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Sensitivity Analysis of the Turning Motion of an Underwater Glider on the Viscous Hydrodynamic Coefficients

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ABSTRACT

Autonomous underwater gliders (AUG) are a class of underwater vehicles that move using a buoyancy engine and forces from wings. Gliders execute turning motion with the help of a rudder or an internal roll control mechanism and the trajectory of the turn is a spiral. This paper analyses the sensitivity of the characteristics of spiral manoeuvre on the hydrodynamic coefficients of the glider. Based on the dynamics model of a gliding fish whose turn is enabled by a rudder, the effect of hydrodynamic coefficients of the hull and the rudder on the spiral motion are quantified. Local sensitivity analysis is undertaken using the indirect method. The order of importance of hydrodynamic coefficients is evaluated. It is observed that the spiral path parameters are most sensitive to the side force created by the rudder and the effect of the drag coefficient is predominant to that of the lift coefficients. This study will aid in quantifying the effect of change of geometry on the manoeuvrability of AUGs.

Keywords: Hydrodynamic coefficient (HDC); Underwater glider; Maneuverability; Spiral path; Turning; Sensitivity analysis; Hydrodynamics; FSOLVE algorithm

NOMENCLATURE

α	Angle of attack of the glider with the flow in rad
β	Side slip angle in rad
δ	Rudder angle in rad
γ	Roll control mass angle in rad
ω_b	Rotational velocity of the glider in rad/s
$\omega_1\ \omega_2\ \omega_3$	Elements of the ω_b matrix
ω_{3i}	Angular velocity of the glider in rad/s
φ	Glider roll angle in rad
θ	Glider pitch angle in rad
H^*	Original hydrodynamic coefficient
H	Varied HDC
R^*	Original response
R	Varied response
c_m	Multiplier for variation
D	Drag force in Newtons
K_{L0}	Lift Force coefficient with respect to V^2
K_{M0}	Pitch moment coefficient with respect to V^2
K_{q1}	Roll moment coefficient with respect to ωV^2 in rad^{-1}
K_{q2}	Pitch moment coefficient with respect to ωV^2 in rad^{-1}
K^{α}_{D}	Drag Force coefficient with respect to αV^2 in rad^{-2}
K^{α}_{L}	Lift Force coefficient with respect to αV^2 in rad^{-1}
K^{α}_{MP}	Pitch moment coefficient with respect to αV^2 in rad^{-1}
K^{β}_{MR}	Roll moment coefficient with respect to βV^2 in rad^{-1}
K^{β}_{MY}	Yaw moment coefficient with respect to βV^2 in rad^{-1}
K^{β}_{SF}	Side Force coefficient with respect to βV^2 rad^{-1}
K^{δ}_{D}	Drag Force coefficient with respect to δV^2 in rad^{-2}
K^{δ}_{MY}	Yaw moment coefficient with respect to δV^2 in rad^{-1}
K^{δ}_{SF}	Side Force coefficient with respect to δV^2 in rad^{-2}
K_{D0}	Drag force coefficient with respect to V^2
k	Unit normal vector in vertical direction
L	Lift force in Newtons
M	Mass matrix with elements $m_1\ m_2\ m_3$
m_0	Excess mass which is the difference between total mass and buoyancy
m_b	Ballast mass
m_h	Hull mass
m_p	Point mass for pitch control
m_r	Point mass for roll control
r_p	position of pitch control mas in m
r_r	position of roll control mass in m
S	Surface area in m^2
V	Velocity of the glider in m/s
v_1, v_2, v_3	Elements of the v_b matrix
v_b	Translational velocity of the glider

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1. INTRODUCTION

The autonomous underwater glider is a type of underwater vehicle that propels using a change in buoyancy. The buoyancy change is executed by either a buoyancy engine or variation in hull form[1,2]. This change in buoyancy, coupled with forces from wings helps the vehicle in traversing the water bodies in a saw-tooth pattern. Also, an external lifting surface like a rudder or an internal actuation mechanism helps the glider in executing a turning motion. Since the glider cannot maintain level depth in any of the motions (as it does not maintain weight buoyancy balance), the turning motion of a glider is a helical or spiral path. Underwater gliders are extensively used for oceanographic survey purposes and ocean observations. The turning motion is used for column survey in lakes, obstacle avoidance during transit and change in direction during patrol/ survey.

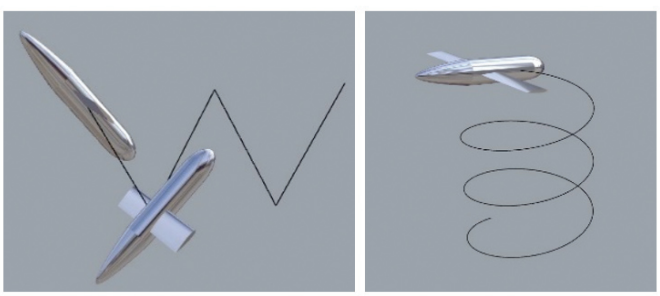
This paper attempts to study the spiral maneuver of an underwater glider and the sensitivity of the turning motion on the viscous hydrodynamic coefficients of the vehicle. A literature review bringing out the dynamics of the glider with participating forces and moments is discussed. Further, the definition of sensitivity analysis and implementation purposes and methods are examined.

