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Determination of Moving Tank and Missile Impact Forces on a Bridge Structure

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ABSTRACT

A method to determine the moving tank and missile impact forces on a bridge is developed. The present method is an online adaptive recursive inverse algorithm, which is composed of the Kalman filter and the recursive least square estimator (RLSE), to estimate the force inputs on the bridge structure. The state equations of the bridge structure were constructed by using the model superposition and orthogonal technique. By adopting this inverse method, the moving tank and missile impact force inputs acting on the bridge structure system can be estimated from the measured dynamic responses. Besides, this work presents an efficient weighting factor applied in the RLSE, which is capable of providing a reasonable estimation results. The results obtained from the simulations show that the method is effective in determining the moving tank and missile impact forces, so that the acceptable results can be obtained.

Keywords: Missile impact, Kalman filter, RLSE, recursive least square estimator, bridge structure, impact studies, impact forces

NOMENCLATURE

$\bar{A}$	Cross section of the beam
A,B	Constant matrices
B_s	Sensitivity matrix
C	Damping matrix
E	Elastic modulus
F	Model force
F_K	Moving tank force input
F_m	Missile impact force input
H	Measurement matrix
I	Unit matrix
I_a	Moment of inertia
K	Total number of tank input forces
K_a	Kalman gain
K_b	Correction gain
K_n	Model stiffness matrix
k	Time (discretised)
l	Length of the beam element
M	Total number of missile impact forces
M_n	Model mass matrix
M_s	Sensitivity matrix
N	Total number of estimation time steps
P	State's error covariance matrix
P_b	Filter's error covariance matrix
Q	Process noise covariance matrix
Q_w	Scalar of process noise covariance
R	Measurement noise covariance matrix
R_v	Scalar of measurement noise covariance
r	Weighting factor
S	Innovation covariance
t	Time (continuous)
u	Displacement of beam
X	State vector
Y	Displacement vector
$\dot{Y}$	Velocity vector
$\ddot{Y}$	Acceleration vector
Z	Observation vector
Γ	Input matrix
δ	Kronecker delta
Δt	Sampling time (interval)
ρ_s	Mass per unit length of the beam
σ	Standard deviation
υ	Measurement noise vector
w	Process noise vector

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Φ State transition matrix

Superscripts

$\wedge$ Estimated

$-$ Estimated by filter

T Transpose of matrix

Subscripts

i, j Indices

1. INTRODUCTION

The network of communication lines are connected with the highways and bridges. In the war, the unobstructed network of communication lines can determine if the military effective forces can be transferred quickly. Furthermore, it influences the effectiveness of the military forces and the success or failure of conducting a military operation. The bridges play the most important role in the network of communication lines. It supports the function of the troop manoeuvres and rear-service supplies. Therefore, it probably will be a target attacked by the enemy missiles which would be unfavorable. It is necessary to ensure that the bridge will not be destroyed under the enemy missile impact. The effect of external forces and bridge are important for bridge design and reliability evaluation. Therefore, the dynamic force inputs on the bridge structure must be determined using the estimation methods or measurement techniques. The major method is to determine the dynamic force input applying the measured dynamic responses.

Direct measurement method for the force inputs using precision instruments is expensive and is subjected to bias. According to the simulations, the results are subjected to modelling errors[1-3]. Therefore, it is necessary to find the alternative methods to estimate the force inputs. The inverse estimation method is in fact a force determination algorithm, which is a process of determining the applied loads from the dynamic responses of structures. Inoue[4,5], *et al.* adopted the least square method based on singular value decomposition to improve the estimation precision. Doyle[6-9] used the method in the frequency domain to obtain the histories of forces from the experimentally measured responses, such as velocities, strains, etc. Wang and Kreitinger[10] developed a direct approach, called the sum of weighted acceleration technique (SWAT), to determine the unknown forces. In these investigations , these estimation algorithms were all processed in the batch form, which consumed large computational resources.

Some researchers had studied the determination method for the inverse problem. Hollandsworth and Busby[11] investigated this method experimentally with a force applied at a known location and the accelerometers used as the sensors. The time domain approach was used to model the structure and the forces with a set of second-order differential equations by Law[12] *et al*. The forces were modelled as step functions in a short time interval. The equations of motion were further expressed in the model coordinates, and solved using convolution in the time domain. Finally, the forces were determined using the model superposition principle. Doyle[13] developed a method for determining the location and magnitude of an impact force using the phase difference of the signals measured at two different locations straddling the impact point. Busby and Trujillo[14] reconstructed the force history using a standing wave approach. Druz[15] *et al*. formulated a nonlinear inverse problem and tried to find the location and magnitude of the external force. The model approach determined the forces effectively in the model coordinates by Chan[16], *et al.* Measured displacements were converted into model displacements with an assumed shape function. The forces were then determined by solving the uncoupled equations of motion in the model coordinates. Recently, Huang[17] used an algorithm based on the conjugate gradient method to estimate the unknown external forces in the inverse nonlinear force vibration problem.

An online recursive inverse method to estimate the force inputs of the structure has been presented. The inverse method is based on the Kalman filter and the recursive least square algorithm. The algorithm is an efficient online recursive inverse method to estimate the force inputs. Ma[18,19], *et al.* proposed an inverse method to estimate impulsive loads on the lumped-mass structural system. The method is more economical of the computational resources than batch form process in estimating the external forces of the complex structure system. In this study, the force input estimation method is adopted in dealing with the moving tank and missile impact forces acting on the bridge structure systems as shown in Fig. 1. The precision of this method is demonstrated through several examples with different types of time-varying moving tank and missile impact forces as the unknown inputs. The force inputs can be estimated by applying the simulated noisy system responses into the input estimation algorithm. The estimated values will be compared with the actual inputs to demonstrate the precision of the inverse method.

2. PROBLEM FORMULATION

To illustrate the practicability and precision of the presented approach in estimating the unknown moving tank and missile impact force inputs, numerical simulations of the bridge structure were investigated. As shown in Fig. 2, the bridge structure was modelled as a simple beam

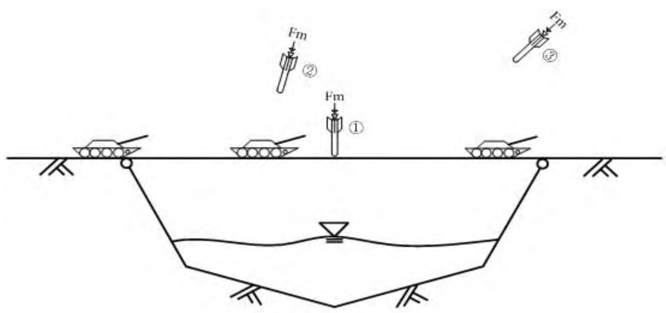

Figure 1. Bridge structure system under moving tank and missile impact loads.

