SHORT COMMUNICATION

Propagation of Strong Magneto-radiative Plane Shock Wave in a Self-gravitating Exponentially-decreasing Medium

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ABSTRACT

A self-similar theoretical model of propagation of strong plane shock waves in an inhomogeneous, magneto-radiative, self-gravitating atmosphere in the direction of decreasing density is considered. The results discussed depend upon the variations of the flow variables which are displayed graphically and the influences of the gravitation and radiation flux have been studied.

Keywords: Plane shock wave, gravitation, radiation flux, magneto-radiative shock waves, exponentially decreasing medium, wave propagation, fluid flow

NOMENCLATURE

x	Lagrangian coordinate
Δ	Scale height
ρ^*	Arbitrary reference density
t	Time
α	Similarity exponent
m	Mass Lagrangian coordinate
M	Total mass ahead of the gas
η	Similarity variable
K	Planck mean absorption coefficient

1. INTRODUCTION

In many astronomical phenomena, the temperature involved is of the order of 10^6 degree. In such a high temperature, one does not know the details of processes by which a shell or very extensive atmosphere surrounding a star is formed, ie, how the star splits out an appreciable amount of its material back to the interstellar space. The phenomenon of doubling of lines suggests violent outburst of gas in some stars. These observations appear to be the indication of propagation of shock waves through the surface layers of the stars. These suggest inclusion of radiation effects in shock wave theory. These effects are important even for supersonic aerodynamics, nuclear explosions, and nuclear energy devices, because the theory deals with very high temperature. Radiation effects were included as in Zel'dovich and Raizer[1] through radiation pressure and radiation energy. The difficulty with radiation gas dynamics or radiation magneto-gas dynamics is that in thermodynamic equilibrium, radiation pressure and radiation energy are given as functions of temperature and frequency of radiation. Ideal thermodynamic equilibrium never exists in the universe. But for a number of astrophysical phenomena, it is nearly in thermodynamic equilibrium. In the case

of shock waves, radiation variables are not in thermodynamic equilibrium even if gas variables are in equilibrium. When the radiation at each point of a medium with a nonuniform temperature is close to equilibrium, then the medium is spoken of as being in a state of local thermodynamic equilibrium between the radiation and the fluid[1]. Further the necessary condition for the existence of local equilibrium also serves as a justification for the use of the diffusion approximation when considering radiative transfer. Diffusion approximations to include radiation flux in energy equation of motion for shock waves are through Rosseland's diffusion approximation and Planck diffusion approximation.

An extensive study of strong magneto-radiative shock wave with the inclusion of the assumption that the radiation is assumed to be in local thermodynamic equilibrium with the fluid, as in Zel'dovich and Raizer[1], were undertaken recently by Singh and Pandey[2], Ganguly and Jana[3], Jana and Ganguly[4], Jana[5,6], and Jana[7], *et al*. In these studies, the influence of radiation flux or radiation pressure, radiation energy was noticeable on the flow variables. In fact, these strengthen the shock.

In this study, the propagation of strong plane shock wave in an exponentially decreasing density with time has been investigated in radiative magneto-gas dynamics with self-gravitation. The radiation pressure and radiation energy have been ignored in comparison to radiation heat flux. Also, it is assumed that the gas is optically thin. This study is important to understand the effects of explosions in stars and the atmosphere of earth.

2. BASIC EQUATIONS AND BOUNDARY CONDITIONS

Let the gas density be distributed in the atmosphere following the exponential law as:

$$\rho_0 = \rho^* e^{\frac{x}{\Delta}} \tag{1}$$

This distribution has the property that the mass of gas concentrated in a column of unit cross-sectional area, from $x = -\infty$, (where $\rho_o = 0$), to $x = X$, is equal to a mass of gas of density $\rho_o(X)$ in a column whose length is Δ,

$$M = \int_{-\infty}^{X} \rho_0(x)dx = \rho_0(X)\Delta \tag{2}$$

one assumes that the shock wave emerges at the boundary of the atmosphere $x = -\infty$, where $\rho_o = 0$, at the time $t = 0$; thus one takes the time prior to emergence as negative. Thus the front velocity is:

$$D = \dot{X} = \alpha\frac{\Delta}{t} \quad , \quad t < 0 , \quad D < 0 \tag{3}$$

where α is the similarity exponent and is a positive constant such that $dX/dt = D$ is negative. Hence the coordinate of the shock front is now given by:

