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SHORT COMMUNICATION

Plasma Frequency Reduction Factor

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ABSTRACT

A simple formula for plasma frequency reduction factor for a solid cylindrical electron beam in a metallic tunnel has been developed by means of a 3-D curve fitting to the standard results of Branch and Mihran with accuracy > 1.7 per cent, over the parametric regime of normalised beam-radius and beam-filling factor applicable for linear-beam microwave tubes. An artificial neural network algorithm was used for the curve-fitting following the approach of universal approximation. The formula is simple and amenable to easy computation, even using a scientific calculator.

Keywords: Artificial neural network, plasma oscillation, plasma frequency reduction factor, solid electron beam, metallic tunnel

NOMENCLATURE

a	Radius of the beam-tunnel
b	Radius of the electron beam
β_e	Electronic propagation constant
η_e	Charge-to-mass ratio of an electron at rest
ε_0	Permittivity of free-space
I_n	Modified Bessel function of first-kind of order n
J_n	Ordinary Bessel function of first-kind of order n
K_n	Modified Bessel function of second-kind of order n
R	Plasma frequency reduction factor
ρ_0	Charge density of the un-modulated electron beam
ω_p	Plasma frequency
ω_q	Reduced plasma frequency

1. INTRODUCTION

Plasma frequency reduction factors are widely used for incorporating the influence of the geometry of the electron beam and the proximity of the metallic tunnel of the RF interaction structure on the space charge field in finite electron beams[1-4]. In most practical situations, the electron beams having finite transverse cross section are placed close to a metallic interaction structure (Fig. 1), and as a result, the space-charge field ceases to be purely radial and, therefore, the axial electric field intensity, and hence the restoring force on electrons reduce as compared to their values in the case of an infinite transverse cross section of the beam, manifesting a reduction in the value of plasma frequency. Introduction of the plasma frequency reduction factor conveniently incorporates the screening effect of conducting surfaces on space charge, thereby simplifying the equation of wave-particle motion and its solution.

Conventional computation of the plasma frequency reduction factor involves numerical solution of a transcendental equation involving Bessel functions, and is not amenable to analytical solution[1-4]. Moreover, numerical solution can not illustrate the physical dependence of parameters, and hence, a closed-form formula[4-6] is ever welcome. Rowe[4], also stated emphatically in this regard "this creates joy in the hearts of the computer people but brings nightmares to the klystron engineer." In fact, use of an approximate plasma frequency reduction factor could provide abundance of physical insight in klystron and TWT interaction phenomena[3-6]. This paper is aimed at deriving a closed-form expression for plasma frequency reduction factor for a solid cylindrical electron beam in a metallic tunnel, applicable for a linear beam microwave tube ($0.6 \leq \beta_e b \leq 0.9$ and $0.5 \leq b/a \leq 0.8$), β_e being the electronic propagation constant and $\beta_e b$ being the normalised beam radius), which would be easy to use in theoretical studies and time saving for computer simulations.

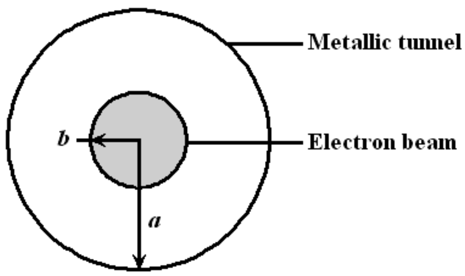

Figure 1. Schematic of the problem.

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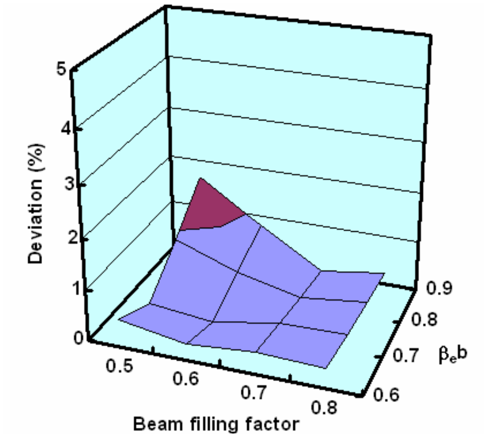

Figure 2. Percentage deviation in the computed values of plasma frequency reduction factor using Eqn (4) wrt (Table 1).