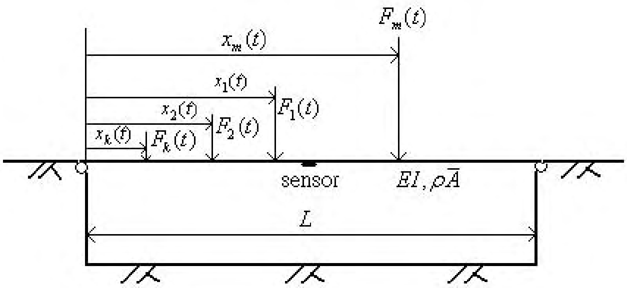

Figure 2. The bridge structure model under the moving multiple tank and missile impact force inputs.

with a total span, L, the flexural stiffness constant, EI_a, mass per unit length, ρ, and the viscous proportional damping coefficient, C. The beam was assumed to be a Bernoulli-Euler beam, in which the effects of shear deformation and rotary inertia were not taken into account. Considering that the group of tank forces are moving from left to right at a constant speed, and the bridge are under continuous missile impact, the equation of motion[20] can be expressed as:

$$\rho\bar{A}\frac{\partial^2 u(x,t)}{\partial t^2} + C\frac{\partial u(x,t)}{\partial t} + EI_a\frac{\partial^4 u(x,t)}{\partial x^4} = p(x,t) \quad (1)$$

where

$$p(x,t) = \sum_{k=1}^{K} F_k(t)\delta(x - x_k(t)) + \sum_{m=1}^{M} F_m(t)\delta(x - x_m(t))$$

where $\bar{A}$ is the cross section of the beam, $F_k(t)$ is the tank forces, $u(x, t)$ is the displacement of beam. $\delta(t)$ is the Dirac delta function, $x_k(t) = v_k t$ represents the position of the k^{th} tank force and v_k is the speed of the k^{th} tank, $F_m(t)$ is the missile impact forces, and $x_m(t)$ is the position of the m^{th} missile impact force. Based on model superposition, the solution of Eqn (1) can be expressed as:

$$u(x,t) = \sum_{m=1}^{\infty} \phi_m(x) Y_m(t) \quad (2)$$

where $\phi_m(x)$ is the m^{th} mode shape function and $Y_m(t)$ is the n^{th} model amplitude of the beam. Thus the response $u(x,t)$ has been expressed as the superposition of the contributions of the individual modes; the m^{th} term in the series of Eqn (2) is the contribution of the n^{th} mode to the response. Substituting Eqn (2) in Eqn (1) gives

$$\sum_{m=1}^{\infty} \rho\bar{A}\phi_m(x)\ddot{Y}_m(t) + \sum_{m=1}^{\infty} C\phi_m(x)\dot{Y}_m(t) + \sum_{m=1}^{\infty} [EI\phi_m''(x)]'' Y_m(t) = p(x,t),$$

Multiplying each term by the m^{th} mode shape function, $\phi_n(x)$, integrating it over the length of the beam, and interchange the order of integration and summation, one gets

$$\sum_{m=1}^{\infty} \ddot{Y}_m(t)\int_0^L \rho\bar{A}\phi_n(x)\phi_m(x)dx + \sum_{m=1}^{\infty} \dot{Y}_m(t)\int_0^L C\phi_n(x)\phi_m(x)dx$$
$$+\sum_{m=1}^{\infty} Y_m(t)\int_0^L \phi_n(x)[EI\phi_m''(x)]''dx = \int_0^L p(x,t)\phi_n(x)dx,$$

By virtue of the orthogonal properties of modes, all the terms in each of the summations on the left side vanish except the one term for which $m=n$, leaving

$$\ddot{Y}_n(t)\int_0^L \rho\bar{A}\left[\phi_n(x)\right]^2 dx + \dot{Y}_n(t)\int_0^L C\left[\phi_n(x)\right]^2 dx$$
$$+Y_m(t)\int_0^L \phi_n(x)[EI\phi_m''(x)]''dx = \int_0^L p(x,t)\phi_n(x)dx,$$

This equation can be rewritten as:

$$M_n\ddot{Y}_n(t) + C_n\dot{Y}_n(t) + K_nY_n(t) = F(t) \quad (3)$$

where $M_n = \int_0^L \rho\bar{A}[\phi_n(x)]^2 dx$ (4)

$$K_n = \int_0^L \phi_n(x) EI_a[\phi_n''(x)]''dx \quad (5)$$

Because of the beam being simply supported, an alternative equation for K_n[21] will be:

$$K_n = \int_0^L EI_a[\phi_n''(x)]^2 dx$$

$$F(t) = \int_0^L p(x,t)[\phi_n(x)]dx \quad (6)$$

where M_n, K_n, and $F(t)$ are the model mass, the model stiffness, and the model force of the n^{th} mode, respectively. $C_n = \alpha M_n + \beta K_n$ represents the model proportional damping coefficient, where α and β are constants with proper units.

Input estimation is based on the state-space analysis method. A state-space model of the beam structure system needs to be constructed before applying the input estimation method. To construct the state-space model, the state variables of the second order dynamic system with n degrees of freedom are represented by a $2n\times1$ state vector, i.e., $X = \left[Y(t) \quad \dot{Y}(t)\right]^T$. From Eqn (3), the continuous-time state equation and measurement equation of the structure system can be written as

$$\dot{X}(t) = AX(t) + BF(t) \quad (7)$$

$$Z(t) = HX(t) \quad (8)$$

where

$$A = \begin{bmatrix} 0_{n\times n} & I_{n\times n} \\ -M_n^{-1}K_n & -M_n^{-1}C_n \end{bmatrix}$$

$$B = \begin{bmatrix} 0_{n\times n} \\ M_n^{-1} \end{bmatrix}$$

$$H = \left[I_{2n\times 2n}\right]$$

$$X(t) = \left[X_1(t) \quad X_2(t) \quad \cdots \quad X_{2n-1}(t) \quad X_{2n}(t)\right]^T$$

where A and B are constant matrices composed of mass, damping and stiffness of the beam structure system. $X(t)$ is the state vector. $Z(t)$ is the observation vector, and H is the measurement matrix.