$$X = \alpha\Delta \text{ In}(-t) + \text{constant} \tag{4}$$

Differentiating Eqn (2) wrt t and using Eqn (1), one gets

$$\frac{dM}{dt} = \rho_0(X)\left(\frac{dX}{dt}\right) = \left(\frac{M}{\Delta}\right)\left(\frac{dX}{dt}\right) = \frac{M}{\Delta}\left(-\frac{\alpha\Delta}{t}\right)$$

$$= M\alpha(-t)^{-1}$$

from which $M = A(-t)^\alpha$ \tag{5}

where A is the constant of integration which characterises the strength of the impact.

Substituting Eqn (5) in Eqn (2), one gets:

$$\rho_0(X) = \frac{M}{\Delta} = \frac{A(-t)^\alpha}{\Delta} \tag{6}$$

In Lagrangian coordinates, the motion is self-similar in usual sense. The Lagrangian coordinate is given by:

$$m = \int_{-\infty}^{x} \rho(x)dx \tag{7}$$

The equations of motion governing the one-dimensional flow behind the shock wave in terms of the mass m and time t are:

$$\frac{\partial}{\partial t}\left(\frac{1}{\rho}\right) - \frac{\partial u}{\partial m} = 0 \tag{8}$$

$$\frac{\partial u}{\partial t}+\frac{\partial p}{\partial m}+h\frac{\partial h}{\partial m}+Gm=0 \tag{9}$$

$$\frac{\partial h}{\partial t}+\rho h\frac{\partial u}{\partial m}=0 \tag{10}$$

$$\frac{1}{\rho}\frac{\partial p}{\partial t}+\gamma\, p\frac{\partial u}{\partial m}+\frac{\partial F}{\partial m}=0 \tag{11}$$

where u, ρ, p, h, F are particle velocity, density, pressure, magnetic field, and radiation flux, respectively. γ is the ratio of the specific heats. G represents the gravitational constant.

Assuming the local thermodynamic equilibrium and using the Planck's diffusion approximation, one has:

$$\rho\frac{\partial F}{\partial m}=4\,K\,\sigma\,T^4 \tag{12}$$

where σ is the Stefan-Boltzman constant and T is the absolute temperature. Now the equation of state for ideal gas is given by:

$$p = \rho RT \tag{13}$$

where R is the gas constant per unit mass.

Now one takes K as a power-law function of the density and temperature as:

$$K = K_0\rho^{\delta}T^{\nu} \tag{14}$$

where K_0 , d and ν are constants.

For a strong shock, the boundary conditions are:

$$p_1=\frac{2\rho_0(X)D^2}{(\gamma+1)},\quad h_1=h_0(X)\frac{(\gamma+1)}{(\gamma-1)}$$

$$F_1=\frac{\rho_0(X)(1-\beta)}{2}(2\beta\theta-\beta-1)D^3,$$

where $\qquad \theta=\dfrac{\gamma}{(\gamma-1)} \tag{15}$

and the subscripts 1 and 0 denote the values of the variables just behind and ahead of the shock, respectively.

The Alf'ven Mach number and the usual Mach number are defined respectively as:

$$M_A{}^2=\frac{\rho_0 D^2}{h_0{}^2}\quad\text{and}\quad M_1{}^2=\frac{\rho_0 D^2}{\gamma\,p_0} \tag{16}$$

where $D = dX/dt$ is the speed of the shock. Alf'ven speed and sound speed are respectively given by

$$\left(\frac{h_0{}^2}{\rho_0}\right)^{\frac{1}{2}}\text{and}\ \left(\frac{\gamma p_0}{\rho_0}\right)^{\frac{1}{2}}.$$

Under the equilibrium condition, from Eqn(9) one has

$$G=-\frac{2D^2}{\Delta m}\left[\frac{1}{\gamma M_1{}^2}+\frac{1}{2M_A{}^2}\right] \tag{17}$$