2. LITERATURE REVIEW

As brought out earlier, gliders have two characteristic manoeuvres, (a) Sawtooth manoeuvre- used for longitudinal or forward motion, (b) Turning motion of the vehicle, which due to lack of propelled force, becomes a spiral or helical path (Fig. 1). Graver et al.[3] discussed the dynamics of the glider motions, and the authors formulated the equations of motion. The turning maneuver involves the forces and moments in the three coordinates as shown in Fig. 2.

2.1 Turning Motion and the Spiral Path Maneuver

The dynamics of turning motion was discussed by various authors and the same was reviewed by the current authors previously[4] and it was proposed that in the top view (in the horizontal plane), the time control problem of the underwater glider is similar to that of a Dubins' car. The glider, cannot execute an in-plane turn (due to continuous variation of buoyancy) and has to traverse a spiral for turning. Hence, turning motion is an essential motion to study for point to point traverse of the AUG. The turning motion of an AUG can be initiated by either an external control surface like a rudder or an internal rotating mass. The dynamics equations for a glider executing the turn with a rudder were reviewed by Zhang[5], *et al.* for a fish-shaped glider and the dynamics for turn initiated using an internal mass was examined by Zhang[6], *et al.* The model assumed was a steady-state system in both conditions. A flowchart for analysing the turning motion using the dynamics model of an underwater glider was proposed by the current authors (Fig. 3)[7].



(a) (b)

Figure 1. Glider characteristic maneuvers .

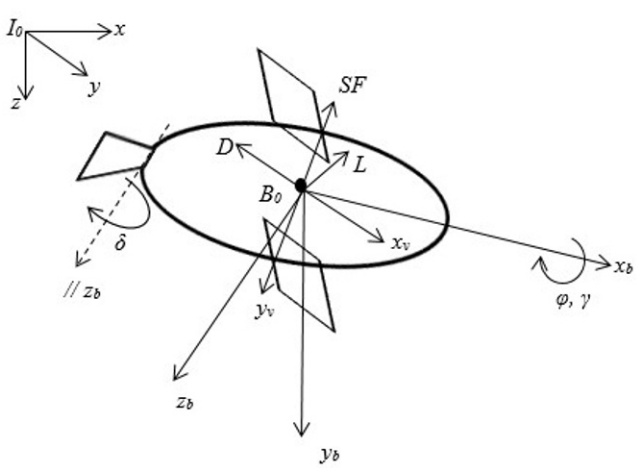


Figure 2. Glider forces and moments along with the Coordinate system used.

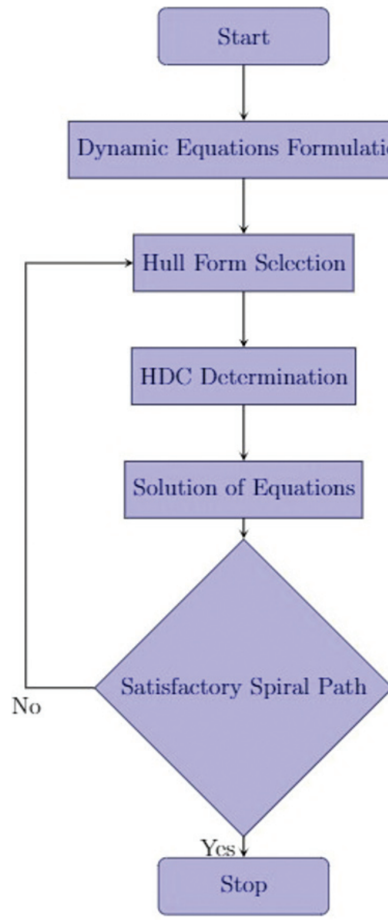


Figure 3. Flowchart for determination of Spiral Path Maneuver[7].

2.2 Dynamics Equations

The dynamics of the underwater glider turning motion, for both internal and external actuation are brought out in Eqns (1) and (2). The parameters for determining the spiralling motion in a steady-state for an underwater glider that are used in these equations are described in the nomenclature.

$$\dot{v}_b = M^{-1}(M * v_b \times \omega_b + m_0 g R^T k + F_{ext}) \tag{1}$$

$$\dot{\omega} = I^{-1}(-I\omega_b + I\omega_b + M v_b \times v_b + T_{ext} + m_r g r_r \times (R^T k) + m_p g r_p \times (R^T k)) \tag{2}$$

In Eqns (1) and (2), the mass matrix M consists of three parts, hull mass m_h, point mass m_r used for roll control, pitch control mass m_p and ballast mass m_b. The buoyancy engine is assumed to be at the origin of the body axis (CG of the glider). The pitch control mass m_p has its motion restricted to longitudinal axis and roll control mass m_r is restricted to move in $y_b - z_b$ plane (Fig. 2). The inputs for the turning motion are excess mass m_0, the position of pitch control mass r_p, angle of roll control mass γ and rudder angle δ.

