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Ballistic Missile Warhead Recognition based on Micro-Doppler Frequency

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ABSTRACT

To elucidate the spinning-precession signatures of ballistic warhead, the model of spinning precession for ballistic missile warhead is established and the mathematics of micro-Doppler signatures caused by spinning-precession is derived. Then the micro-Doppler features are analysed using high-resolution time-frequency transform, and the model predictions match the experimental data well. Based on different mass of warheads and decoys, the feature, which can reflect the mass of the targets, is extracted from the time-frequency plane, proving a new method for recognising warheads and discriminating these from decoys. Finally the validity of the feature extracted in this study is verified by computer simulations even with low signal-to-noise ratio.

Keywords: Spinning precession target recognition, micro-doppler, feature extraction, high-resolution time-frequency transform, warheads, ballistic missile warhead

1. INTRODUCTION

Give-and-take between offence and defence has been going on since one cave man first threw a rock at another. Defence provides value in many ways–its protection, its deterrent effect, and the complications it presents to the offence. Measured in these terms, defence may be worth doing even if today it cannot answer all of tomorrow's threats. Neutralising the threat of an incoming ballistic missile is a difficult task, which makes the ballistic missile defence (BMD) more and more important. The BMD system consists of boosting-phase defence system, midcourse-phase defence system, and reentry-phase defence system. Midcourse phase is the longest phase of flight, which has a good chance to recognise and intercept[1]. However, warheads flying outside the aerosphere in the midcourse phase are usually accompanied by decoys and debris moving with the same velocity, which makes a great challenge for the detection and recognition of ballistic missiles, and has become a major technological choke point. Since many warheads are spin-stabilised, precession, which is a special signature of a spinning conic warhead, will occur if there is a latitudinal disturbance, which is generally unavoidable. Due to the lack of infrared information, the ground-based radar plays an indispensable role in detecting and discriminating ballistic missile targets in midcourse[2]. Effective radar characteristics such as tracks, radr cross section (RCS), matrix of polarisation scattering and images of scattering centres can be obtained in the midcourse of ballistic trajectory. So target features including position, velocity, orbit, shape, and the exterior material's electromagnetic parameters, etc can also be induced. Thus, the missile warheads can be distinguished from light decoys, satellites and space debris. However, the heavy decoys protecting missile warheads in midcourse and reentry phases are similar to the warheads on characteristics of shape, scattering, and radiation. These features can't be effectively used to recognise the real missile warheads. Hence, the discrimination between missile warheads and heavy decoys must be done by other means. A method to estimate the size and shape of targets using the single-range Doppler interferometry was expatiated by Sato[3]. Baker[4] presented a method of extraction for the spinning frequency as the recognition feature of warheads by the techniques of targets motion recognition. The process of ballistic missile releasing decoys outside the aerosphere was analysed and the method of discriminating warheads from heavy decoys was given by Shu-dan and Xuan[5]. Still there is no effective way testified in practice for discriminating warheads from heavy decoys.

In recent years, micro-Doppler effect and its application in radar for target recognition has been researched. Mechanical vibration or rotation of a target, or structures on the target, may induce additional frequency modulations on the returned radar signal, which generate sidebands about the target's Doppler frequency, called micro-Doppler effect[6]. Micro-Doppler can be regarded as a unique signature of the target and provides additional information that is complementary to existing methods. However, the relative intensity of micro-Doppler effect is usually very weak. To exploit these weak features, high-resolution time-frequency analysis with high dynamic range is needed. Recent research has indicated that the time-frequency analysis is a suitable tool to extract the time-varying micro-Doppler signatures[7]. So, it is a reasonable idea to recognise and classify targets based on time-frequency distribution features of micro-dynamics. Micro-Doppler features in the joint time-frequency domain are analysed

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and exploited, providing useful information for target recognition.

This study proposes a method to discriminate the warheads from heavy decoys based on the feature of precession which is extracted from time-frequency distribution. A model has been developed for micro-doppler analysis of spinning precession target.

