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Comparison of Mesh Adaptivity Schemes in Finite Element Simulation of Tube Extrusion Process

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ABSTRACT

In this study, finite element simulation of tube extrusion process has been carried out considering different mesh adaptivity schemes. A comparison of these schemes has been made based on stress, strain distribution, and load-stroke curves. Based on the finite element results, it is observed that the success of the computer simulation is dependent on the mesh refinement criteria.

Keywords: Finite element analysis, tube extrusion process, finite element simulation, mesh adaptivity

1. INTRODUCTION

Extrusion is an important metal forming process having wide application in industrial and domestic sectors[1]. Some of the common extrusion products are rods, tubes, channels, I sections, etc. A schematic tube extrusion setup showing its different elements is shown in Fig. 1. A good quality extruded product results after optimising process, geometrical and material parameters. Laboratory experimentation of extrusion considering these parameters is difficult and time consuming. Application of finite element (FE) simulations of extrusion process is getting wide popularity in recent years due to its ease, economy, and short duration[2-4]. The major problems of FE simulation of extrusion is the mesh distortion. Degree of mesh distortion increases with increase in extrusion ratio. A distorted mesh halts the execution due to numerical reasons like negative jacobian, convergence violation, etc. To overcome such difficulties, mesh refinement also called mesh adaptivity is employed[5].

There are several schemes of mesh adaptivity reported in the literature[6]. The aim of this study is to compare various *h*-refinement schemes for an identical tube extrusion problem. MSC. Marc software was used for FE simulations. Three levels of adaptivity were accounted for each scheme. Assessment of the adaptivity schemes were made based on stress, strain, load-displacement curve, and punch movement.

2. MESH ADAPTIVITY

It is based on error estimation and refines the elements and mesh based on the error in them. The refinement criteria commonly used in an adaptive process[1] are:

- *r*-refinement: where the degrees of freedom of the finite element discretisation are kept constant and the positions of the nodes are relocated.

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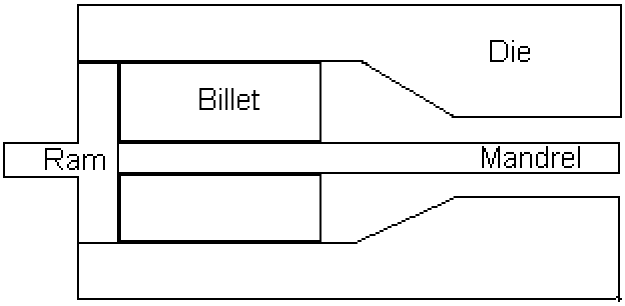

Figure 1. Schematic tube extrusion setup.

- *h*-refinement: where the selected finite elements are subdivided into elements of smaller size.
- *p*-refinement: where the polynomial order *p* of selected elements is increased.
- Suitable combinations of the above three methods can be used.

In this study *h*-refinement criteria given in the Marc software are employed. A brief description of these criteria is given[8].

2.1 Mean Strain Energy Criterion

The element is refined if the strain energy of the element is greater than the average strain energy in a chosen set of elements times a given factor, f_1

$$\text{Element strain in energy} < \frac{\text{Total strain energy}}{\text{Number of elements}} f_1$$

2.2 Zienkiewicz-Zhu Stress Criterion

The error norm is defined[9] as

$$\pi^2 = \frac{\int \left(\sigma^* - \sigma\right)^2 dv}{\int \sigma^2\, dv + \int \left(\sigma^* - \sigma\right)^2 dv}$$

The stress error is

$$X = \int \left(\sigma^* - \sigma\right)^2 dv$$

where σ* is the smoothed stress and σ is the calculated stress.

An element is subdivided if

$\pi > f_1$ and $X_{el} > f_2 * X/\text{NUMEL} + f_3 * X * f_1/\pi/\text{NUMEL}$

where NUMEL is the number of elements in the mesh.

2.3 Zienkiewicz-Zhu Strain Criterion

The error noun is defined[9] as

$$\gamma^2 = \frac{\int \left(E^* - E\right)^2 dv}{\int E^2\, dv + \int \left(E^* - E\right)^2 dv}$$

The strain error is

$$Y = \int \left(E^* - E\right)^2 dv$$

where E* is the smoothed strain error and σ is the calculated strain energy.