The solution of Eqns (7) and (8) can be written as

$$X(t) = \Phi(t - t_0)X(t_0) + \int_{t_0}^{t} \Phi(t-\tau)BF(\tau)d\tau \quad (9)$$

where $\Phi(t)$ is the state transfer matrix, it is defined as

$$\Phi(t) = \exp(At)$$

$F(t)$ is unknown input, the input with sampling interval, Δt can be shown as

$$F(t) = F(k\Delta t),\ k\Delta t \le t \le (k+1)\Delta t$$

Let $t_0 = k\Delta t,\ t = (k+1)\Delta t$ substitution into Eqn (9) gives

$$X\left((k+1)\Delta t\right) = \Phi(\Delta t)X(k\Delta t) + \int_{k\Delta t}^{(k+1)\Delta t} \Phi\left((k+1)\Delta t - \tau\right) BF(k\Delta t)d\tau \quad (10)$$

Definition $\Gamma = \int_{k\Delta t}^{(k+1)\Delta t} \exp\left\{A\left[(k+1)\Delta t - \tau\right]\right\} Bd\tau$, the discrete state equation can be written as

$$X\left((k+1)\Delta t\right) = \Phi(\Delta t)X(k\Delta t) + \Gamma(\Delta t)F(k\Delta t) \quad (11)$$

$$Z(k\Delta t) = HX(k\Delta t) \quad (12)$$

The noise interference exists in the practical circumstances and is not considered in Eqns (7) and (8). To approximate the noise to the truth, the statistical noise interference is added in the state equation and measurement equation of the structure system. This random noise interference was assumed as the Gaussian white noise. The statistical characteristic of the random noise is described in detail with the probability distribution and density function and represented with the mean and variance values[22]. On account of the above reason, Eqn (11) is sampled with the sampling interval, Δt, and created in the statistical system dynamic model of the state vector including process noise input[23]. Then, Eqn (11) becomes

$$X(k+1) = \Phi X(k) + \Gamma[F(k) + w(k)] \quad (13)$$

where

$$X(k) = \begin{bmatrix} X_1(k) & X_2(k) & \cdots & X_{2n-1}(k) & X_{2n}(k) \end{bmatrix}^T$$

$$\Phi = \exp(A\Delta t)$$

$$\Gamma = \int_{k\Delta t}^{(k+1)\Delta t} \exp\left\{A\left[(k+1)\Delta t - \tau\right]\right\} Bd\tau$$

$$F(k) = \begin{bmatrix} F_1(k) & F_2(k) & \cdots & F_{n-1}(k) & F_n(k) \end{bmatrix}^T$$

$$w(k) = \begin{bmatrix} w_1(k) & w_2(k) & \cdots & w_{n-1}(k) & w_n(k) \end{bmatrix}^T$$

where $X(k)$ represents the state vector. Φ is the state transition matrix. Γ is the input matrix. Δt is the sampling interval. $w(k)$ is the process noise vector which is assumed to be the Gaussian white noise with zero mean and variance, $E\left\{w(k)w^T(k)\right\} = Q\delta_{kj}$, where $Q = Q_W \times I_{2n\times 2n}$. Q is the discrete time process noise covariance matrix. δ_{kj} is the Kronecker delta function.

To additionally consider the measurement noise, Eqn (12) is then expressed as

$$Z(k) = HX(k) + \upsilon(k) \quad (14)$$

where

$$Z(k) = \begin{bmatrix} Z_1(k) & Z_2(k) & \dots & Z_{2n}(k) \end{bmatrix}^T$$

$$\upsilon(k) = \begin{bmatrix} \upsilon_1(k) & \upsilon_2(k) & \dots & \upsilon_{2n}(k) \end{bmatrix}^T$$

where $Z(k)$is the observation vector. $\upsilon(k)$ represents the measurement noise vector. Also, $\upsilon(k)$ is assumed to be the Gaussian white noise with zero mean and variance, $E\left\{\upsilon(k)\upsilon^T(k)\right\} = R\delta_{kj}$, where $R = R_V \times I_{2n\times 2n}$. R is the discrete time measurement noise covariance matrix, and H is the measurement matrix.

3. ADAPTIVE WEIGHTED RECURSIVE INPUT ESTIMATION METHOD

Adaptive weighted recursive input estimation method is a process of determining the load inputs by applying the measurements of the system responses. The presented method consists of two parts, the Kalmam filter and the estimator. The Kalman filter is used to generate the residual innovation sequence. The residual innovation sequence connotes bias or systematic error from the unknown time, varying input item, and variance or random error form the measurement. The estimator then computes the onset time histories of the excitation forces by applying the residual innovation sequence into the adaptive weighted recursive least square algorithm. The detailed derivation of this technique can be referred in Tuan[24], *et al*. The equations of the Kalman filter are as follows:

$$\overline{X}(k/k-1) = \Phi\overline{X}(k-1/k-1) \quad (15)$$

$$P(k/k-1) = \Phi P(k-1/k-1)\Phi^T + \Gamma Q\Gamma^T \quad (16)$$

$$\overline{Z}(k) = Z(k) - H\overline{X}(k/k-1) \quad (17)$$

$$S(k) = HP(k/k-1)H^T + R \quad (18)$$

$$K_a(k) = P(k/k-1)H^T S^{-1}(k) \quad (19)$$

$$\overline{X}(k/k) = \overline{X}(k/k-1) + K_a(k)\overline{Z}(k) \quad (20)$$

$$P(k/k) = \left[I - K_a(k)H\right]P(k/k-1) \quad (21)$$

In Eqns (15) to (21), the superscript,–, represents the estimation value of the filter. $\overline{X}(k/k-1)$ denotes state estimation. $P(k/k-1)$ is the state estimation error covariance. $\overline{Z}(k)$ is the bias innovation caused by the measurement noise and input disturbance. $S(k)$, represents the innovation covariance. $K_a(k)$ is the Kalman gain. $\overline{X}(k/k)$ is the state filter, $P(k/k)$ represents the state filter error covariance. The parameters of the Kalman filter, such as the state transition matrix Φ, the measurement matrix H, the discrete-time process noise covariance matrix Q, and the discrete-time measurement noise covariance matrix R, must be obtained before filtering. The initial values of X_0 and P_0 are chosen. With the continuous input of the observation vector, the output of Kalman filter can be obtained online. The estimation value $\overline{X}(k/k-1)$ and the state estimation error covariance $P(k/k-1)$ of the structure system can be obtained immediately. The equations of the adaptive weighted recursive least square algorithm are as follows.

$$B_s(k) = H\left[\Phi M_s(k-1) + I\right]\Gamma \quad (22)$$

$$M_s(k) = \left[I - K_a(k)H\right]\left[\Phi M_s(k-1) + I\right] \quad (23)$$

$$K_b(k) = \gamma^{-1}P_b(k-1)B_s^T(k)\left[B_s(k)\gamma^{-1}P_b(k-1)B_s^T(k) + S(k)\right]^{-1} \quad (24)$$

$$P_b(k) = \left[I - K_b(k)B_s(k)\right]\gamma^{-1}P_b(k-1) \quad (25)$$

$$\hat{F}(k) = \hat{F}(k-1) + K_b(k)\left[\bar{Z}(k) - B_s(k)\hat{F}(k-1)\right] \quad (26)$$

where $\bar{Z}(k)$ denotes innovation, $K_b(k)$ is the correction gain, $B_s(k)$ and $M_s(k)$ are sensitivity matrices, P_b represents the error covariance of the estimated input vector, and $\hat{F}(k)$ is the estimated input vector. The weighting factor γ is employed to compromise between the tracking speed and the loss of estimation precision. The detailed derivation of this function can be referred in Tuan[25] *et al*. The adaptive weighting function is shown below.

$$\gamma(k) = \begin{cases} 1 & \left|\bar{Z}(k)\right| \le \sigma \\ \dfrac{\sigma}{\left|\bar{Z}(k)\right|} & \left|\bar{Z}(k)\right| > \sigma \end{cases} \quad (27)$$

According to Eqns (22) to (27), the estimator computes the Kalman gain, $K_a(k)$, the innovation covariance, *S(k)*, and the innovation, $\bar{Z}(k)$. By substituting Eqn (27) in Eqns (24) and (25) for the weighting factor, γ, the adaptive weighted recursive least square estimator can be developed.