3. SIMILARITY SOLUTIONS

Let one assumes the solutions of the fundamental Eqns (8)-(14) in the similarity form [c.f. Zel'dovich and Raizer[1]]:

$$u=\frac{2}{(\gamma+1)}\frac{\alpha\Delta}{t}U(\eta),$$

$$\rho=\frac{(\gamma+1)}{(\gamma-1)}\rho_0(X)W(\eta)=\frac{(\gamma+1)}{(\gamma-1)}\frac{M}{\Delta}W(\eta)$$

$$p=\frac{2}{(\gamma+1)}\frac{\alpha^2\Delta^2}{t^2}\rho_0(X)P(\eta)=\frac{2}{(\gamma+1)}\frac{\alpha^2\Delta}{t^2}M\,P(\eta)$$

$$h=\frac{\sqrt{2}}{\sqrt{(\gamma+1)}}\frac{\alpha\Delta}{t}\sqrt{\rho_0(X)}H(\eta)=\frac{\sqrt{(2\Delta)}}{\sqrt{(\gamma+1)}}\frac{\alpha}{t}\sqrt{M}\,H(\eta)$$

$$F=\frac{2}{(\gamma+1)}\rho_0(X)\frac{\alpha^3\Delta^3}{t^3}Q(\eta)=\frac{2}{(\gamma+1)}\frac{\alpha^3}{t^3}M\Delta^2 Q(\eta) \tag{18}$$

where the dimensionless reduced functions U, W, P, H, and Q depend on γ and on the similarity variable η, the dimensionless distance measured from the shock front.

The Lagrangian coordinate m as defined in Eqn (7) can be written with the help of Eqn (18) as:

$$m = \int_{-\infty}^{x} \rho(x)dx = \text{constant}.M. \int_{\eta}^{\infty} W(\eta)d\eta \qquad (19)$$

Thus U, W, P, H, and Q are functions of the similarity variable

$$\eta = \frac{m}{M} = \frac{m}{A(-t)^{\alpha}} \qquad (20)$$

Substituting Eqn (18) in the fundamental Eqns (8)-(14), one gets:

$$-\frac{\eta W'}{W^2} + \frac{1}{W} + \frac{2U'}{(\gamma-1)} = 0 \qquad (21)$$

$$U + \alpha(\eta U' - P' - HH') + \alpha(\gamma+1)\left[\frac{1}{\gamma M_1^2} + \frac{1}{2M_A^2}\right] = 0 \qquad (22)$$

$$\left[\left(\frac{\alpha}{2}-1\right) + \frac{2\alpha W U'}{(\gamma-1)}\right]H - \alpha\eta H' = 0 \qquad (23)$$

$$\frac{(\gamma-1)}{W}\left[-\eta\alpha P' + (\alpha-2)P\right] + \alpha\left[2\gamma PU' + (\gamma+1)Q'\right] = 0 \qquad (24)$$

$$Q' = N\left(\frac{2^{3+v}(\gamma-1)^{4+v}}{(\gamma+1)^{7+2v}}\right)W^{(\delta-5-v)}P^{4+v} \qquad (25)$$

where $N = \dfrac{4k_0\sigma\Delta}{R^{4+v}}$ and $\delta = 1$, $v = -2.5$

The above set of differential Eqns (21)-(25) can be put in the following forms:

$$U' = \frac{(\gamma-1)}{W}\left[\eta U - H^2\left(\frac{\alpha}{2}-1\right) + \eta\alpha(\gamma+1)\left\{\frac{1}{\gamma M_1^2} + \frac{1}{2M_A^2}\right\} - (\alpha-2)P\right]$$

$$-\frac{\alpha N 2^{3+v}(\gamma-1)^{4+v}}{(\gamma+1)^{6+2v}}W^{(\delta-5-v)}P^{4+v}$$

$$\div\left[-\frac{\eta^2\alpha(\gamma-1)}{W} + 2\alpha\left(H^2 + \gamma P\right)\right] \qquad (26)$$

$$W' = \frac{1}{\eta}\left[W + \frac{2W^2 U'}{(\gamma-1)}\right] \qquad (27)$$

$$P' = \frac{U}{\alpha} + \eta U' - \frac{H^2}{\alpha\eta}\left\{\left(\frac{\alpha}{2}-1\right) + \frac{2\alpha WU'}{(\gamma-1)}\right\}$$

$$+ (\gamma+1)\left\{\frac{1}{\gamma M_1^2} + \frac{1}{2M_A^2}\right\} \qquad (28)$$

$$H' = \left[\left(\frac{\alpha}{2}-1\right) + \frac{2\alpha WU'}{(\gamma-1)}\right]\frac{H}{\alpha\eta} \qquad (29)$$