$\gamma > f_1$ and $\text{Yel} > f_4 * Y/\text{NUMEL} + f_5 * Y * f_1/\gamma/\text{NUMEL}$

If f_2, f_3, f_4 and f_5 are input as zero, then $f_2 = 1.0$

2.4 Zienkiewicz-Zhu Plastic Strain Criterion

The plastic strain error norm is defined[9] as

$$\alpha^2 = \frac{\int \left(\varepsilon^{P*} - \varepsilon^{P}\right)^2 dv}{\int {}_{\varepsilon}P^2\, dv + \int \left(\varepsilon^{P*} - \varepsilon^{P}\right)^2 dv}$$

The plastic strain error is $A = \int \left(\varepsilon^{P*} - \varepsilon^{P}\right)^2 dv$. The allowable element plastic strain error is $\text{AEPS} = f_2 * A/\text{NUMEL} + f_3 * A * f_1/\alpha/\text{NUMEL}$.

The element will be subdivided when $\alpha > f_1$ and $A_{el} > AEPS$.

2.5 Equivalent Values Criterion

This method is based on either relative (rel) or absolute (abs) testing using either the equivalent Von Mises stress, the equivalent strain, equivalent plastic strain or equivalent creep strain. An element is subdivided if the current element value is a given fraction of the maximum (relative) or above a given absolute value.

$$\sigma_{vim} > f_1 \ \sigma^{max}_{vm} \ \text{or} \ \sigma_{vim} > f_2$$

$$\varepsilon_{vm} > f_3 \ \sigma^{max}_{vm} \ \text{or} \ \varepsilon_{vm} > f_4$$

2.6 Node within a Box Criterion

An element is subdivided if it falls within the specified box. If all the nodes of the subdivided elements move outside the box, the elements are merged back together.

2.7 Nodes in Contact Criterion

An element is subdivided if one of its nodes is associated with a new contact condition. In the case of a deformable-to-rigid contact, this implies that the node has touched a rigid surface. For deformable-to-deformable contact, the node can either be a tied node or a retained node.

Three levels of mesh adaptivity were considered for each criterion. Mesh density increases with increment to adaptivity level. The f's in the above equations are factors required for different criteria.

3. FINITE ELEMENT ANALYSIS OF TUBE EXTRUSION

In this study, a hollow cylindrical billet of 24 mm inner diameter (ID) and 70 mm outer diameter (OD) is extruded to tube of 24 mm ID and 8 mm thickness through a die having thickness of 20 mm. Extrusion ratio is 4.22. The length of the die is 150 mm. Both die and billet are considered as deformable. The finite element modelling of the die-billet setup was carried out using 4-noded axisymetric elements. There are 1080 elements and 1190 nodes in the FE model (Fig. 2). Die backer and punch are considered as rigid whereas die is considered as deformable.

Material properties of the billet and die materials are given in Table 1. Materials for die and tube are die steel and aluminum, respectively. Displacement boundary condition was applied through the punch. Both die and billet are considered as elastic rigid plastic. The finite element analysis was carried out using MSC Marc Software. The punch movement was taken as 60 mm. Three levels of mesh adaptivity were considered in each scheme. Large plastic analysis was carried out in 195 incremental steps. Negligible friction between die and billet was considered in each simulation. Identical material and processing parameters were considered for each case.

Table 1. Material properties of billet and die materials

Material properties	Billet	Die
Young's modulus (MPa)	70000	220000
Yield strength (MPa)	80	1200
Poisson's ratio	0.3	0.3
Hardening component	0	0

4. RESULTS AND DISCUSSION

A comparative chart of various refinement criteria in terms of stress, strain, and maximum punch movement are given in Table 2. The results are:

- Von Mises Stress

The maximum von Mises stresses occur in the die. As the level of adaptivity is enhanced, stresses

Figure 2. Finite element model.

Table 2. Comparative chart of various refinement criteria in terms of stress, strain, and maximum punch movement