2. MATHEMATICAL MODEL FOR SPINNING-PRECESSION

For ballstic missile attacking, zero angle of attack (AOA) is required before the reentry phase. That means, the orientation of the missile warhead and the direction of its velocity should be accordant. So when missile warheads and decoys are released, these usually spin to keep the orientation[8]. It is known in geostatics theory that a spinning rigid body will precess if there is latitudinal disturbance. Generally, this disturbance is unavoidable during releasing. Therefore, missile warheads and heavy decoys mostly keep precessing until reentering the aerosphere. In the spinning precession model of a cone object in Fig.1, the radar is stationary and located at $(-R_0, 0, 0)$ in the radar coordinate system (x, y, z). The top of the cone object is located at the origin O of the target co-ordinate system (X, Y, Z), and Z-axis points at the geometrical axis all along. The target local co-ordinate system shares the same origin O with the radar coordinate system.

Assuming that the half-cone angle and precession angle of the cone target are $\alpha (0 < \alpha < \pi/2)$ and $\theta (0 \le \theta \le \pi/2)$, while the precession angular velocity and spinning angular velocity are w and Ω, respectively. Two point scatters A and B are located at (X_1, Y_1, Z_1) and (X_0, Y_0, Z_0). One can see from Fig. 1 that $X_1 = -X_0$, $Y_1 = Y_0 = 0$, $Z_1 = Z_0$. The movement can be considered as, first, a precession from A and B to A` and $B$`, and then a spinning with an angular velocity Ω around Z'-axis. At time t, the relation between radar coordinate and cone target coordinate system can be expressed as

$$\begin{cases} x(t) = (Z(t)\sin\theta + X(t)\cos\theta)\cos wt \\ y(t) = v_R t + (Z(t)\sin\theta + X(t)\cos\theta)\sin wt \\ z(t) = Z(t)\cos\theta - X(t)\sin\theta \end{cases} \quad (1)$$

where

$$\begin{cases} X(t) = X_0 \cos\Omega t \\ Y(t) = X_0 \sin\Omega t \\ Z(t) = Z_0 \end{cases}$$

By substituting the above Eqn. into Eqn.(1), one can obtain

$$\begin{cases} x(t) = Z_0 \sin\theta + X_0 \cos\Omega t\cos\theta)\cos wt \\ y(t) = v_R t + (Z_0 \sin\theta + X_0 \cos\Omega t\cos\theta)\sin wt \\ z(t) = Z_0 \cos\theta - X_0 \cos\Omega t\sin\theta \end{cases} \quad (2)$$

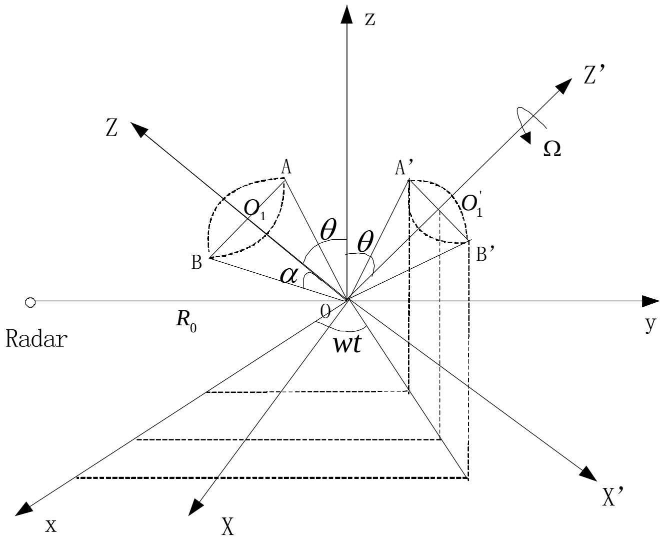

Figure 1. The spinning-precession model of a cone object.