F_{ext} denotes all external forces: external hydrodynamic forces (lift force (L), drag force (D), and side force(SF)) acting on the underwater glider, and T_{ext} is the total hydrodynamic moment (the roll moment M_1, the pitch moment M_2, and the yaw moment M_3) caused by F_{ext}. M^{-1} and I^{-1} indicate the inverse of the Mass Matrix and the Moment of Inertia Matrix. The matrices include the added mass and the added moment of inertia of the body.

The coefficients used in these equations are called hydrodynamic coefficients and are used to define the forces and moments in the velocity frame (the water flow frame) as below: -

$$D = f(K_D) = \frac{1}{2}\rho S V^2 (K_{D0} + K_D^{\alpha} \times \alpha^2 + K_D^{\delta} \delta^2) \quad (3)$$

$$SF = f(K_{SF}) = \frac{1}{2}\rho S V^2 (K_{SF}^{\beta} + K_{SF}^{\delta} \delta) \quad (4)$$

$$L = f(K_L) = \frac{1}{2}\rho S V^2 (K_{L0} + K_L^{\alpha} \alpha) \quad (5)$$

$$M_1 = f(K_{M1}) = \frac{1}{2}\rho S V^2 (K_{MR}^{\beta} \beta + K_{q1} \omega_1) \quad (6)$$

$$M_2 = f(K_{M2}) = \frac{1}{2}\rho S V^2 (K_{M0} + K_{MP}^{\alpha} + K_{q2} \omega_2) \quad (7)$$

$$M_3 = f(K_{M3}) = \frac{1}{2}\rho S V^2 (K_{MY}^{\beta} \beta + K_{q3} \omega_3 + K_{MY}^{\delta} \delta) \quad (8)$$

The expansion of Eqns (1) and (2) gives the non-linear dynamics equations. Since the model is steady-state, all acceleration terms are ignored. The equations, with known inputs m_0, r_p, γ and δ can be solved to obtain the outputs. The outputs include the six characteristics of the glider spiral path that define the attitude of the glider (Pitch angle θ, Roll angle ϕ), the speed of the glider (Velocity V, rotational velocity ω) and the flow experienced by the glider (angle of attack α, side slip angle β). The Radius of the spiral path is calculated using these parameters. Solution methods involve standard non-linear equation solvers like Newton Raphson methods. A previous study by the current authors used the above equations in conjunction with optimisation algorithms to predict the spiral motion characteristics of an underwater glider[8]. The first study used the FSOLVE algorithm and the results were validated with the work of Feitian Zhang et. al. The results are shown in Fig. 4. The second study compared FSOLVE with a standard Newton-Rhapson method and brought out that FSOLVE is a preferable solver[9].

The values used as inputs for glider characteristics in the current study are shown in Table 1. These include the mass and moment of inertia of the values of the glider. These are taken from the robotic gliding fish.

Table 1. Values of Parameters used in steady-state spiralling equations[5]

Mass and moment of inertia parameters	
m_1	3.88 kg
m_2	9.9 kg
m_3	5.32 kg
m_p	0.8 kg
I_1	0.8 kg.m^2
I_2	0.05 kg.m^2
I_3	0.08 kg.m^2
Viscous hydrodynamic coefficients	
K_{D0}	0.45
K_D^{α}	17.59 rad^{-2}
K_{SF}^{β}	-2 rad^{-1}
K_{SF}^{δ}	1.5 rad^{-1}
K_{L0}	0.075
K_L^{α}	19.58 rad^{-1}
K_{M0}	0.0076 m
K_{q1}	-0.1 m.s/rad
K_{q2}	-0.5 m.s/rad
K_{q3}	-0.1 m.s/rad
K_{MY}^{β}	5 m/rad
K_{MY}^{δ}	-0.2 m/rad
K_{MR}^{β}	-0.3 m/rad
K_{MP}^{α}	0.57 m/rad

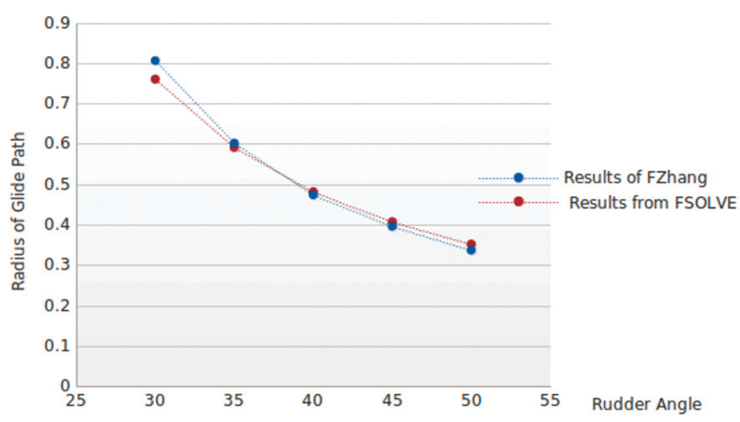


Figure 4. Validation of FSOLVE results with published results. The maximum error was found to be 5 %.