Scheme	Adaptivity level	Equivalent Von Mises stress (MPa)	Plastic strain	Punch load (N)	Punch movement (mm)
Mean strain energy	1	1.294×10^3	2.612	1.35×10^7	60
	2	1.294×10^3	2.612	1.35×10^7	60
	3	1.294×10^3	2.612	1.35×10^7	60
Zienkiewicz-Zhu stress criterion	1	1.294×10^3	2.612	1.35×10^7	60
	2	1.515×10^3	2.938	8.55×10^6	50
	3	1.226×10^3	5.955	2.47×10^7	60
Zienkiewicz-Zhu strain criterion	1	1.242×10^3	4.604	2.38×10^7	60
	2	1.262×10^3	2.883	8.37×10^6	48.75
	3	1.222×10^3	5.038	2.52×10^7	60
Zienkiewicz-Zhu plastic criterion	1	1.238×10^3	2.851	2.21×10^7	60
	2	1.238×10^3	2.851	2.25×10^7	58.75
	3	1.369×10^3	2.999	9.56×10^6	50
Equivalent stress (rel) criterion	1	1.226×10^3	2.330	2.36×10^7	60
	2	1.226×10^3	2.330	2.36×10^7	60
	3	1.226×10^3	2.330	2.36×10^7	60
Equivalent stress (abs) criterion	1	1.226×10^3	2.330	2.36×10^7	60
	2	1.226×10^3	2.330	2.36×10^7	60
	3	1.226×10^3	2.330	2.36×10^7	60
Equivalent strain (rel) criterion	1	1.226×10^3	2.330	2.36×10^7	60
	2	1.226×10^3	2.330	2.36×10^7	60
	3	1.226×10^3	2.330	2.36×10^7	60
Equivalent strain (abs) criterion	1	1.227×10^3	2.319	2.36×10^7	60
	2	1.223×10^3	2.334	2.36×10^7	60
	3	1.226×10^3	2.330	2.36×10^7	60
Equivalent plastic strain (rel) criterion	1	1.226×10^3	2.330	2.36×10^7	60
	2	1.226×10^3	2.330	2.36×10^7	60
	3	1.226×10^3	2.330	2.36×10^7	60
Equivalent plastic strain (abs) criterion	1	1.226×10^3	2.330	2.36×10^7	60
	2	1.226×10^3	2.330	2.36×10^7	60
	3	1.226×10^3	2.330	2.36×10^7	60
Node within box criterion	1	1.294×10^3	2.612	1.35×10^7	60
	2	1.294×10^3	2.612	1.35×10^7	60
	3	1.294×10^3	2.612	1.35×10^7	60
Node in contact criterion	1	1.319×10^3	2.615	1.34×10^7	60
	2	1.285×10^3	2.721	1.23×10^7	60
	3	1.310×10^3	3.700	1.18×10^7	60

get saturated in the range 1260-1270 MPa where full punch movement takes place. Most of the billet portion was totally yielded during extrusion. A typical picture of the von Mises stress in the case of node within a box criteria (level 1) is shown in Fig. 3. Maximum stress in the die is around the yield strength of the die material, which is reached in all the cases.

Maximum stress against each criterion is given in Table 2. It can be observed that mean energy, equivalent stress/strain, and node within a box criterion give consistent stresses independent of adaptivity levels. Maximum stress prediction by mean energy and node within a box criterion is 5.26 per cent higher than those of equivalent criteria.

- Equivalent Plastic Strain

Plastic strain values for different adaptivity schemes are given in Table 2. Maximum plastic strain in mean energy and node within a box criterion is 2.612 and equivalent stress/strain criterion is 2.33 for all adaptivity levels. For other criteria, it is not constant. Zienkiewicz Zhu criterion has highest ups and down. A typical plastic strain distribution for node within a box criterion is shown in Fig. 4. Like von Mises stresses, mean energy, equivalent stress/strain, and node within a box criterion give consistent strain independent of adaptivity levels. Maximum strain prediction by mean energy and node within a box criterion is 10.8 per cent higher than those of equivalent criteria.

- Load-stroke Curve

Load-stroke curve considering different adaptivity schemes are shown in Figs 5, 6, and 7. Curves are identical up to 50 mm displacement, because till

Figure 3. Equivalent von mises stress distribution (MPa).

Figure 4. Equivalent plastic strain distribution.

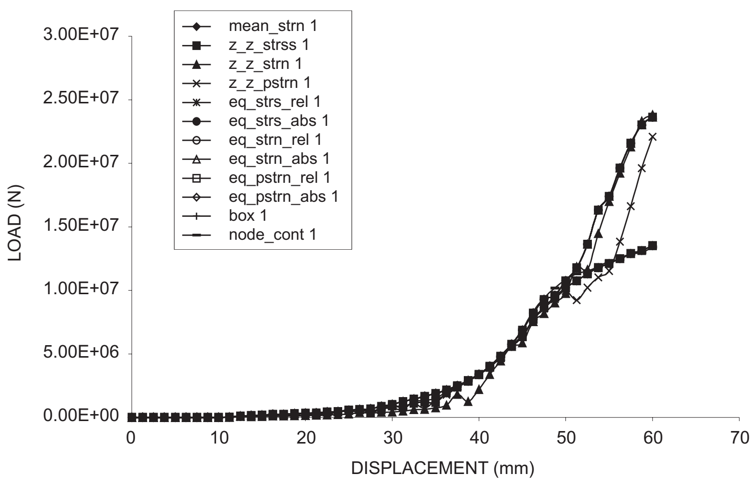

Figure 5. Load-displacement curve, adaptivity level 1.

this, it is used for filling the die cavity. Zinekiewicz and Zhu criterion experiences sudden ups and downs in the load value. It can be observed, although stress and strain are lower in case of equivalent stress/strain criterion, load requirements are on the higher side. Maximum load requirement in equivalent stress/strain criterion is 42.8 per cent higher than the mean energy and node with a box criterion. Mean energy and node within a box criterion give realistic prediction of the load requirement, hence, these are the most suitable for tube extrusion simulation. This is fully supported by stress and strain distribution mentioned above.