The procedure to estimate the unknown inputs using the inverse method is summarised as follows

(a) Construct and the system discrete-time state-space model, Eqns (7) and (13), and measure the system response, *X(k)*.
(b) Use the Kalman filter, Eqns (15) to (21), to obtain the innovation, $\bar{Z}(k)$, the innovation covariance, $S(k)$, and the Kalman gain, $K_a(k)$.
(c) Use the adaptive weighted recursive least square algorithm, Eqns (22) to (26) to estimate the unknown force, $\hat{F}(k)$.

A flow chart of the computation for the application of the recursive input estimation algorithm is given in Fig. 3.

4. RESULTS AND DISCUSSIONS

To verify the availability and precision of the presented approach in estimating the unknown moving tank and missile impact force inputs, the bridge structure is modelled as a simple beam with a total span, L = 30 m, the flexural stiffness constant, $EI = 1.27914\times10^{11}$ Nm^2, mass per unit length, $\rho = 1.2\times10^4$ kgm^{-1}, and the model proportional damping coefficient, $C_n = \alpha M_n + \beta K_n$, where $\alpha = 0.01$ and $\beta = 0.001$, and the mode shape function, $\Phi_n = \sin(\pi x_k / L)$. The initial condition of the error covariance is given as $p(0/0) = diag[10^4]$ for the KF and $p_b(0) = 10^4$ for the adaptive weighted recursive least square estimator. The simulation conditions are set as follows. The sampling interval, is $\Delta t = 0.01$ s. The sensitivity matrix, $M(0)$, is null. The weighting factor is an adaptive weighting function. The error used to quantify the deviation between the estimated and actual moving force inputs is defined as per cent root mean square difference (PRD)[26]:

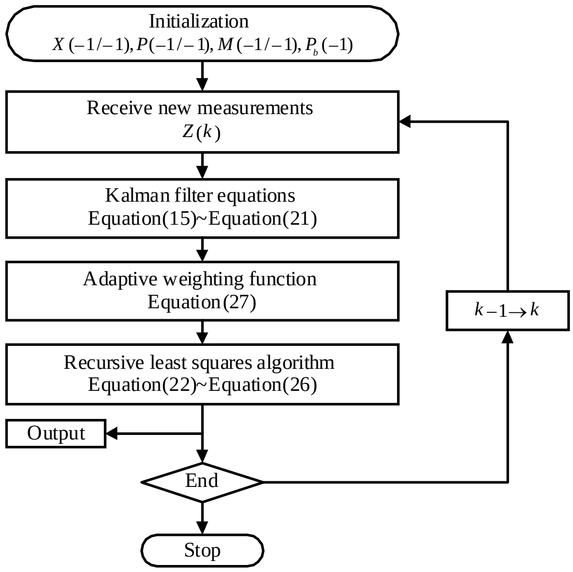

Figure 3. Flowchart of the adaptive input estimation algorithm.

$$Error(\%) = \frac{\sqrt{\sum_{i=1}^{N}\left[F(t_i) - \hat{F}(t_i)\right]^2}}{\sqrt{\sum_{i=1}^{N}\left[F(t_i)\right]^2}} \times 100\%$$

where N is the total number of estimation time steps. $F(t_i)$ and $\hat{F}(t_i)$ represent the actual and estimated forces at time t_i, respectively.

- **Case 1: Singular moving tank and missile impact force input estimation**

The singular tank and missile impact force input is simulated as follows.

A tank with a static weight $F_k = 500$ KN is acting on the bridge structure. The tank has a constant velocity $v_k = 5$ ms^{-1} when passing through the bridge. According to Eqn (6), the time-varying moving tank force input, $F_k(t)$, and the missile impact force input, $F_m(t)$, are superposed as $F(t) = F_k(t) + F_m(t)$. The moving tank force input, $F_k(t)$, is expressed as

$$F_k(t) = \begin{cases} F_k \sin(\pi v_k t / L) & t_i \le t \le t_o \\ 0 & others \end{cases} \quad (28)$$

The missile impact force input, $F_m(t)$, is expressed as

$$F_m(t) = \begin{cases} 10^6 \times \sin(\omega(t - t_s)) \times \sin(\pi x_m / L), \omega = 8 & t_s \le t \le t_e \\ 0 & others \end{cases} \quad (29)$$

where t_i represents the initial time when the tank enters the bridge. There is a time delay for 0.3 s in order to clearly determine the simulation results. $t_o = L/v$ represents the terminal time when the tank leaves the bridge. $x_{m=1\sim3}$ represents the impact position of the missile. The initial and final time of the missile impact are t_s = 1s, and t_e=1.39

s, respectively. The dynamic responses of the bridge structure can be obtained by using a numerical method with the system noise and the measurement noise. The Kalman estimation parameters used in the numerical model are given as follows.

The covariance matrix of the process noise, $Q = Q_w \times I_{2n\times 2n}$, where $Q_W = 10^4$. The covariance matrix of the measurement noise, $R = R_V \times I_{2n\times 2n}$, where $R_V = \sigma^2 = 10^{-14}$.

Figure 4 shows the time history of the singular moving tank and missile impact force input estimation result. The displacement is measured at the middle of the bridge. The simulation result shows that the missile impact force input causes stronger vibration of the bridge. The estimating capability is good, since the estimation values converge to the actual values rapidly. Because the values of $p(0/0)$ and $p_b(0)$ are usually unknown, the estimator can be initialised with large values of $p(0/0)$ and $p_b(0)$, such as 10^4. This condition will have the effect of treating the initial errors as very large values, so that the estimator will ignore the first few estimates. The error (PRD) of the estimated force input is 20.44 per cent approximately . The influence caused by the process and measurement noises on the estimation results is considered. The estimation results demonstrate the validity of the presented inverse estimation algorithm in coping with the singular tank and missile impact force inputs.

The process noise covariance, $Q_W = 10^4$. The measurement noise covariance is adjusted as $R_V = \sigma^2 = 10^{-12}$. The slow response of on-line estimation is shown in Fig. 5. The error (PRD) value of the estimated force input is enlarged to approximately 48.83 per cent. In this case, although the measurement error influences the estimation resolution, the results are still acceptable. To obtain better estimation results, the values of the process noise covariance is adjusted as $Q_W = 10^9$ and $R_V = \sigma^2 = 10^{-12}$. The relatively faster response of on-line estimation is shown in Fig. 6. The results reveal a very good estimating capability, since the error (PRD) of the estimated force input is reduced apparently (10.03 per cent). Next, the first impact position of the missile at middle span will enlarge the vibration of the bridge. The initial and final time of the missile impact are t_s=1s and t_e=1.39 s , respectively. The second impact position of the missile at $L/3$ of the span will reduce the vibration of the bridge. The initial and final time of the second missile impact are t_s=3 s and t_e=1.3.39 s, respectively. The estimation result is displayed in Fig. 7. The error (PRD) of the estimated force input is 9.8 per cent, approximately.