Thus, the range between the point scatter B and radar is

$$R(t) = \sqrt{x^2(t) + (R_0 + y(t))^2 + z^2(t)} \quad (3)$$

where, $x(t)$, $y(t)$, $z(t)$ are shown in Eqn. (2).

When $X_0, Z_0 << R_0$, $R(t)$ can be expressed as

$$R(t) = R_0 + v_R t + (Z_0 \sin\theta + X_0 \cos\Omega\ t\cos\theta)\sin wt \quad (4)$$

The returned radar signal of point scatter B is

$$\begin{aligned} s_r(t) &= \rho(X_0, Z_0)\exp\left\{-j2\pi f\,\frac{2R(t)}{c}\right\} \\ &= \rho(X_0, Z_0)\exp\{-j\Phi(t)\} \\ &= \rho(X_0, Z_0)\exp(-j\frac{4\pi f}{c}[R_0 + v_R t \\ &\quad +(Z_0 \sin\theta + X_0 \cos\Omega t\ \cos\theta)\sin wt] \end{aligned} \quad (5)$$

By compensating the transition, the returned radar signal of point scatter B is

$$\begin{aligned} s_r(t) &= \rho(X_0, Z_0)\exp(-j\frac{4\pi f}{c}(Z_0 \sin\theta \\ &\quad + X_0 \cos\Omega t\cos\theta)\sin wt) \end{aligned} \quad (6)$$

where, $\rho(X_0, Z_0)$ is the reflectivity of the point scatter B, $\Phi(t) = 2\pi f\,(2R(t)/c)$. The Doppler frequency f_d of the returned signal is

$$\begin{aligned} f_d &= \frac{1}{2\pi}\frac{d\Phi(t)}{dt} = \frac{2f}{c}\frac{dR(t)}{dt} \\ &= \frac{2f}{c}[v_R + (Z_0 \sin\theta + X_0 \cos\Omega t\cos\theta)w\cos wt \\ &\quad - X_0\Omega\sin\Omega t\cos\theta\sin wt] \end{aligned} \quad (7)$$

The first term of Eqn. (7) which is caused by transition, is traditional Doppler frequency, while the latter terms caused by spinning and precession are micro-doppler frequency.

$$
\begin{aligned}
f_{micro} &= \frac{2f}{c}[(Z_0\sin\theta + X_0\cos\theta\cos\Omega t)w\cos wt \\
&\quad - X_0\Omega\cos\theta\sin\Omega t\sin wt] \\
&= \frac{2f}{c}[Z_0 w\sin\theta\cos wt + X_0 w\cos\theta\cos\Omega t\cos wt \\
&\quad - X_0\Omega\cos\theta\sin\Omega t\sin wt] \\
&= \frac{2f}{c}\{Z_0 w\sin\theta\cos wt + \frac{1}{2}X_0 w\cos\theta[\cos(\Omega - w)t + \cos(\Omega + w)] \\
&\quad - \frac{1}{2}X_0\Omega\cos\theta[\cos(\Omega - w)t - \cos(\Omega + w)t]\} \\
&= \frac{2f}{c}[Z_0 w\sin\theta\cos wt - \frac{1}{2}X_0(\Omega - w)\cos\theta\cos(\Omega - w)t \\
&\quad + \frac{1}{2}X_0(\Omega + w)\cos\theta\cos(\Omega + w)t]
\end{aligned} \tag{8}
$$

From Eqn.(8), one can see that f_{micro} is the sum of three sinusoidal, thus it is periodic.