2.3 Hydrodynamic Coefficients for an Underwater Glider

As discussed in the previous section, the forces and moments experienced by the glider due to the interaction of the vehicle with the flow can be modelled using the HDCs. The coefficients in the dynamics equations are of two types: in-viscid and viscous coefficients. While viscous terms arise from Lift, Drag and Side Slip forces, in-viscid terms are the added mass/ inertia and coupling terms that result when a body is accelerated through a fluid at rest. There are a total of 14 viscous coefficients and 36 in-viscid coefficients.

The HDCs in this study are obtained from CFD[5]. A detailed study of the use of CFD for the estimation of HDCs was also undertaken by Singh[10]. The hydrodynamic coefficients seen in glider dynamic equations are slightly different to HDCs seen in ships and submarine etc. This is because the forces and moments are seen concerning the flow velocity axis. For submarines and ships, the maneuvering model expands the forces and moments with respect to the body fixed axis[11,12]. The difference between the velocity frame and body fixed frame is that the axis are defined with respect to the water flow direction as the steady flow is developed on the glider. The values of HDCs are dependent on glider fuselage shape, the layout of wings etc. The effect of wings on the hydrodynamic coefficients of the glider has been studied by various authors. The studies include the change in wing form, layout, sweep angle, and change in longitudinal position of the wing across the glider[13].

2.4 Sensitivity Analysis

Sensitivity analysis (SA) is defined as the investigation of the variation in the output of a numerical model and how it can be attributed to the variations of its input factors[14]. Saitelli [15], *et al.* discussed the purposes of sensitivity analysis as follows: -

- Ranking (or Factor Prioritisation) aims at generating the ranking of the input factors $x_1, x_2, \ldots, x_M$ according to their relative contribution to the output variability;
- Screening (or Factor Fixing) aims at identifying the input factors if any, which have a negligible influence on the output variability;
- Mapping aims at determining the region of the input variability space

Various methods of sensitivity analysis are used in the fields of engineering. These include the Direct method, indirect method, sensitivity normalisation etc. In the current study, we will be using the indirect method. A study on the sensitivity of maneuvering characteristics on hydrodynamic coefficients was undertaken for ships by Debabrata Sen[16]. The definition of sensitivity index used in this study is analogous to a similar index defined by De Kat and Paulling[17]. The methodology used involves characterising *R* which signifies some measure of the maneuvering performance of the vehicle. *R* is defined as

$$R = f(H, O) \tag{9}$$

H represents the hydrodynamic coefficients and *O* represents other vehicle parameters related to its mass properties, initial conditions, control parameters for different maneuvers etc. The purpose of the sensitivity analysis is to relate the changes *R*, to the changes *H* within the operating domain of the glider. Further, a sensitivity index *S* is defined as

$$S = \frac{\frac{(R - R^*)}{R^*}}{\frac{(H - H^*)}{H^*}} = \frac{\frac{\Delta R}{R^*}}{\frac{\Delta H}{H^*}} \tag{10}$$

where *H** represents the basic set of coefficient values and *R** are corresponding maneuvering parameters. *S*, hence, provides a measure of the changes in response *R* resulting from corresponding changes in input coefficients *H*. For example, *S*=0.5 will indicate that an ε% change in the input coefficient *H* concerning its base value *H** will result in a 0.5ε% change in *R* measured concerning *R**. Lower values of S, thus, will indicate low sensitivity of output response to input hydrodynamic coefficient and a higher value of *S* will indicate higher sensitivity. The case where *S*=0 implies that *R* is independent of *H* and changes in input coefficients will not influence at all on output responses. *H* consists of several hydrodynamic coefficients h_j, j=1,.., *M*. In the studies for ships and submarines, any definitive maneuver is quantified by several parameters, i.e.., *R* consisting of r_i, i=1,..., *N*. *M* and *N* are the number of coefficients in the dynamic model and different maneuvering response parameters. For the current study, the viscous HDCs in Table 1 are varied to arrive at the turning path parameters.