$$Q' = N\left(\frac{2^{3+v}(\gamma-1)^{4+v}}{(\gamma+1)^{7+2v}}\right)W^{(\delta-5-v)}P^{4+v} \qquad (30)$$

where $N = \dfrac{4k_0\sigma\Delta}{R^{4+v}}$ and $\delta = 1$, $v = -2.5$

The boundary conditions are given by:

$$U(1) = 1, \ W(1) = 1, \ P(1) = 1, \ H(1) = \frac{(\gamma+1)^{1.5}}{\sqrt{2}(\gamma-1)M_A}$$

$$Q(1) = \frac{1}{2}[2\beta\theta - \beta - 1], \text{ where } \theta = \frac{\gamma}{(\gamma-1)} \qquad (31)$$

Applying the conservation equations to the shock front, one has

$$\beta = \frac{\rho_0}{\rho_1} = \frac{(\gamma-1)}{(\gamma+1)}\left[1 + \frac{(F_1 - F_0)}{\rho_1(X)}\right] \qquad (32)$$

Here, the heat flux F is positive in the direction of shock propagation. When the second term in the above bracket is small, this discloses.

$$\beta = \frac{(\gamma - 1)}{(\gamma + 1)} \qquad (33)$$

which is the Hugoniot shock density ratio.

4. RESULTS AND DISCUSSIONS

The differential Eqns (26)-(30) with the help of the boundary conditions [Eqn (31)] have been integrated numerically by the wellknown Runga-Kutta method for $M_A^2 = 5$, $\delta = 1$, $\nu = -2.5$, $M_1^2 = 2$, $N = 0,10, 25$, $\gamma = 1.2$.

Other values of γ may be taken for calculation and it lies between $1 < \gamma < 2$ for gases.

The values of γ, β and α have been borrowed from Zel'dovich and Raizer[5] and are given in Table 1.

Table 1. Values of ratio of the specific heats (γ), Hugoniot shock density ratio (β) and similarity exponent (α)

γ	β	α
1.2	0.0909	6.48
1.4	0.1660	5.45
1.6	0.2307	4.90

The variations of the flow parameters with distance are shown graphically in the Figs 1-10.

The following six cases have been considered:

Case I: *Magneto-gas dynamic plane shock waves propagating in exponentially decreasing medium.* (*N=0*)

In this case, propagation of magneto-gas dynamic plane shock waves has been considered in the absence of radiation flux and gravitation in an exponentially decreasing medium and the line pattern used for this case is as follows:
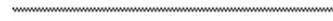
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Case II: *Magneto-gas dynamic self-gravitating plane shock waves propagating in exponentially decreasing medium.*(*N=0*)

In this case, propagation of magneto-gas dynamic plane shock waves has been considered in the absence of radiation flux and in the presence of gravitation in an exponentially decreasing medium and the line pattern used for this case is as follows:

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Case III: *Magneto-radiative plane shock waves propagating in exponentially decreasing medium.* (*N=10*)

In this case, propagation of magneto-radiative plane shock waves has been considered in the presence of radiation flux and in the absence of gravitation in an exponentially decreasing medium and the line pattern used for this case is as follows:

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Case IV: *Magneto-radiative self-gravitating plane shock waves propagating in exponentially decreasing medium.* (*N=10*)

In this case, propagation of magneto-radiative plane shock waves has been considered in the presence of radiation flux and gravitation in an exponentially decreasing medium and the line pattern used for this case is as follows:

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Case V: *Magneto-radiative plane shock waves propagating in exponentially decreasing medium.* (*N=25*)

In this case, propagation of magneto-radiative plane shock waves has been considered in the presence of radiation flux and in the absence of gravitation in an exponentially decreasing medium and the line pattern used for this case is as follows:

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Case VI: *Magneto-radiative self-gravitating plane shock waves propagating in exponentially decreasing medium.* (*N=25*)

In this case, propagation of magneto-radiative plane shock waves has been considered in the presence of radiation flux and gravitation in an exponentially decreasing medium and the line pattern used for this case is as follows:

