Kinematic Analyses of Metallic Plate Perforation by Penetrators with Various Nose Geometries

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ABSTRACT

This study analyses kinematics of a metallic plate perforation by a penetrator with truncated ogive nose geometry to find solutions also to blunt, conical, ogive, and hemi-spherical nosed penetrators. Plugging, ductile hole enlargement, dishing, and petal forming failure modes are used in the analyses. Acceleration throughout perforation is calculated by using the related failure mode, analytical model, and the target-penetrator interaction geometry. Depending on the failure model; back lip and front lip formation during ductile hole enlargement, plug formation during plugging, and deflection of target plate during dishing is also analysed. Analyses are based on projectile’s equation of motion, momentum and energy equations, and projectile-target plate interactions. The analyses results for selected cases, with the impact velocity range 215-863 m/s, are compared with the test data. The residual velocity estimation for a strike velocity is close to the related test data with an error of 0.3-2.2 %, except for conical nosed penetrators at impact velocities approaching the ballistic limit velocity.

Keywords: Perforation kinematics; Penetrator-plate interaction; Nose geometry; Limit velocity; Terminal ballistics

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>A</td>
<td>The displacement of stress wave in radial direction</td>
</tr>
<tr>
<td>b</td>
<td>The target plate thickness</td>
</tr>
<tr>
<td>c</td>
<td>Velocity of stress wave</td>
</tr>
<tr>
<td>c₁</td>
<td>Parameter for static target resistance stress</td>
</tr>
<tr>
<td>c₂, c₃</td>
<td>Parameter for dynamic target resistance stress</td>
</tr>
<tr>
<td>dₚ</td>
<td>Shank diameter of the projectile</td>
</tr>
<tr>
<td>D</td>
<td>Flexural rigidity of the target plate per unit width</td>
</tr>
<tr>
<td>εₛ</td>
<td>Strain energy stored by per unit volume of the target plate</td>
</tr>
<tr>
<td>E</td>
<td>Young modulus</td>
</tr>
<tr>
<td>Eₜ</td>
<td>The energy dissipated through friction</td>
</tr>
<tr>
<td>Eₚ</td>
<td>The energy lost by the projectile</td>
</tr>
<tr>
<td>Eₛ</td>
<td>Target plate’s strain energy</td>
</tr>
<tr>
<td>Eₜ</td>
<td>Energy absorbed in target</td>
</tr>
<tr>
<td>F</td>
<td>The target material resistance to penetration</td>
</tr>
<tr>
<td>Fₛ</td>
<td>Impact force on the target panel</td>
</tr>
<tr>
<td>h</td>
<td>Depth of penetration of the penetrator’s nose tip at time t</td>
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<tr>
<td>K</td>
<td>Bulk modulus</td>
</tr>
<tr>
<td>L</td>
<td>Nose length of the penetrator</td>
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<tr>
<td>Lₚ</td>
<td>Effective length</td>
</tr>
<tr>
<td>mₚ</td>
<td>Penetrator’s mass</td>
</tr>
<tr>
<td>mₚₜ</td>
<td>Plug mass at time t</td>
</tr>
<tr>
<td>M₟</td>
<td>Bending moment (radial)</td>
</tr>
<tr>
<td>Mₕ</td>
<td>Bending moment (tangential)</td>
</tr>
<tr>
<td>n</td>
<td>Target plate’s strain hardening exponent</td>
</tr>
<tr>
<td>Pₖ</td>
<td>The pressure required for cavity expansion</td>
</tr>
<tr>
<td>P₀</td>
<td>The required pressure to yield the target plate</td>
</tr>
<tr>
<td>Q₟</td>
<td>Shear force per unit length at distance r</td>
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<tr>
<td>Rₜ</td>
<td>Target’s resistance stress to penetration</td>
</tr>
<tr>
<td>Rₜₑᶠ</td>
<td>Target plate’s effective resistance stress</td>
</tr>
<tr>
<td>Rₜₑᶠₛ</td>
<td>Resistance stress of the target plate under compression</td>
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<tr>
<td>rₖ</td>
<td>Cavity radius</td>
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<td>rₚ</td>
<td>Shank radius of the penetrator</td>
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<td>rₚₚ</td>
<td>Plastic zone size</td>
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<td>rₜ</td>
<td>Distance of penetrator’s tip to the target plate’s rear surface</td>
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<td>rₜₑ</td>
<td>Radius at the truncated nose tip</td>
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<tr>
<td>SCE</td>
<td>Spherical cavity expansion</td>
</tr>
<tr>
<td>t</td>
<td>Time</td>
</tr>
<tr>
<td>V</td>
<td>The penetrator’s instantaneous velocity</td>
</tr>
<tr>
<td>Vₜ</td>
<td>Penetrator’s instantaneous velocity at penetration depth h</td>
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1. INTRODUCTION