Finally, the tank constant velocity passed through the bridge is set as $v_k = 7\ \text{ms}^{-1}$. The first and second missile impact simulation is set in the same condition as the previous one. The third missile impact is at $2L/3$ of the span. The initial and final time of the missile impact are t_s=7 s and t_e=7.39 s , respectively. Under the third missile impact, all of tanks have passed the bridge. The vibration simulation of the bridge and the force input estimation are presented in Fig. 8. The error (PRD) of the estimated force input is 10.45 per cent approximately. The results are still acceptable. Fig. 9 shows comparison of the inverse estimation using the adaptive and constant weighting functions under the singular tank and single missile impact force inputs. ($Q_W = 10^9$, $R_V = 10^{-10}$ and $x_1 = L/2$). The simulation results demonstrate that the constant weighted input estimator with $\gamma = 0.05$ has better target tracking capability when the unknown input is larger. Howerer, the constant weighted input estimator with $\gamma = 0.05$ is not effecient in reducing the noise effect. Although the constant weighted input estimator with $\gamma = 0.90$ has more effective noise reduction capability, it is not effective in tracking the target. In other words, the proposed

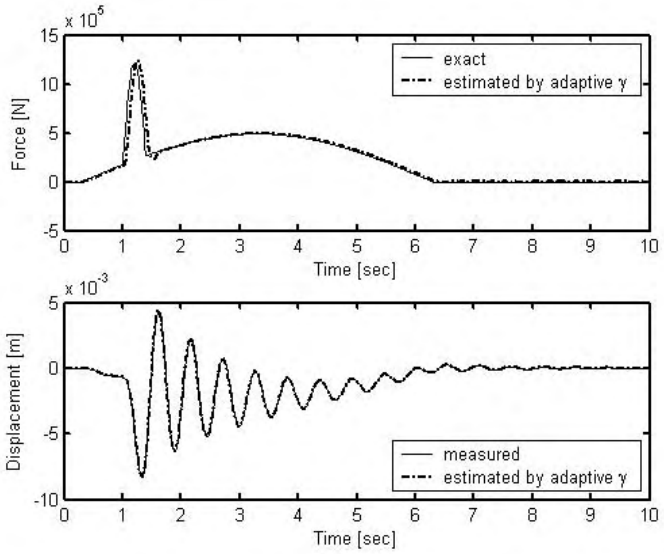

Figure 4. Results of the force input and displacement estimation under the singular tank and single missile impact force inputs. ($Q_W = 10^4$, $R_V = 10^{-14}$, x_1=L/2, and error= 20.44 per cent).

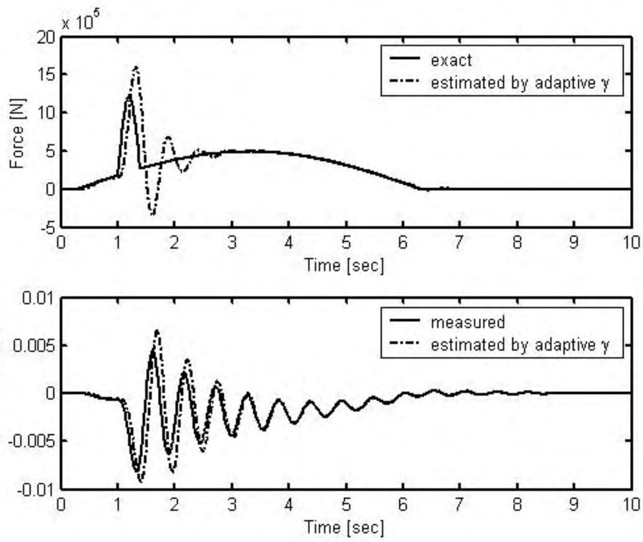

Figure 5. Results of the force input and displacement estimation under the singular tank and single missile impact force inputs. ($Q_W = 10^4$, $R_V = 10^{-12}$, x_1=L/2, and error= 48.83 per cent).

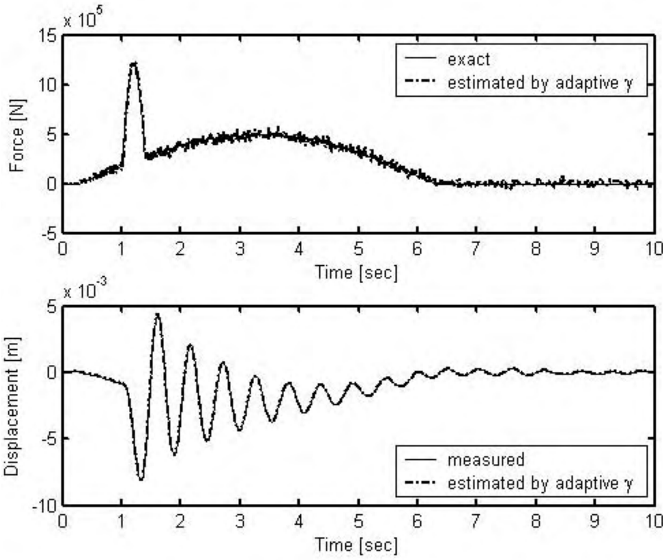

Figure 6. Results of the force input and displacement estimation under the singular tank and single missile impact force inputs. ($Q_W = 10^9$, $R_V = 10^{-12}$, $x_1 = L/2$, and error= 10.03 per cent).

method has the property of faster convergence in the initial response, better target tracking capability and more effective noise reduction.

- **Case 2: Multiple moving tank and missile impact force input estimation**

The multiple tank and missile impact force inputs are simulated in this section. According to Eqn (6), the time-varying moving tank force input, $F_k(t)$, and the missile impact force input, $F_m(t)$, are superposed as $F(t)=F_k(t)+F_m(t)$. The moving tank force input, $F_k(t)$, is expressed as

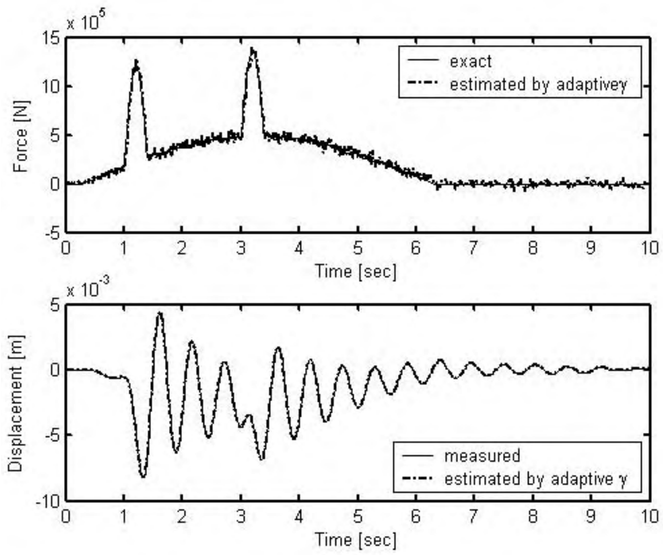

Figure 7. Results of the force input and displacement estimation under the singular tank and double missile impact force inputs. ($Q_W = 10^9$, $R_V = 10^{-12}$, $x_1 = L/2$, $x_2 = L/3$ and error= 9.80 per cent).