For a particular type of definitive maneuver with a given combination of initial conditions and control-surface parameters, one can deduce the sensitivity index S, which will be a matrix with elements S_{ij} denoting sensitivity of the i^{th} response parameter to the j^{th} coefficient. The different values of response parameters r_i can be found from simulating the trajectory. To determine S_{ij} for a k% change in the j^{th} coefficient, we need to perform the trajectory simulation by changing the coefficient to the required amount and determine all the output parameters. The calculation for sensitivity index can be undertaken using the following methodology: -

- Multiply each hydrodynamic coefficient h_m by a factor c_m
- The S_{ij} value for a k% change is achieved by simulating the trajectory with the following setting

$$\begin{aligned} c_m &= 1 \forall m; except - m = j \\ c_m &= (1 + \frac{k}{100}); for - m = j \end{aligned} \tag{11}$$

- The base value of response *R** can be found by setting c_m =1 for all h_m. From Eqns (10) and (11), it can be seen that

$$S_{ij} = \frac{\frac{(r_i - r_i^*)}{r_i^*}}{(\frac{c_m}{100}) - 1} \tag{12}$$

It is evident that sensitivity index values are not only a function of different *i* and *j*, but also are dependent on the value of the change in multiplier *k*. Debabrata sen[16] considered the overshoot maneuver for two submerged geometries (a submarine and an axisymmetric body) in the horizontal and vertical plane and the turning circle maneuver in the horizontal plane. The study was aimed to understand the sensitivity of simulated trajectory on changes in different hydrodynamic coefficients. From his analysis, he found that: -

- Most of the non-linear hydrodynamic coefficients had little influence on the trajectory.
- Linear coefficients (representing damping forces and

moments for the submarine and inertial forces and moments for axisymmetric bodies) are the most sensitive.

- For both geometries, the most influential terms were those associated with moments.

Ray[18], *et al.,* quantified the sensitivity of trajectory simulation to uncertainty levels in various HDCs. The authors brought out the uncertainty in various steps of maneuvering study for an axisymmetric body and a SUBOFF body. The steps being:- (a) mathematical model for the trajectory simulation, (b) HDC measurement using a model test, (c) full-scale trial. Further, they discussed that the Sensitivity studies have been carried out for HDCs of underwater vehicles to determine their relative importance and hence, the error bounds permissible for their values.

Sensitivity studies on underwater gliders have not been studied extensively. In the context of energy consumption, Song[19], *et al.* studied the performance of an underwater glider by establishing two models, an energy consumption model (ECM) and a gliding range model (GRM). The accuracy of the models was established using field trials and then they quantified the impact of fourteen glider parameters on ECM and GRM parameters using a sensitivity index calculated through the Sobol sensitivity analysis method. The results indicated that the gliding angle is the most influencing parameter on ECM, followed by velocity, diving depth, and drag coefficient K_{D0}. For the GRM, the order is diving depth, velocity, K_{D0} and gliding angle.

The sensitivity analysis for underwater gliders with a turning circle/ spiral path maneuver as the definitive maneuver is warranted as the gliders do not have propelled motion like submarines and the speed of operation of the vehicles is very slow. These lead to a considerable deviation in the dynamics of the vehicle. This study is believed to give an understanding of the effect of viscous HDC on the spiral path and the relative importance of HDCs for the maneuver. Ranking of HDCs by arriving at order of their importance is considered as the prime purpose at this juncture.

3. METHODOLOGY

The turning maneuver and its sensitivity to underwater glider HDCs are pertinent to analyse because while individual geometry of the glider fuselage, the shape of wings, position of wings etc. affect the performance of the vehicle, the study of sensitivity to the hydrodynamic coefficients presented in nomenclature will help the designer in identifying which parameter to focus on for optimisation and then take steps to modify the geometry/ other factors affecting the identified parameter.

The definitive maneuver for sensitivity analysis is taken as the spiral path/ helical path maneuver. The trajectory simulation is carried out using the procedure discussed in section 2.2. The trajectory simulation requires three inputs i.e. excess mass m_0, rudder angle δ and position of pitch control mass r_p. Each combination of the three input variables gives one type of spiral path. The output parameters, required to characterise the spiral path are seven. They are $\theta, \phi, \omega_{3i}, V, \alpha, \beta$. The combination of V, θ and α can give the speed in the vertical direction of the glider while executing the spiral path V_{vert} and radius of spiral path R. For the glider studied, seventeen combinations are calculated which result in a stable spiral. The inputs and outputs for these seventeen conditions are taken from published and validated literature[8]. The trajectory simulation method used is the solution of the equations by the FSOLVE algorithm.

We consider for each input condition (fixed values of m_0, δ and r_p, the values of the seven output parameters as R^*, or the base response. The viscous hydrodynamic coefficients for the glider used in Table 2 were used as base hydrodynamic coefficient H^*. Then, the values of HDCs were changed by multiplying each of the coefficients by a fixed value c_m and the trajectory is again simulated. Output parameters of the simulated trajectory obtained were designated as R. Then the sensitivity for this change was calculated using Eqn (12). Maximum sensitivity computed for the range of variation of HDC was considered.