- Punch Movement

Maximum punch displacements for each scheme are given in Table 2. Nine out of 12 schemes could undergo full punch movement of 60 mm. Zienkiewicz-Zhu[9] criterion gives minimum punch displacement.

Based on these four parameters it can be inferred that mean energy and node within a box criterion are the best for the tube extrusion simulation. Although

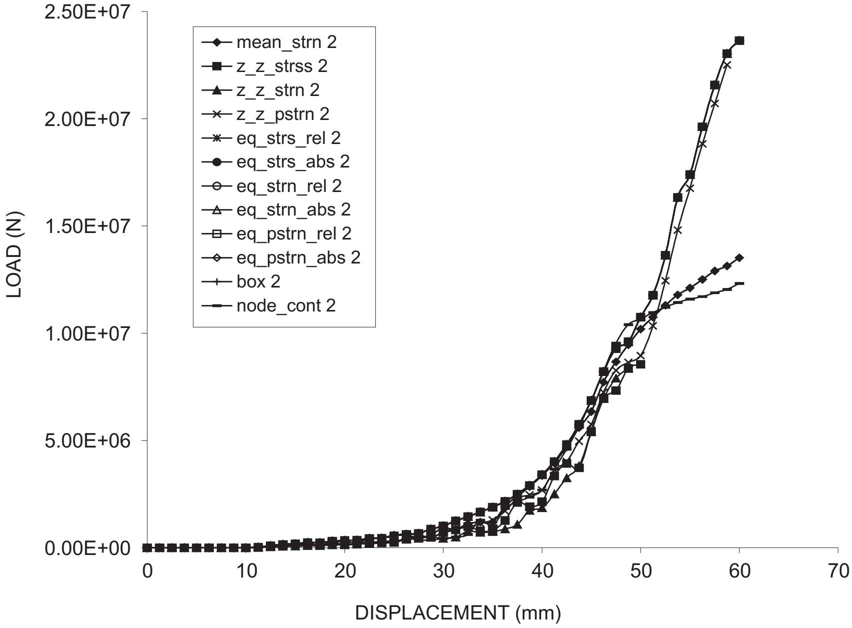

Figure 6. Load-displacement curve, adaptivity level 2.

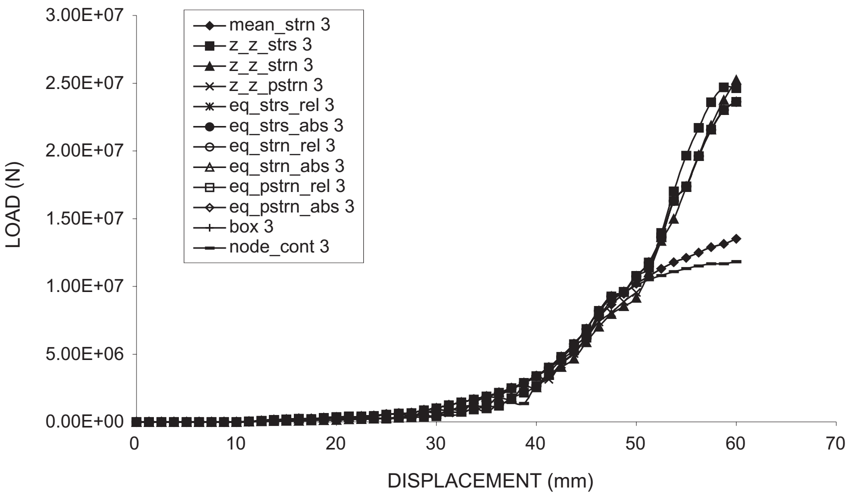

Figure 7. Load-displacement curve, adaptivity level 3.

equivalent stress/strain criterion are consistent in strain, strain, and load predictions, its load predictions are not accurate.

5. CONCLUSIONS

In this study, a comparison of various mesh adaptivity schemes in simulation of a tube extrusion problem is made. Three levels of adaptivity are accounted for in each case. Comparisons are made on stresses, strains, load-stroke curve, and punch movement. It is observed that mean strain energy criterion and node within a box criterion are the best out of the 12 schemes considered in this study. This study will help in the selection of right adaptivity for the success of extrusion simulation.

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