Objective of the study is to analyse perforation kinematics of metallic target plates at different thicknesses, impacted by the penetrators at different impact velocities, with different masses, and nose geometries, with the intention to obtain acceleration–time, and instantaneous velocity – time graphs for various penetrator-target plate combinations, by studying penetrator nose and target plate interaction, using the appropriate failure mechanisms that require only common test data, and to verify the analytical solutions by using the test data.

Penetration of a plate by a penetrator which has a defined nose profile is studied widely\(^4\). They are reviewed by the references, such as\(^2,3,6\). However, there is a need for a common model that can be applied to the penetrators with various nose geometries.

1.1 Projectile’s Equation of Motion

With the assumption of constant cross-sectional area for the penetrator’s body, analysis of perforation of target plates by the penetrators can be made by using projectile’s equation of motion, as given below\(^4\).

\[
\rho_p L_{\text{eff}} \frac{dV}{dt} = -R_t = -(c_1 + c_2V + c_3V^2) \tag{1}
\]

Eqn (1) yields to Resal equation\(^5\) when \(c_2=0\), and to Poncelet equation\(^7\) when \(c_3=0\).

1.1.1 Resistance to Penetration

Eqn (1) implies that \(R_t\) is constant or function of \(V\). \(R_t\) is a function of \(V_t\) as \(R_t = k_1V_t^2\), where \(k_1\) is 0.5, 0.5 or 1.33\(^2\), 1.92\(^8\), 2.0\(^9\), 3-6\(^10\).

1.1.2 Equation of Motion (Poncelet Equation)

Penetration \(h\) can be calculated through integral of Eqn (1) and taking first term of serial expansion of logarithm within limits from \(V\) to zero\(^1\).

\[
h = \frac{p_0V_s^2}{L_{\text{eff}}} \tag{2}
\]

It is shown that effect of \(V\) on \(R_t\) is negligible also at very thin plates\(^11\).

When \(h=b\), \(V\) becomes \(V_o\), which can be calculated approximately from Eqn (2) as:

\[
V_o \approx \sqrt{\frac{2c_1b}{\rho_p L_{\text{eff}}}} \tag{3}
\]

Using \(R_{\text{eff}}\) in place of \(c_1\), as proposed by Rosenberg and Dekel\(^12\) yields to the same energy loss of the projectile. This approach implies that \(c_2 = 0\) and \(C_t = R_{\text{eff}}^{-1}\).

1.1.3 Balance of Energy

Energy transfer between penetrator and target could be shown as\(^13\).

\[
E_p = E_i = E_s + E_j + W_p \tag{4}
\]

\(E_j\) could be neglected.

1.1.4 The Strain Energy Stored During Penetration

The stress-strain relationship for elastic and plastic deformation could be written as:

\[
\sigma = \begin{cases} \frac{E\varepsilon}{n} & \text{for } \varepsilon \leq \varepsilon_y \\ \frac{E\varepsilon}{\varepsilon_y} & \text{for } \varepsilon > \varepsilon_y \end{cases} \tag{5}
\]

Substituting into the following one would yield to \(e_y^s\):

\[
e_y^s = \int_0^{e_y} \sigma \varepsilon d\varepsilon = \int_0^{e_y} E\varepsilon \varepsilon d\varepsilon + \int_0^{e_y^s} \frac{E\varepsilon}{\varepsilon_y} \varepsilon d\varepsilon = \frac{\varepsilon_y^2}{2} + \frac{\varepsilon_y^{1+n}}{1+n} \tag{6}
\]

\(E_s\) is found by taking the integral of \(e_y\) over the volume.

Equating \(E_p\) to the case with \(V_o\) and dividing both sides of the equality with \(V_o^2\) yields to the following normalised equation.

\[
\frac{V}{V_o} = \sqrt{\frac{V_s^2}{V_o^2} - 1} \tag{7}
\]

1.2 Basic Failure Modes

Basic failure modes are briefly explained as follows.