The study is undertaken by calculating R for H for each condition by:-

(a) Change individual HDC value while keeping all others constant (Local SA);

(b) Change one force or moment coefficient while keeping the others constant, e.g. the drag coefficient K_D (which is a function of K_{D0} and K_D^{α}) is changed while keeping all other force and moment coefficients unchanged;

The study is limited to the following conditions: -

(a) SA is conducted for a steady-state turning maneuver.

(b) The yaw angle of the glider is considered to change constantly while roll and pitch angles are considered to be constant in the steady spiral maneuver.

(c) The turn is initiated by the rudder alone and the role of roll control mass has not been included in the same.

(d) HDCs under consideration are the viscous coefficients only.

(e) The method used is the indirect SA method or the brute force method (same as that used by Sen, D[16]).

(f) Range of change of HDC was attempted for -15% to +15%, and the values where the algorithm predicted values near the original range, with convergence, the result was accepted and published here.

4. RESULTS

The results of SA for the change in individual HDCs was undertaken for the 14 HDCs mentioned in Table 1. The sample results of sensitivity for one input condition from published literature are discussed for an explanation. In this condition, the inputs and outputs for the system are shown in Table 2. The change in HDC varied (in this case, the drag coefficient at zero Lift, K_{D0}) is presented in Table 3.

Considering the change percentage of K_{D0} from -15% to +15% with an interval of 5%, and solving the trajectory equations, we obtained the output parameters as shown in

Table 2. Input and output values

Parameters	m_0	r_p	δ	θ	φ	ω_{3i}	V	α	β	R
Values	25	0.5	45	-48.3	-40.6	0.464	0.268	-1.61	4.87	0.396

Table 3. Values of K_{D0}

% change in K_{D0}	-15%	-10%	-5%	+5%	+10%	+15%
Value of K_{D0}	0.3825	0.405	0.4275	0.4725	0.495	0.5175

Fig. 5. The red marked data point is the base output parameter R^* and the other points are the parameters (R) for changed HDC. The value of sensitivity for these output parameters concerning the variation in input HDC (in this case K_{D0}) is plotted in Fig. 6. which gives the effect of change of K_{D0} on the spiral path characteristics. We notice that the sensitivity of the pitch angle θ varies from 0.67 to 0.51 for -15% to +15% change. Three parameters display negative values of sensitivity. In such a case, the absolute value of sensitivity would be considered for establishing the maximum value. The negative sign indicates the inverse relation between the HDC and the output parameter. Thus in Fig. 7, parameter R of the spiral path is the one that gets maximum affected as the sensitivity values of R for change in K_{D0} from 0.89 to 0.69. It is recognised that change in K_{D0} affects the radius of the spiral path (R) maximum and the angular velocity of the glider ω_{3i} in spiral minimum. The maximum sensitivity values of the seven output parameters for all the changes in K_{D0} are selected and tabulated and shown in Table 4.

This method is repeated for the other 11 HDCs and absolute results are shown in Table 5. The interpretation of results can be studied row-wise and column-wise.

Table 4. Maximum sensitivity of output parameters for change in K_{D0}

Output parameter	Maximum sensitivity value
θ	-0.673
φ	0.210
ω_{3i}	-0.064
V	-0.359
α	0.288
β	0.231
R	-0.889

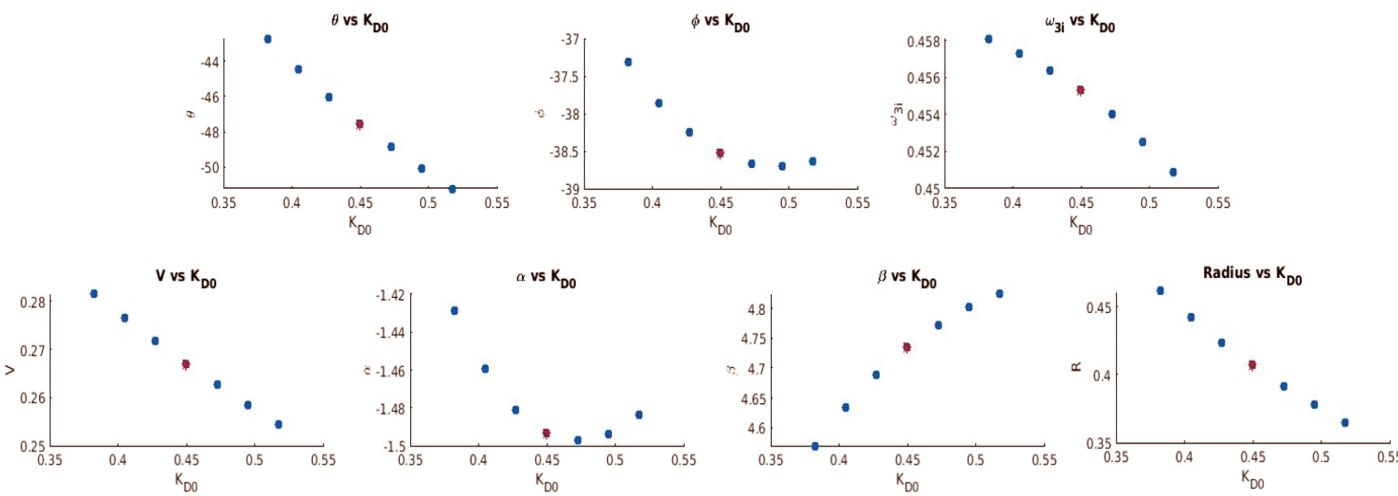


Figure 5. Output parameters for varying HDC K_{D0}. The x-axis in the above figures is the HDC with a variation of -15 % to +15%.

