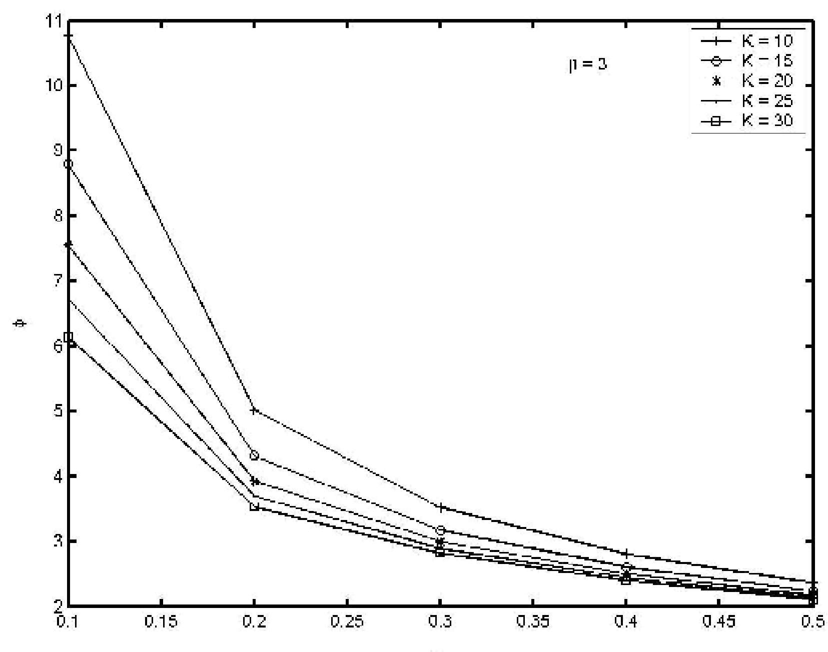

Figure 9. Effect of porosity (*K*) on the phase parameter.

The effect of the porosity of the bounding surface on the phase parameter of the fluid is illustrated in Fig. 9. It is seen that as the porosity of the plate increases, the phase parameter decreases. Further, it can also be noticed that the phase parameter and the frequency of excitation are inversely related to each other.

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