1.2.1 Ductile Hole Enlargement

Ductile hole enlargement is the penetration mode for penetrators with pointed nose if \(b/d \geq 0.1\). SCE theory is widely used in describing ductile hole enlargement. Static SCE modes are analysed by various researchers\(^13-17\). Bishop et al.\(^14\) consider \(P_e\) as the required work for generating a cavity of unit volume. \(P_e\) is approximately equal to \(R_t\). The following equation for \(P_e\) is proposed by Hill\(^15\).

\[
P_e = \frac{2}{3} Y_t \left[ 1 + \ln \left( \frac{E}{3Y_t(1-\nu)} \right) \right] \tag{8}
\]
Satapathy\textsuperscript{18} proposes the following Eqn to consider thickness effect on $P_c$:

$$R = P_c = \frac{2Y_p}{3}\left[1 - \left(\frac{r_p}{r_t}\right)^3\right] - \frac{2}{3}Y_t\ln\left[\frac{Y_t}{2\mu}\left(1 + \frac{4\mu}{3\lambda + 2\mu}\left(\frac{r_p}{r_t}\right)^3\right)\right]$$

(9)

$r_{pl}$ is calculated as:

$$r_{pl} = \left[\frac{Y_p}{6\mu} + \left(\frac{Y_p}{6\mu}\right)^3 + \frac{4}{3}\frac{Y_t}{3\lambda + 2\mu}\left(\frac{r_p}{r_t}\right)^3\right]^{\frac{1}{3}}$$

(10)

$r_c$ is found by using Tate equation\textsuperscript{19},

$$r_c = \left[1 + 2\rho_p\left(V_s - V_t\right)^2\right] R_t$$

(11)

$V$ is calculated by using:

$$Y_p + \frac{1}{2}\rho_p\left(V_t - V_s\right)^2 = \frac{1}{2}\rho V^2 + R_t$$

(12)

where $Y_p = 1.7\sigma_p$ and $R_t$:

$$R_t = \sigma_t\left[\frac{2}{3} + \ln\left(\frac{0.57E_s}{\sigma_t}\right)\right]$$

(13)

### 1.2.1.1 Energy Balance

The penetrator’s kinetic energy is used for the work of volume change. The energy balance at $h$:

$$\frac{1}{2}m_p(V_t^2 - V_h^2) = R_t\Delta v_h$$

(14)

$V_h$ is calculated by studying interaction of the penetrator’s nose with target plate, as explained in section 2.1. The analysis is based on a truncated (blunt) ogive nose geometry. Solutions to other nose geometries are obtained from the blunt ogive analysis.

### 1.2.2 Plugging

The references\textsuperscript{20-21} provide basic theory on plugging. Plugging usually occurs when $V_s$ is close to $V_0$. Conservation of momentum for the plug attached to the penetrator:

$$m_p V_s = (m_p + m_{plug}) V_t$$

(15)

Conservation of energy, with the assumption of adiabatic process, can be written as:

$$\frac{1}{2}m_p V_s^2 = \frac{1}{2}(m_p + m_{plug}) V_t^2 + W_p + E_s$$

(16)

When $V_s = V_0$, $V_t = 0$, $W_p = m_p V_s^2 / 2$. Substituting these values into Eqn (16) and neglecting $E_s$ results in\textsuperscript{10}:

$$\frac{V_s}{V_0} = \sqrt{\frac{1}{1 + m_{plug}/m_p} \left(\frac{V_s^2}{V_0^2} - 1\right)}$$

(17)

For the blunt nosed penetrators, Eqn (17) takes the form:

$$\frac{V_s}{V_0} = \sqrt{\frac{1}{1 + m_{plug}/m_p} \left(\frac{V_s^2}{V_0^2} - 1\right)}$$

(18)

For thin plates ($b \leq 2r_p$), Eqs (17) and (18) convert to Eqn (7) if $m_{plug} = 0$.

### 1.2.3 Dishing

Dishing, analysed by Woodward and Cimporeu\textsuperscript{22}, is due to stretching and bending of the plate around the impact point. Impact creates stress waves with velocity $c$, and imposes a transverse $F_s$ on the target, which is equal to $R_t$. Figure 1 shows the internal loads at $r$ from the strike point.