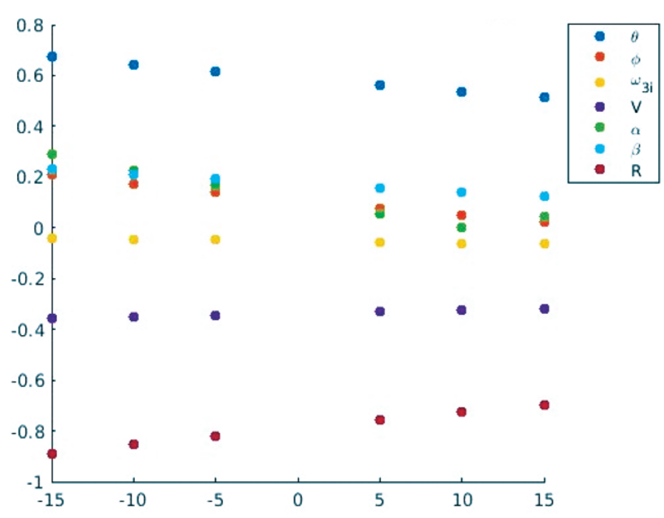


Figure 6. The sensitivity of output parameters for a varying percentage of HDC K_{D0}. The x-axis is the variation percentage.

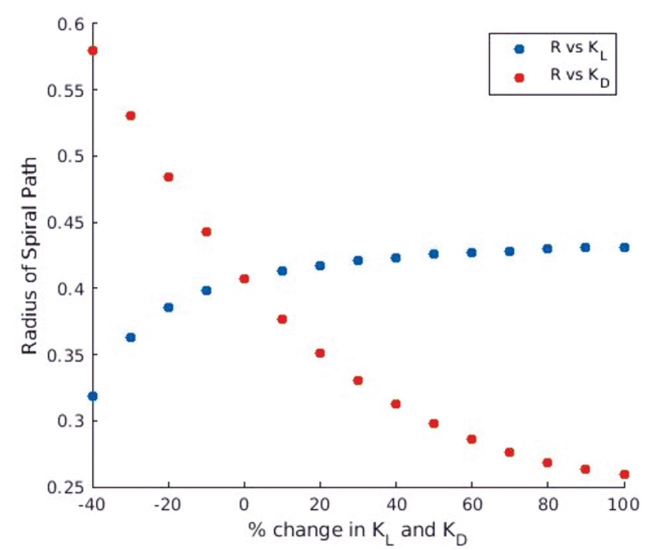


Figure 7. The radius of the Spiral path for change in K_L and K_D.

Table 5. Maximum Sensitivity of Output Parameters on individual Hydrodynamic Coefficients studied in Condition 3

Output	K_{D0}	K^{β}_{SF}	K_{L0}	K^{β}_{MR}	K^{β}_{MY}	K_{q1}	K_{q3}	K^{α}_{D}	K^{δ}_{SF}	K^{α}_{L}	K_{M0}	K^{α}_{MP}	K^{δ}_{MY}	K_{q2}
θ	0.67	0.05	0.03	0.23	0.52	0.33	0.03	0.02	0.505	0.24	0.03	0.06	0.17	0.42
φ	0.21	0.28	0.03	0.23	1.0	0.34	0.05	0.003	2.14	0.27	0.08	0.15	0.35	1.04
ω_{3i}	0.06	0.22	0.02	0.30	0.98	0.43	0.05	0.004	1.56	0.13	0.03	0.06	0.35	0.43
V	0.36	0.01	0.00	0.07	0.11	0.09	0.007	0.011	0.12	0.007	0.01	0.01	0.03	0.07
α	0.29	0.82	0.25	0.83	2.74	1.20	0.14	0.003	5.82	2.29	0.16	0.33	0.94	2.22
β	0.23	0.25	0.01	0.19	0.54	0.28	0.03	0.005	1.94	0.13	0.04	0.08	0.17	0.53
R	0.89	0.23	0.03	0.40	1.44	0.57	0.07	0.022	2.23	0.28	0.05	0.10	0.46	0.66

- **Row-Wise**- The cells in orange colour indicate the HDC that affects the output parameter the most. That is Pitch angle θ and velocity of the glider V are most affected by drag coefficient K_{D0} and parameters φ, ω_{3i}, α, β and R are most sensitive to changes in K^{δ}_{SF}.
- The cell in green indicates that the Velocity of the glider, V, is not affected by the coefficient K_{L0}. This behaviour is also observed in sensitivity values of output parameters on HDCs for various input conditions.
- **Column-wise**- The cells in blue colour indicate the output parameter that is maximum affected by a change in that individual HDC column. We see that the angle of attack α is the most sensitive parameter when we individually change any HDC except Drag related coefficients K_{D0} and K^{α}_{D}. These coefficients affect the radius of the glide path.
- The cell in brown colour indicates the row wise and column-wise maximum sensitive cell. That is, the angle of attack is most affected by the Side force coefficient and amongst the variables affected by the side force coefficient, the angle of attack displays the maximum sensitivity.