If there is no spinning, Eqn. (8) can be simplified as

$$
\begin{aligned}
f_{micro} &= \frac{2f}{c}(Z_0 w\sin\theta\cos wt + X_0 w\cos\theta\cos wt) \\
&= \frac{2f}{c}[(Z_0\sin\theta + X_0\cos\theta)w\cos wt] \\
&= \frac{2f}{c}[\sqrt{X_0^2 + Z_0^2}(\frac{Z_0}{\sqrt{X_0^2 + Z_0^2}}\sin\theta \\
&\quad + \frac{X_0}{\sqrt{X_0^2 + Z_0^2}}\cos\theta)w\cos wt] \\
&= \frac{2f}{c}[\frac{Z_0}{\cos\alpha}(\cos\alpha\sin\theta + \sin\alpha\cos\theta)w\cos wt] \\
&= \frac{2f}{c}[\frac{Z_0}{\cos\alpha}\sin(\theta + \alpha)w\cos wt]
\end{aligned} \tag{9}
$$

The above equation. is the micro-Doppler frequency induced only by precession. From the process of ballistic missile releasing warheads and decoys, one can see that the spinning angular frequency of these are the same or similar. Thus, the difference of micro-Doppler frequencies between warheads and decoys is indicated only by the frequency produced by precession. Since the heavy decoys are similar to the warheads on characteristics of shape, scattering and radiation, it is difficult to distinguish warheads from heavy decoys using imaging. Considering the differences on mass between warheads and heavy decoys, the characteristic of mass can be used to distinguish warheads from heavy decoys. Liu[9], *et al* show that the precession angles of warheads or heavy decoys are inversely proportional to the density while the precession period is proportional to the density. One can see from Eqn. (9) that the amplitude of micro-Doppler frequency is smaller than that of heavy decoys as the precession angle and anglular frequency of the warheads are smaller than those of heavy decoys because of their larger density.

3. TIME-FREQUENCY ANALYSIS OF MICRO-DOPPLER SIGNATURES

Due to lack of localised time information, the Fourier transform, however, cannot provide more complicated time-varying frequency modulation information. A joint time-frequency analysis that provides localised time-dependent frequency information is needed for extracting time-varying motion dynamic features. To analyse the time-varying frequency characteristics of the micro-doppler modulation and visualise the localised joint time and frequency information, the signal must be analysed using a high-resolution time-frequency transform, which characterises the temporal and spectral behaviour of the analysed signal.

Time-frequency transforms include linear transforms, such as the short-time Fourier transform (STFT), and bilinear transforms, such as the Wigner-Ville distribution (WVD). With a time-limited window function, the resolution of the STFT is determined by the window size. There is a trade-off between the time resolution and the frequency resolution. A larger window has higher frequency resolution but a poorer time resolution. It is difficult to find a suitable time-limited window to suit different time series for a time-varying nonstationary signal, which is the most deficiency for STFT. The bilinear WVD has better joint time-frequency resolution than any linear transform. It suffers, however, from a problem of cross-term interference. If a signal contains more than one component in the joint time-frequency domain, its WVD will contain cross terms that occur halfway between each pair of auto-terms. The magnitude of these oscillatory cross terms can be twice as large as the auto terms. To reduce the cross-term interference, the filtered WVD has been used to preserve the useful properties of the time-frequency transform with a slightly reduced time-frequency resolution and largely reduced cross-term interference. The usual types of kernel functions, which lead to the smoothed Wigner, the pseudo Wigner and the smoothed pseudo Wigner-Ville distribution, can be designed to reduce the cross-term interference problem of the WVD. Other high-resolution time-frequency transform includes the adaptive time-frequency transform. It decomposes a signal into a family of basis functions, such as the Gabor function, which is well localised in both the time and the frequency domain and adaptive to match the local behaviour of the analysed signal.

In principle, any time-frequency transform can be used to analyse micro-Doppler modulations. However, a desired time-frequency transform should satisfy the requirements on high resolution in both the time and frequency domains and low cross-term interference. The smoothed pseudo Wigner-Ville distribution and the adaptive time-frequency transform are all good candidates for analysing micro-Doppler modulations. In the micro-Doppler signature study, the smoothed pseudo Wigner-Ville distribution is used to reduce the cross-term interference and achieve higher resolution.