$Q_r$ is calculated for a clamped-edge circular plate as\textsuperscript{1}:

$$Q_r = D\frac{d}{dr}\left(\frac{d^2 w}{dr^2} + \frac{1}{r}\frac{dw}{dr} - r^2\right) = \frac{F}{2\pi r}$$

(19)

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right] = \frac{Q_r}{D}$$

(20)

where $D$ is:

![Figure 1. Penetrator’s impact at a panel: (a) Overall view, (b) Stressed region, (c) Internal loads at $r$, (d) Internal loads at $r_0$.](image-url)
c is calculated as
\[ c = \sqrt{\frac{K}{\rho_1}} = \sqrt{\frac{1}{E}} \cdot \sqrt{\frac{3(1-2\nu)}{\rho_1}} \]  
(22)

Ballistic design requires high specific stiffness to transmit stress wave far from the impact point\(^1\). The deflection \( w \) is calculated as follows by using Eqns (19) & (20):
\[ w = \frac{F_s}{16\pi D} \left( 2r^2 - \frac{a^2}{3} - r^2 \right) \]  
(23)

Where \( a = ct. \) \( w \) is maximum at \( r=0 \):
\[ w_{\text{max}} = \frac{F_s a^2}{16\pi D} \]  
(24)

\( M_r \) and \( M_\theta \) are found to be:
\[ M_r = D \left( \frac{d^2w}{dr^2} + \frac{v}{r} \frac{dw}{dr} \right) = \frac{F_s}{4\pi} \left[ 1 + (1+\nu)\ln \frac{r}{a} \right] \]  
(25)
\[ M_\theta = D \left( \frac{v}{r} \frac{d^2w}{dr^2} + \frac{1}{r} \frac{dw}{dr} \right) = \frac{F_s}{4\pi} \left[ \nu + (1+\nu)\ln \frac{r}{a} \right] \]  
(26)

\( M_r \) and \( M_\theta \) at \( r=0 \) could be found as:\(^2\)
\[ M_r = M_\theta = \frac{F_s}{4\pi} \left[ (1+\nu)\ln \frac{a}{0.325b} \right] \]  
(27)

The stresses at \( r=0 \):
\[ \sigma_{rr} = \sigma_{\theta\theta} = \frac{-12z}{b^2} M = \frac{-3zF_s}{\pi b^2} \left[ (1+\nu)\ln \frac{a}{0.325b} \right] \]  
(28)

The bending stress is maximum at \( r=0 \):
\[ \sigma_{rr}^{\text{max}} = \sigma_{\theta\theta}^{\text{max}} = \frac{6}{b^2} M = \frac{-3F_s}{2\pi b^2} \left[ (1+\nu)\ln \frac{a}{0.325b} \right] \]  
(29)

\( F_s \) is found by equating \( E_s \) to projectile’s kinetic energy at \( V_0 \):
\[ E_s = E_i = E_s + W_p = \frac{1}{2} m_p V_0^2 = \frac{1}{2} F_s w_{\text{max}} = \frac{F_s a^2}{32D} \]  
(30)
\[ F_s = \sqrt{16D m_p V_0^2} = \frac{V_0}{a} \sqrt{\frac{4 Eb^3 m_p}{3(1-\nu^2)}} \]  
(31)

1.2.4 Petal Formation
Petal formation is observed with sharp-nosed projectiles when \( b/d < 0.1 \). It occurs when \( \sigma_{rr} \) and \( \sigma_{\theta\theta} \) reach \( R_r \) and \( V_s \) is close to \( V_0 \).

2. Methodology
Kinematics of plate perforation by a penetrator with blunt ogive nose is analysed to find solutions to the penetrators with all common nose geometries such as ogive, hemi-spherical, conical, and blunt. Acceleration and velocity histories during perforation are obtained by first deciding on the failure mode, and then using the related analytical model and the penetrator-target plate interaction geometry.

2.1 Analyses of Perforation
2.1.1 Blunt Ogive Nosed Penetrator
Perforation through hole enlargement or plugging of a target plate by a truncated ogive projectile is idealized and schematically shown in Fig. 2, with the four cases that might occur: 1) \( h < b \) and \( L \), 2) \( b < h < L \), 3) \( h > b \) and \( L \), 4) \( L < h < b \). There are two coordinate systems: a system moving with impactor \( (r \) and \( x) \), and a fixed system \( (h) \)\(^2\). During hole enlargement, lips are created, whose volume is assumed to be equal to the volume swept by the projectile. First, the back lip is observed, and its volume increases till \( h \) reaches \( b \). Then, the front lip emerges and enlarges with increasing \( h \).