$$F_k(t) = \begin{cases} F_k \sin(\pi v_k t / L) & t_i \le t \le t_o \\ 0 & others \end{cases} \quad (30)$$

The missile impact force input, $F_m(t)$, is expressed as

$$F_m(t) = \begin{cases} 10^6 \times \sin(\omega(t - t_s)) \times \sin(\pi x_m / L), \omega = 8 & t_s \le t \le t_e \\ 0 & others \end{cases} \quad (31)$$

where t_i represents the initial time when the tank enters the bridge. There is a time delay for 0.3 s in order to clearly determine the simulation results. $t_o = L / v$ represents the terminal time when the tank leaves the bridge. The time interval between the entrances of the tanks is 0.2 s . $x_{m=1\sim3}$ represents the impact position of the missile. The initial

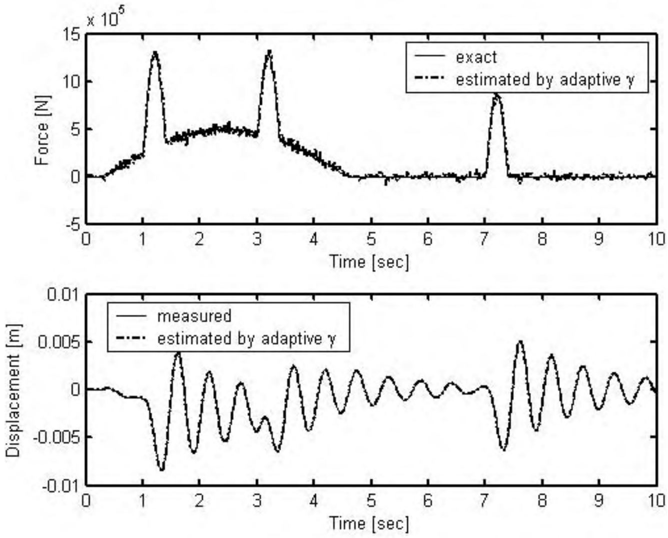

Figure 8. Results of the force input and displacement estimation under the singular tank and triple missile impact force inputs. ($Q_W = 10^9$, $R_V = 10^{-12}$, $x_1 = L/2$, $x_2 = L/3$, $x_3 = 2L/3$, and error= 10.45 per cent).

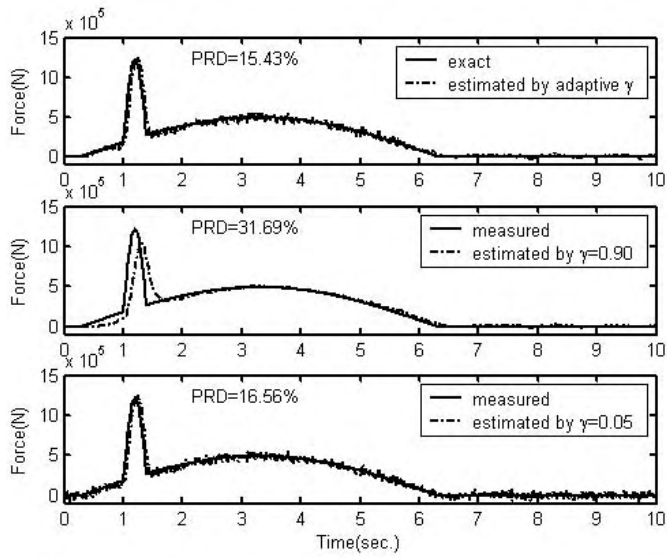

Figure 9. Comparison of the inverse estimation using the fuzzy, and constant weighting functions under the singular tank and single missile impact force inputs. ($Q_W = 10^9$, $R_V = 10^{-10}$, $x_1 = L/2$).

and final time of the missile impact are t_s= 2 s , and t_e=2.39 s, respectively. The multiple tank and missile impact force inputs are simulated as follows. Each tank has a static weight $F_{k=1\sim3} = 500$ KN acted on the bridge structure and a constant velocity $v_{k=1\sim3} = 5\ \text{ms}^{-1}$ when passing through the bridge. The dynamic responses of the bridge can be obtained by using a numerical method with system noise and measurement noise taken into account. The Kalman estimation parameters used in the numerical model are given as follows:

The covariance matrix of process noise, $Q_W = 10^9$. The covariance matrix of measurement noise, $R_V = \sigma^2 = 10^{-12}$. Fig.10 shows the histories of the multiple moving tank and missile impact force input estimation results and the displacement, which is measured at the middle of the span. Because of the uniform movement of each tank, the smooth time histories of the multiple moving tank force input estimation are presented. The simulation results show that the missile impact force input will cause stronger vibration of the bridge. The results reveal a very good estimating capability, since the estimation values converge to the actual values rapidly. The error (PRD) of the estimated force input is 4.29 percent, approximately . It demonstrates good performance in tracking the unknown force inputs of a complex structure system.

Assuming that the velocity of each tank is different, the first being $v_1 = 8\ \text{ms}^{-1}$, the second $v_2 = 6\ \text{ms}^{-1}$, and the third $v_3 = 4\ \text{ms}^{-1}$. The impact position of the missile is at the middle of the span. The initial and final time of the missile impact are t_s=2 s and t_e=2.39 s, respectively. The Kalman estimation parameters are set as $Q_W = 10^9$ and $R_V = \sigma^2 = 10^{-12}$. The time histories of the actual and estimated force inputs are shown in the (a) part of Fig. 11. The simulated curves of force inputs are not smooth due to the tanks with different velocities and the abrupt missile impact. The error (PRD) of the estimated force input is 5.13 percent approximately. The time histories of the measured and estimated displacement are presented in (b) part of Fig.10. The simulation results show that the missile impact force input will cause stronger vibration of the bridge.

The velocity and weight of each tank is assumed as fixed under the same simulated condition as in Fig. 9. The Kalman estimation parameters are set as $Q_W = 10^9$ and $R_V = \sigma^2 = 10^{-12}$. The time histories of the actual and estimated force inputs are shown in the (a) part of Fig.12. The error (PRD) of the estimated force input is 5.07 per cent approximately. The first impact position of the missile at the middle of the span enlarges the vibration of the bridge. The initial and final time of the first missile impact are t_s=2 s and t_e=2.39 s, respectively. The second impact position of the missile at L/3 of the span will reduce the vibration of the bridge. The initial and final time of the second missile impact are t_s= 4 s and 4.39 s, respectively. The third missile impact is at 2L/3 of the span. The initial and final time of the missile impact are t_s=7 s and t_e=7.39 s, respectively. When the third missile impact occurs, the first and second tanks have already passed the bridge. On account of the light load acted on the bridge, the missile impact force input will cause even stronger vibration of the bridge. This situation is presented in (b) part of Fig.13.