4. FEATURE EXTRACTION

After releasing decoys, ballistic missiles accompanied by a minatory cluster including heavy decoys and debris are flying in the similar ballistic trajectory with the same velocity, which makes a great challenge for the detection and discrimination of ballistic missiles when these are in the same shape and electromagnetic scattering characteristic. Based on the analysis in Section 2, one can derive that the warheads with larger mass have smaller precession angular and precession rate in angular frequency, which corresponds to smaller amplitude of micro-Doppler frequency caused by precession. While the amplitude of micro-Doppler frequency for the decoys with smaller mass is larger.

As the signal is represented in the joint time-frequency functions in time-frequency distribution, the time-varying frequency characteristics of the micro-Doppler can be found from the joint time-frequency domain signature. The warheads can be distinguished from the decoys by extracting the amplitude feature in time-frequency distribution since the amplitudes of their micro-Doppler frequencies are different. The features of the warheads with larger mass which have smaller precession angular, and precession rate in angular frequency are smaller than that of the decoys.

5. SIMULATION STUDY OF MICRO-DOPPLER MODULATIONS INDUCED BY SPINNING-PRECESSION

Assuming that radar operates at 10 GHz and transmits a pulse waveform with a PRF of 2,000 Hz. Suppose the target has two strong scatters: scatter *A* is located at (– 0.1411, 0, 0.8); scatter *B* is located at (– 0.1411, 0, 0.8) with a spin angular velocity $\Omega = 2\pi rad/s$, precession angular $\theta = \pi/9$ and precession angular velocity $w = \pi/2$. The simulated micro-doppler modulation signature is shown in Fig. 2, which is identical to the theoretical micro Doppler modulation calculated by Eqn.(8) shown in Fig. 3.

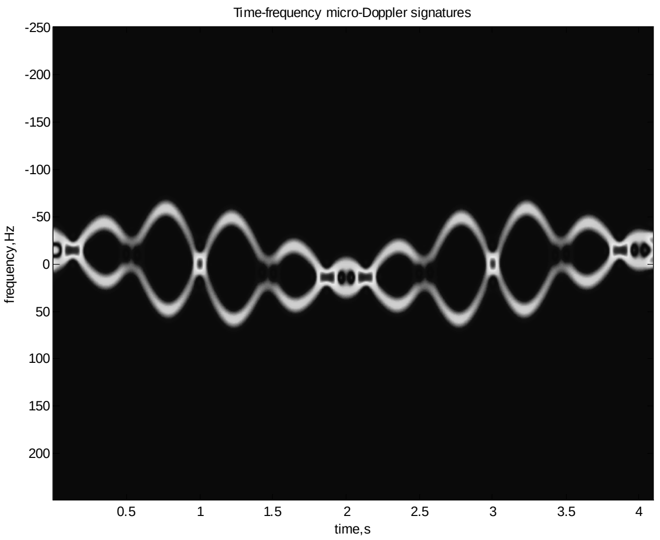

Figure 2. Time-frequency analysis of spinning-precession target.

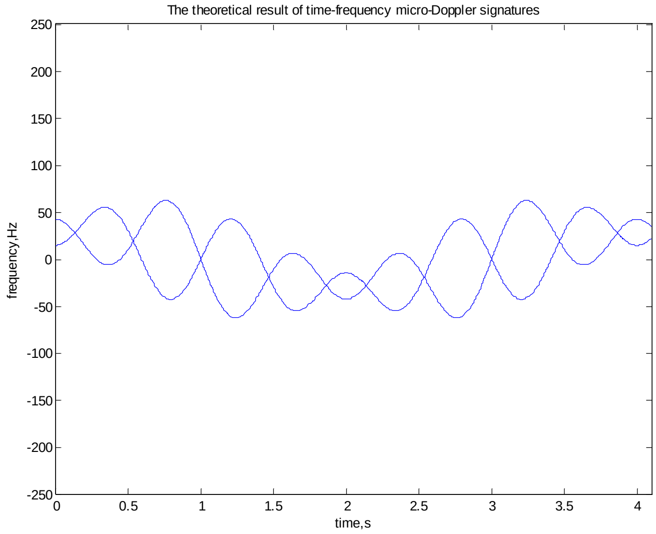

Figure 3. Theoretical result of micro-doppler frequency.