Nose profile of the impactor is written as follows, with the coordinate origin at the nose tip:
\[ r = \varnothing(x) = \sqrt{r_o^2 - (L-x)^2} + r_p - r_o \]  
(32)
\[ r_p = \frac{r_o^2 + (L + x)^3}{2r_p} \]  
(33)
where \( x_i \) is found as:

\[
x_i = L - \sqrt{2r_p (r_p + r_t) - (r_p - r_t)^2}
\]  

(34)

Assuming that the penetrator is rigid, the penetration will create a displacement of target's material, whose displaced volume is equal to the penetration volume of the penetrator into the target. The displaced volume due to differential penetration depth \((dh)\) is:

\[
dv = \pi \phi (h) dh = \pi \left( \sqrt{r_p^2 - (L-h)^2} + r_p - r_o \right) dh \quad \text{if } h \leq L
\]  

(35)

Integration of Eqn (35) will give the total amount of displaced volume \((\Delta v_h)\) at \( h \). For the case \( h \leq b, \ h \leq L \), \( \Delta v_h \) is calculated as:

\[
\Delta v_h = \int_{h=0}^{h} \pi \left( \sqrt{r_p^2 - (L-h)^2} + r_p - r_o \right) dh = \pi (A + B + C)
\]  

(36)

\[
A = h \left[ r_o^2 - L^2 + h \left( L - \frac{h}{3} \right) \right]
\]  

(37)

\[
B = \left( r_p - r_o \right) \left( h - L \right) \sqrt{r_p^2 - (L-h)^2} + r_p^2 \tan^{-1} \left( \frac{h-L}{\sqrt{r_p^2 - (L-h)^2}} \right)
\]  

(38)

\[
\left( r_p - r_o \right) \left[ L \sqrt{r_p^2 - (L-h)^2} - r_p^2 \tan^{-1} \left( \frac{h-L}{\sqrt{r_p^2 - (L-h)^2}} \right) \right]
\]

\[
C = h \left( r_p^2 - r_o^2 \right)^2
\]  

(39)

\[
\Delta v_h \text{ for } b < h < L:
\]

\[
\Delta v_h = \int_{h=b}^{L} \pi \left( \sqrt{r_p^2 - (L-h)^2} + r_p - r_o \right) dh = \pi (A + B + C)
\]  

(40)

\( A, B \) and \( C \) values in Eqn (40) are calculated by substituting the related limit values. \( \Delta v_h \) for \( h > b \) and \( L \):

\[
\Delta v_h = \int_{h=b}^{L} \pi \left( \sqrt{r_p^2 - (L-h)^2} + r_p - r_o \right) dh + \pi r_p^2 (h-L) = \pi (A + B + C) + \pi r_p^2 (h-L)
\]  

(41)

\( A, B \) and \( C \) values in Eqn (41) are calculated by substituting the related limit values.

For \( L < h < b \), \( A, B, C \) values are found by using the limit values from \( h-L \) to \( h \).

\( V_h \) can be found from Eqn (14) as:

\[
V_h = \sqrt{\frac{m_p V_s^2 - 2 Y^2 \Delta v_h}{m_p}}
\]  

(42)
Figure 5. $V_r - V_s$ comparison charts for different thicknesses: (a) Values, (b) Normalized values.
Figure 6. \( V_r - V_s \) comparison charts for different nose geometries: (a) Values, (b) Normalized value.
The normalized velocity values perfectly fit into the idealized Recht - Lipson curve. Residual velocity predictions with Model I and V are with an error of 0.3-2.2 % for all velocities. Model II predicts the results with good estimations. Model III underestimates . The best calculations are with Model IV.

3.2 Test Case II

The test data from the literature is on conical nosed steel penetrators impacting at the AA5083-H116 aluminium plates of thickness 15-30 mm with various values.

 graphs with the values calculated for different thicknesses by using different models, and the test data are shown in Fig. 5.

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In the case of projectiles having hemispherical nose, Models III, V, and I results are good at all velocities except the ones approaching . In the case of projectiles having hemispherical nose, Models III, IV and I estimations are good at all velocities.

4. CONCLUSIONS

Analyses of plate perforation by a truncated ogive nosed penetrator provide solutions also to the penetrators with common nose geometries such as ogive, hemispherical, conical, and blunt.

Kinematic analyses have been made with various test cases that are available in the literature. calculations are in line with the test data for most of the models used. In general, estimations are good at the velocities not approaching .

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