The Kalman estimation parameters are set as $Q_W = 10^9$ and $R_V = \sigma^2 = 10^{-12}$. The weight of each tank is assumed as fixed under the same simulated condition as in Fig.11. The time histories of the actual and estimated force inputs are shown in (a) part of Fig. 13. The error (PRD) of the estimated force input is 6.00 per cent, approximately. The

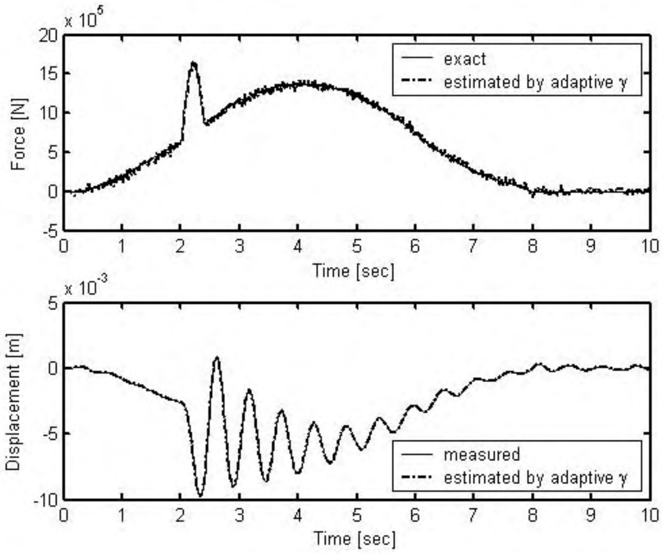

Figure 10. Results of the force input and displacement estimation under the multiple tank and single missile impact force inputs. ($F_1 = F_2 = F_3$ = 500(KN), $v_1 = v_2 = v_3$ = 5 m/s, $Q_W = 10^9$, $R_V = 10^{-12}$, and error= 4.29 per cent).

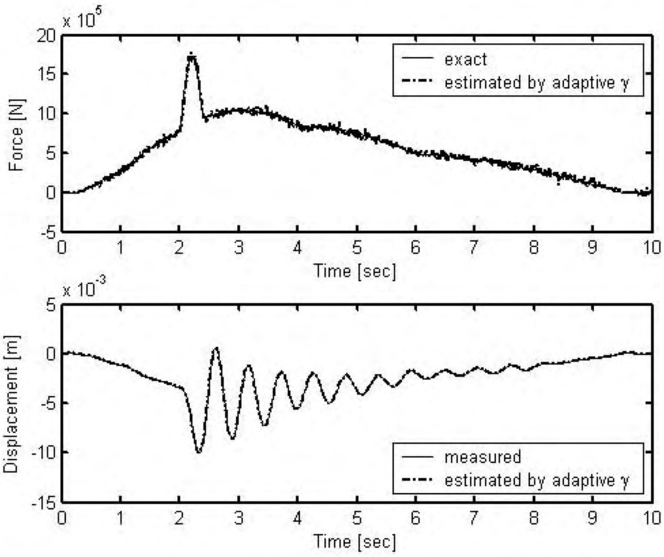

Figure 11. Results of the force input and displacement estimation under the multiple tank and single missile impact force inputs. ($F_1 = F_2 = F_3$ = 500 (KN), v_1 = 8, v_2 = 6, v_3 = 4 m/s, $Q_W = 10^9$, $R_V = 10^{-12}$, and error = 5.13 per cent).

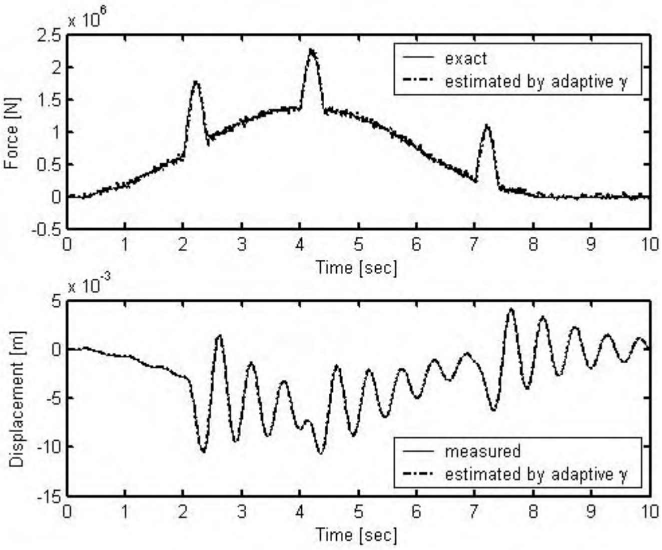

Figure 12. Results of the force input and displacement estimation under the multiple tank and triple missile impact force inputs. ($F_1 = F_2 = F_3$ = 500(KN), v_1, v_2, v_3 = 5 m/s, $x_1=L/2$, $x_2=L/3$, $x_3=2L/3$, $Q_W = 10^9$, $R_V = 10^{-12}$, and error = 5.07 per cent).

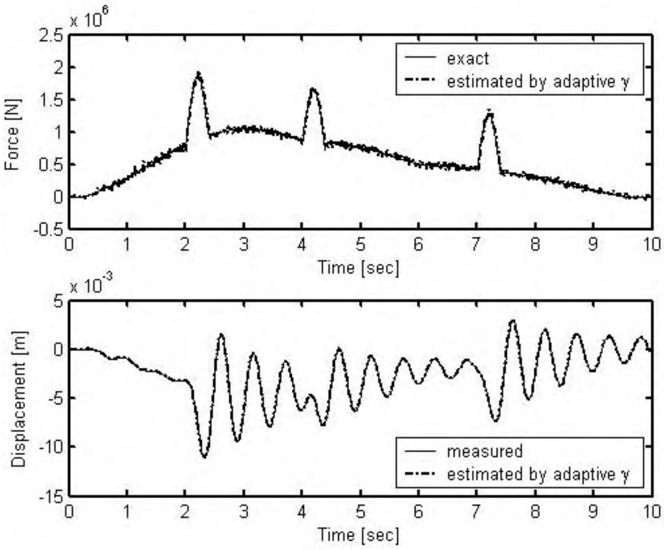

Figure 13. Results of the force input and displacement estimation under the multiple tank and triple missile impact force inputs. ($F_1 = F_2 = F_3$ = 500(KN), v_1 = 8, v_2 = 6, v_3 = 4 m/s, $x_1=L/2$, $x_2=L/3$, $x_3=2L/3$, $Q_W = 10^9$, $R_V = 10^{-12}$, and error = 6.00 per cent).