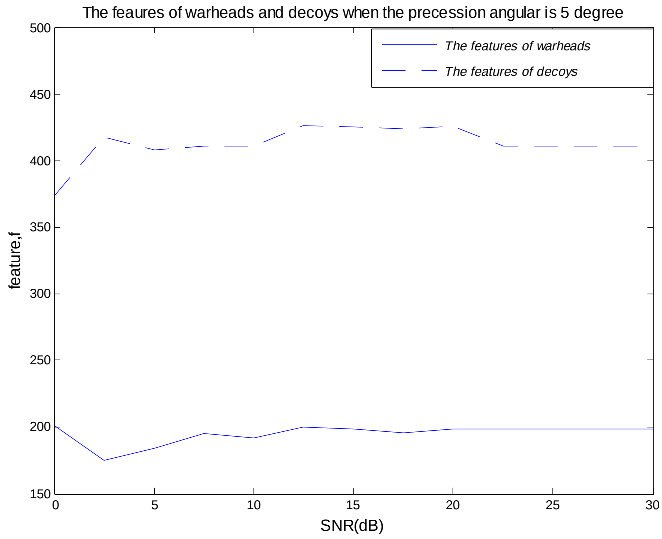

Figure 4. The feature values correspond to 5 degree precession angular.

It is assumed that the warheads have the same electromagnetic characteristic with the decoys. For the conic warhead, it is supposed that the half-cone angle is $\alpha = \pi/18$ and the length is $L = 0.8m$, which is the same for the decoys. The precession angular velocity for the warhead is $w = \pi/12$ where as for the decoy is $w = 5\pi/2$. Figure 4 shows the features for 5 degree precession angle with different SNR, where the real line is the features of the warhead and the broken line is the features of the decoy. Figure 5 shows the features for 10 degree precession angle with different SNR. From the two figures, one can see that the features of the warheads are much smaller than those of the decoys, which can be used to distinguish the warheads from the decoys. One can also see that the features increase as the precession angles increase, which is identical to the theoretical analysis.

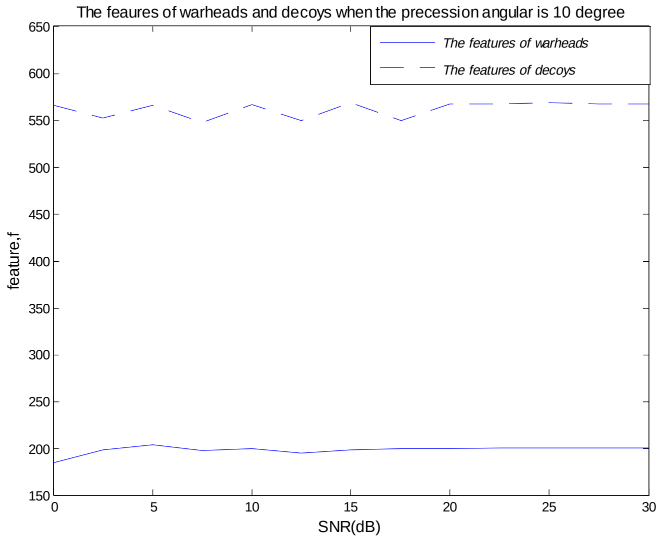

Figure 5. The feature values correspond to 10 degree precession angular.

6. SUMMARY

This study establishes the model of spinning precession for ballistic missile warheads, derived mathematical formulas solving micro-doppler modulations induced by spinning-precession, and applied high-resolution time-frequency transform to micro-Doppler frequency analysis. The mass difference of the warheads and decoys results in the discrepancy of the precession angular, which represents the difference of amplitudes in the time-frequency distribution. By extracting the micro-Doppler signature of amplitudes, the warheads can be distinguished from the heavy decoys. The simulation results confirmed that the presented method is effective even with low SNR.

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