Here, J_n is the ordinary Bessel function of first-kind of order n, and I_n and K_n are the modified Bessel functions of first- and second-kind, respectively, of order n. The transcendental Eqn (3) has been solved numerically and the values of plasma frequency reduction factors over the parametric regime of interest for a linear beam device are shown in Table 1.

The problem has now been to find a closed-form formula for R, without resorting to numerical solution of a transcendental equation and also without sacrificing the accuracy. For this purpose, an artificial neural network (ANN) algorithm based on universal approximation theorem[8-9] was followed and a 3-D curve fitting to the standard results of Branch and Mihran (Table 1) was developed which was applicable for the parametric regime of linear beam microwave tubes ($0.6 \le \beta_e b \le 0.9$ and $0.5 \le b/a \le 0.8$) as

Table 1. Branch and Mihran's[1] plasma frequency reduction factors

$\beta_e b$	Beam filling factor (*b/a*)					
	0.4	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
0.5	0.3468	0.3229	0.2978	0.2727	0.2481	0.2247
0.6	0.4004	0.3763	0.3495	0.3216	0.2936	0.2666
0.7	0.4483	0.4251	0.3976	0.3678	0.3372	0.3072
0.8	0.4908	0.4696	0.4422	0.4113	0.3787	0.3461
0.9	0.5286	0.5094	0.4831	0.4519	0.4179	0.3832
1	0.5622	0.5454	0.5206	0.4897	0.4549	0.4186

2. FORMULATION AND RESULTS

Plasma oscillation frequency (ω_p) for an infinite unbounded cloud of electrons[1-4] is given by

$$\omega_p^2 = |\eta||\rho_0|/\varepsilon_0 \tag{1}$$

Here, η is the charge-to-mass ratio of an electron at rest, ρ_0 is the electronic charge density in the unbounded plasma and ε_0 is the permittivity of free space. In any finite electron beam inside a metallic drift tube, the plasma oscillation frequency (ω_p), given by Eqn (1) reduces. The reduced plasma frequency (ω_q) is defined by a plasma frequency reduction factor (R), given[1-4] as

$$R = \left(\frac{\omega_q}{\omega_p}\right) = \left(1 + \left(\frac{T_0}{\beta_e}\right)^2\right)^{-\frac{1}{2}} \tag{2}$$

Here, β_e is the beam propagation constant, and T_0 is the eigen-value of the beam-tunnel coupled geometry solvable from the wave equation $\nabla_\perp^2 E_z + T_0^2 E_z = 0$, E_z being the axial space charge field[1-4]. The solution of the eigen-value T_0 can be numerically solved[1-3] from the transcendental equation given as

$$T_0 \frac{J_1(T_0 b)}{J_0(T_0 b)} = \beta_e \frac{K_0(\beta_e a) I_1(\beta_e b) + K_1(\beta_e b) I_0(\beta_e a)}{K_0(\beta_e b) I_0(\beta_e a) - K_0(\beta_e a) I_0(\beta_e b)} \tag{3}$$

$$R = \left(1 + \frac{7.5214(b/a)^2 - 4.3178(b/a) + 2.4895}{\beta_e^2 b^2}\right)^{-\frac{1}{2}} \tag{4}$$

The percentage deviations of the computed values of the plasma frequency reduction factor using the present formula Eqn (4) wrt Branch and Mihran's results (Table 1) are plotted in Fig. 2 over the regime of operation as specified in Table 1. The present simple formula predicts values of plasma frequency reduction factor with accuracy better than 1.7 per cent.

3. CONCLUSIONS

A closed-form formula for plasma frequency reduction factor is developed using an ANN algorithm, which is simple, amenable to easy computation, and yet accurate enough, for practical range of parameters. It is hoped that the present handy formula would help in the design of linear beam microwave tubes.

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