Further, a sensitivity study of only the lift and drag coefficients was undertaken. The results are plotted in Fig. 7. The radius of the spiral path showed inverse proportionality to the drag of the vehicle and a direct proportionality to the lift of the vehicle i.e. as the drag coefficient K_D is increased, the radius of the spiral path is found to decrease.

4.1 Analysis of Results

The analysis of results for the sensitivity calculations undertaken are as follows: -

1. The Side force coefficient due to change in rudder angle δ, K^{δ}_{SF} act as the most prominent HDCs. The change in their values affect the spiral path parameters the most, as brought out in Table 5; (For example, for a 10% change in, K^{δ}_{SF}, a maximum of 58.5% change in the angle of attack α of spiral path can be expected.)
2. The Drag coefficient K_{D0} is the second most prominent HDC that effects the Pitch angle of the glider, the velocity of the glider and as a consequence, affetcs the spiral path radius. (For example, for a 10% change in , K_{D0}, a maximum of 3.6% change in the Velocity V can be expected).
3. The velocity of the glider in the spiral maneuver is not affected much by the change in the Lift coefficients (green cell highlighted in Table 5);
4. The radius of the spiral path is inversely proportional to changes in K_D and directly proportional to K_L (as brought out in Fig. 7) for a turn initiated by rudder;
5. The order of importance of HDCs for each spiral maneuver has been arrived at (based on the absolute value of sensitivity of the HDC over that parameter) and the same is shown in Table 6. Speed of turn is maximum dependent on the Drag of the body (and is related inversely, i.e. as drag increases, the velocity of the glider decreases).
6. The angle of attack to the glider, while executing the turn is the most sensitive output parameter in the glider dynamics model (as indicated by the blue cells);
7. The roll angle φ, the rotational velocity ω and Radius of spiral path are most affected by the side force due to rudder. This indicates the dominance of the rudder forces on the turn initiated by rudder.
8. The radius of spiral is inversely proportional to the force coefficient from rudder. That is the more the rudder force, lesser is the spiral path radius.

Table 6. Order of importance of HDC for each parameter of spiral maneuver (1 being the most important)

Parameter	K_D	K_L	K_{SF}	K_{M1}	K_{M2}	K_{M3}
θ	1	6	3	2	4	5
ϕ	6	5	1	3	2	4
ω_{3i}	6	5	1	2	4	3
V	1	6	3	2	4	5
α	6	4	1	2	3	5
β	5	6	1	2	3	4
R	3	6	1	2	5	4

5. CONCLUSION

A sensitivity analysis of the turning motion of an underwater glider on the viscous Hydrodynamic Coefficients is presented. While such studies exist for AUVs and ships, the relevant insights for an underwater glider are not available. The study focused on the spiral path maneuver as the results for the longitudinal saw-tooth maneuver are available in the literature. The results obtained show a heavy sensitivity of the glider's trajectory on the rudder developed forces. The order of importance of HDCs for each output parameter (spiral path characteristics) was estimated.

The results shown here are indicative and may not be taken as definitive quantitative values as: -

- The values depend on the form of the equation of motion.
- The base values of the different coefficients will also affect the sensitivity values.

However, the general trend in the importance of various HDCs is found to be accurate and the study in itself is opined to give the designer better knowledge to choose the glider hull form at the initial design level. The current study is limited to turn initiated by the rudder of the glider. Similar SA can be undertaken for turn initiated by an internal rotating mass.

The authors believe that a major takeaway from this work also includes a tool for designers to understand the effect of change in geometry on the spiral path trajectory of an underwater glider. As observed that the increase in drag leads to a reduction in spiral path radius, a designer can choose a geometry change to modify the drag coefficient. Gliders with futuristic geometries are in design and usage phase. One of such geometries is the blended wing gliders[20,21]. These gliders require thorough study of the geometric parameters. A change in geometry will affect the other glider parameters like the inertia of the vehicle and added mass. Such studies are existing for AUVs and submarines[22] and the need for study for AUGs exist. A further study concerning the sensitivity of the glider on the geometric parameters individually and in conjunction with HDCs will cover the entire spectrum of the effect of change in hull shape. The study will be undertaken by comparing the change of geometry of the vehicle to the HDCs of importance.

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