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From Figs 1-4, one may conclude for *case* I that behind the shock front towards the centre of the symmetry, radial velocity increases, density decreases,
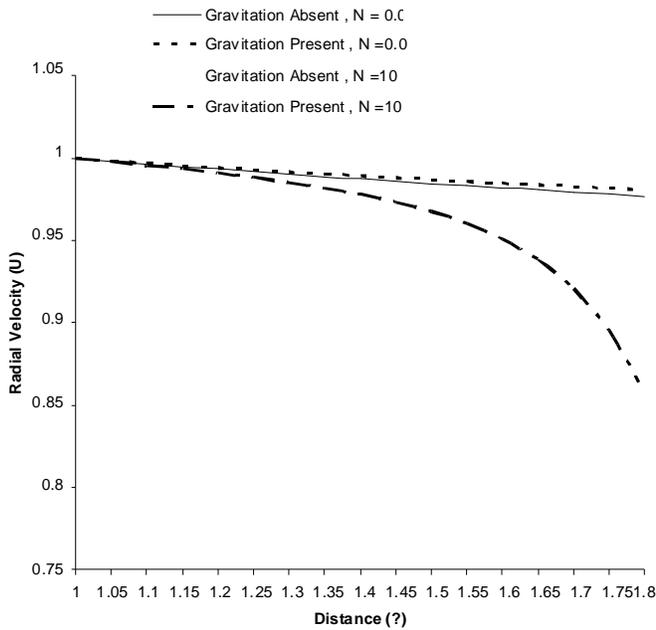
magnetic field increases , and the pressure decreases.

In *case* II, one observes that radial velocity increases, density decreases, magnetic field decreases, and pressure decreases.

From Figs 1-5, for the *case* III , one concludes that radial velocity increases, density increases rapidly and then decreases, magnetic field increases rapidly, pressure decreases, and the radiation flux decreases behind the shock front.

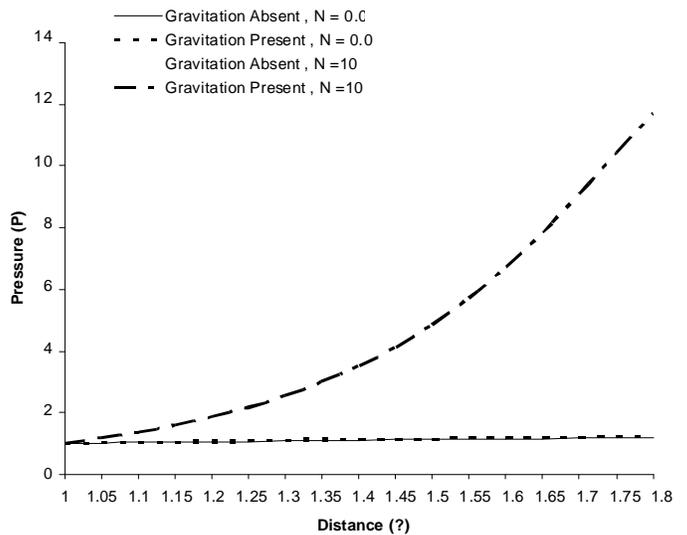
Figure 1. Variation of radial velocity with distance.
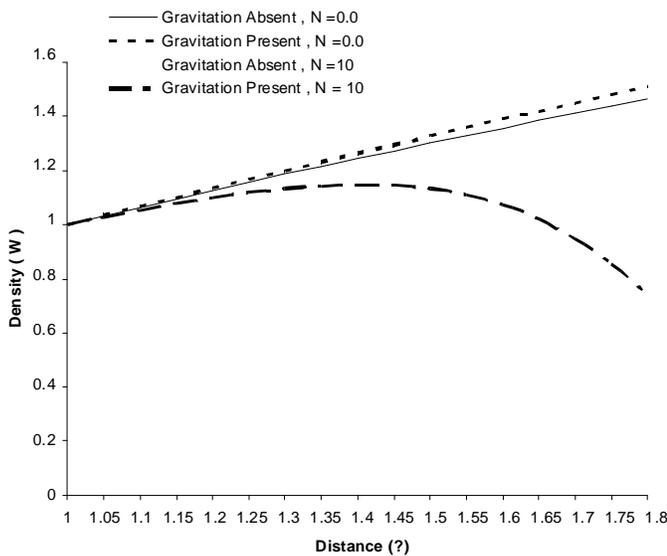
Figure 2. Variation of density with distance.
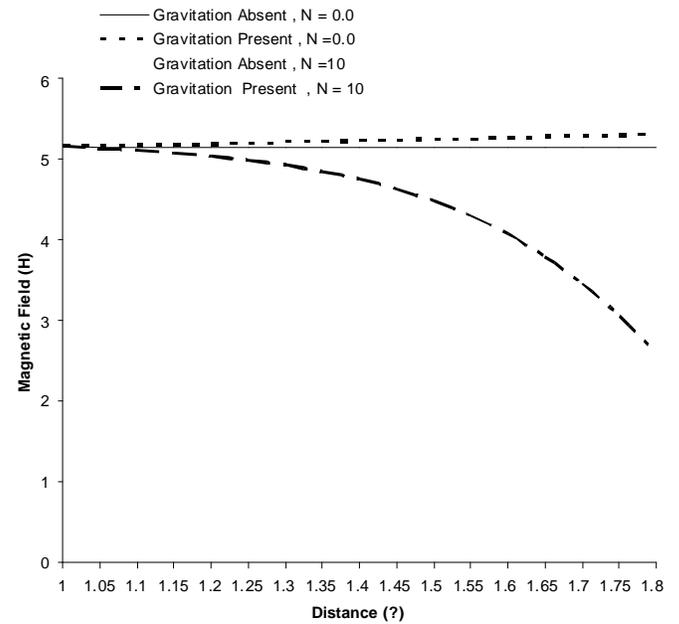
Figure 3. Variation of magnetic field with distance.