Table 1. The per cent root mean square difference (PRD) with different simulate variables

Q_W	R_V	PRD	Forces style
10^4	10^{-14}	20.44%	Singular tank and missile impact forces
10^4	10^{-12}	48.83%	Singular tank and missile impact forces
10^4	10^{-12}	10.03%	Singular tank and missile impact forces
10^9	10^{-12}	9.80%	Singular tank and 2 missiles impact forces
10^9	10^{-12}	10.45%	Singular tank and 3 missiles impact forces
10^9	10^{-12}	4.29%	3 tanks (uniform motion) and 3 missiles impact forces
10^9	10^{-12}	5.13%	3 tanks (non-uniform motion) and 3 missiles impact forces
10^9	10^{-12}	5.07%	3 tanks (uniform motion) and 3 missiles impact forces
10^9	10^{-12}	6.00%	3 tanks (non-uniform motion) and 3 missiles impact forces

time histories of the measured and estimated displacement are presented in (b) part of Fig 13.

The above simulation results demonstrate that the measurement, process noise covariance and forces input situation affect the per cent root mean square difference (PRD). From Figs 4-5 it is seen that smaller measurement noise covariance, i.e., the precise measure instrument has excellent estimated results. Figs. 6-8 illustrate that the superior performance in tracking unknown input forces with the larger process noise covariance, Q_W. The slight state variations due to the singular tank and missile impact force inputs are simulated in Case1. Therefore, the larger PRD estimated results on account of the relative strong effect of measure error is shown in Figs 6-8. On the contrast, the smaller PRD estimated results are shown in Figs 10-13. From the above simulation results, the proposed method has good performance in tracking the unknown moving tank and missile impact force inputs of the bridge structure.The tabulation of per cent root mean square difference (PRD) with different simulate variables are shown in Table 1.

5. CONCLUSIONS

This work has presented an online adaptive recursive inverse algorithm to estimate the time-varying unknown moving tank and missile impact force inputs of the bridge structure systems. This algorithm includes the Kalman filter (KF) and the adaptive weighted recursive least square estimator (RLSE). The estimation results demonstrate that the online inverse method proposed in this research can be successfully applied to the determination of the excitation forces. The method has the rapid adaptive capability and great performance in tracking the moving forces, by adequately choosing the precise measurement devices, and adopting the adaptive weighting factor, γ. The future work on this study will also involve the estimation of different types of structure systems with the consideration of nonlinearity.

REFERENCES

1. Cantineni, R. Dynamic behaviour of highway bridges under the passage of heavy vehicles. Swiss Federal Laboratories for Materials Testing and Research (EMPA)

Report, 1992, No. 220, pp 240.
2. Green, M.F. & Cebon, D. Dynamic response of highway bridges to heavy vehicle loads: theory and experimental validation. *J. Sound Vibr.*,1994, **170**, 51-79.
3. Yang, Y. B. & Yau, J.D., Vehicle-bridge interaction element for dynamic analysis. *J. Struct. Engg.*, ASCE, 1997, **123**, 1512-518.
4. Inoue, H.; Ikeda, N.; Kishimoto, K.; Shibuya, T. & Koizumi, T. Inverse analysis of the magnitude and direction of impact force. *JSME Inter. J. Series A*, 1995, **38**, 84-91.
5. Inoue, H.; Kishimoto, K.; Shibuya, T. & Koizumi, T. Estimation of impact load by inverse analysis. *JSME Inter. J. Series I*, 1992, **35**, 420-27.
6. Doyle, J.F. Further Developments in determining the dynamic contact law. *Experimental Mechanics*, 1984, **24**, 265-70.
7. Doyle, J. F. Determining the contact force during the transverse impact of plates. *Experimental Mechanics*, 1987, **27**, 68-72.
8. Doyle, J. F. Force identification from dynamic responses of a bimaterial plate. *Experimental Mechanics*, 1993, **33**, 64-69.
9. Doyle, J. F. A wavelet deconvolution method for impact force identification. *Experimental Mechanics*, 1997, **37**, 403-08.
10. Wang, M. L. & Kreitinger, T. J. Identification of force from response data of a nonlinear system. *Soil Dyna. Earthquake Engg.*, 1994, **13**, 267-80.
11. Hollandsworth, P.E. & Busby, H.R. Impact force identification using the general inverse technique. *Inter. J. Impact Engg.*, 1989, **8**, 315-22.
12. Law, S.S.; Chan, T.H.T. & Zeng, Q.H. Moving force identification: A time domain method. *J. Sound Vibr.*, 1997, **201**, 1-22.
13. Doyle, J.F. & Eisenberger, M. An experimental method for determining the location and time of initiation of an unknown dispersing pulse. *Experimental Mechanics*, 1987, **27**, 229-33.
14. Busby, H.R. & Trujillo, D.M. Solution of an inverse dynamics problem using an eigenvalue reduction technique. *Computers and Structures* 1987, **25**, 109-17.
15. Druz, J.; Crisp, J. D. & Ryall, T. Determining a dynamic force on a plate-an inverse problem. *AIAA Journal*, 1991, **29**, 464-70.
16. Chan, T.H.T.; Law, S.S.; Yung, T.H. & Yung, X.R. An interpretive method for moving force identification. *J. Sound Vibr*., 1999, **219**, 503-24.
17. Hang, C.H., An inverse non-linear force vibration problem of estimating the external forces in a damped system with time-dependent system parameters. *J. Sound Vibr.*, 2001, **244**, 749-65.
18. Ma, C.K.; Tuan, P.C.; Lin, D.C. & Liu, C.S. A study of an inverse method for the estimation of impulsive loads. *Int. J. Sys. Sci.*, 1998, **29**, 663-72.
19. Ma, C.K.; Chang, J.M. & Lin, D.C. Input forces estimation of beam structures by an inverse method. *J. Sound Vibr*., 2003, **259**, 387-07.
20. Tommy, H. T. C. & Demeke, B. A., Theoretical study of moving force identification on continuous bridges. *J. Sound Vibr.*, 2006, **295**, 870-83.
21. Anil, K. C. Dynamics of structures, theory and applications to earthquake engineering. Prentice-Hall Inc, London, 1995. 597 p.
22. Fun, T., Random vibration of engineering. *In* Defense Industries Publication, Peking, 1995. pp 364.
23. Bogler, P. L. Tracking a maneuvering target using input estimation. *IEEE Trans. Aeros.Electr. Syst*, 1987, **23**, 298-310.
24. Tuan, P.C.; Ji, C.C.; Fong, L.W. & Huang, W.T. An input estimation approach to on-line two dimensional inverse heat conduction problems. *Numer. Heat Transf. B*, 1996, **29**, 345-63.
25. Tuan, P.C. and Huang, W.T. Adaptive robust weighting input estimation method for the 1-D inverse heat conduction problem. *Numeri. Heat Transfer B*, 1998, **34**, 439-56.
26. Ma, C.K.; Chang, J.M. & Lin, D.C. Input forces estimation of beam structures by an inverse method. *J. Sound Vibr*., 2003, **259**, 387-407.

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