Figure 4. Variation of pressure with distance.

Figure 5. Variation of radiation flux with distance (N=25).

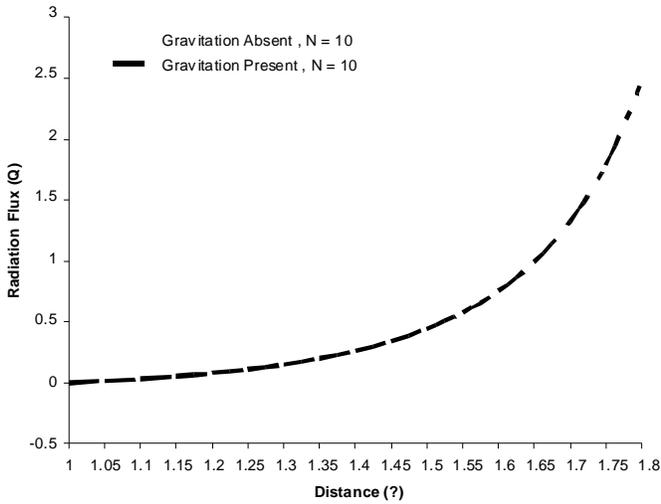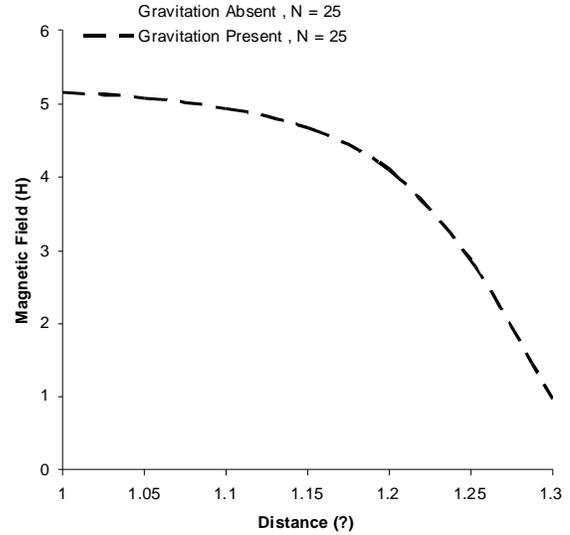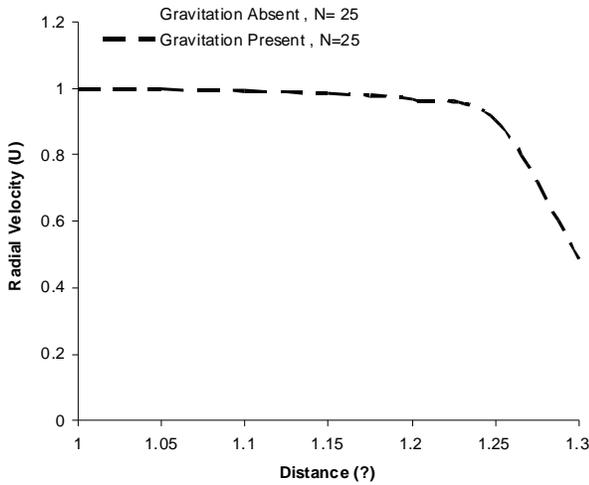**Figure 6. Variation of radial velocity with distance (N=25).**

Figure 7. Variation of density with distance (N=25).

Figure 8. Variation of magnetic field with distance (N=25).
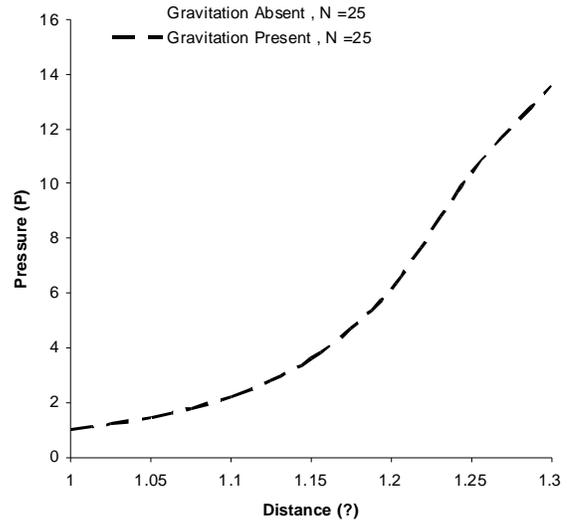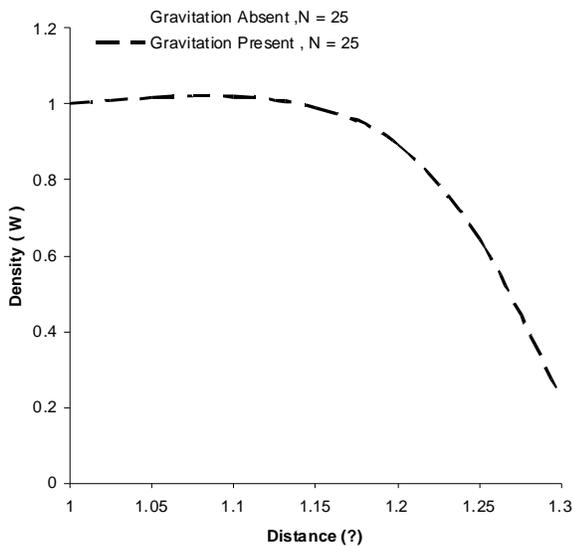
Figure 9. Variation of pressure with distance (N=25).

In *case* IV, one observes that radial velocity increases, density initially increases then decreases, magnetic field increases, pressure decreases, and the radiation flux decreases.

In *case* V, from Figs 6-10, one observes that radial velocity increases, density increases rapidly, magnetic field increases rapidly, pressure decreases continuously, and the radiation flux decreases rapidly.

In *case* VI, one observes that radial velocity increases, density increases rapidly, magnetic field increases rapidly, pressure decreases, and the radiation flux decreases.
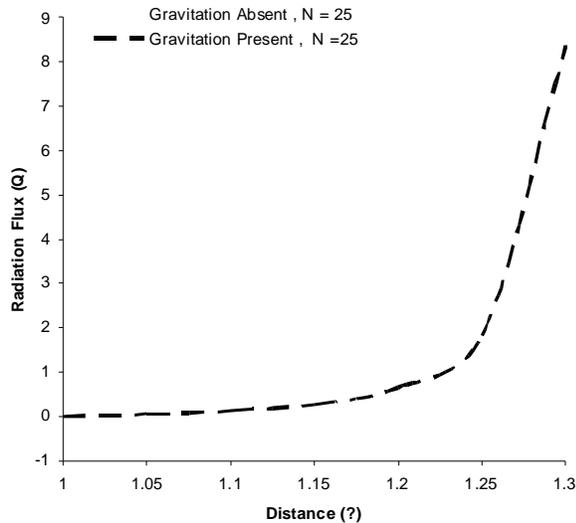
Figure 10. Variation of radiation flux with distance (N=25).

Moreover, comparing the line patterns of *case* I and *case* III, one concludes that the influence of radiation flux is to decrease radial velocity, density, magnetic field and increase pressure.

Similarly, comparing the line patterns of *case* I and *case* II, one concludes that the influence of gravitation is to increase radial velocity, density, pressure, and magnetic field.

However, as seen in the cases V and VI, it is to be noted that for higher values of *N* as 25 onwards except for pressure, the influence of gravitation seems to be negligible upon the variation of flow parameters.

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