# **Decoy's Operational Jamming Effect**

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#### **ABSTRACT**

The number of decoys deployed is very important for the operational effect in practical war actions, and the more number of decoys deployed, the better is the jamming effect. However, in the development of decoys, the cost-benefit factor is most important. So it is important to study the problem of decoy's cost-effectiveness. In this paper, a mathematical model has been established to study the problem of the cost-effectiveness of decoys.

**Keywords:** Decoy, cost-effectiveness, jamming

#### 1. INTRODUCTION

In the modern warfare, the use of precision-guided weapons make the targets face bigger threat. 'If the target is detected, it would be destroyed', is becoming a reality. As an important resource for camouflage, decoy plays an important role in a war<sup>1-3</sup>. The usage of decoys ensures that the precision-guided weapon can't attack the target precisely, which means that the decoy can jam effectively, or to say that the decoy can make jamming effective.

How to get the best jamming effect is an important issue. Various factors can influence the decoy's jamming effect, such as the decoy's simulative degree to the true target, the ratio of decoys to targets, and so on. Every factors influence the jamming but how to get a better jamming effect is very important for the usage of the decoy.

On the basis of the hypothesis that the three variations—the cost ratio of target-to-decoy, the probability that the target is detected, and the

probability that the decoy is taken as target, are independent of each other. Some problems such as the jamming effect, demand allocation and cost-effectiveness are discussed according to the mathematical model established with its numerical simulation.

#### 2. DECOY'S JAMMING EFFECT

Suppose the number of targets is m and the number of decoys is n, the probability that the camouflage target is detected is  $p_d$  and the probability that the decoy is taken as target is  $p_f$ . Then the number of targets taken as true by enemy is  $mp_d + np_f$ . Suppose the probability that the target is attacked after being detected is  $p_{ad}$  and the probability that the target is attacked is  $p_a$ . Then

$$p_{ad} = mp_d / (mp_d + np_f) \tag{1}$$

$$p_a = p_d p_{ad} = p_d \frac{mp_d}{mp_d + np_f}$$
 (2)

In the modern warfare, the usage of precisionguided weapon ensures that the target will be damaged as soon as being attack. So it is rational that the probability of attacked is equal to the probability of damage (the following analysis is based on this assumption). On the condition that the probability of attack is less than a little value such as 5 per cent, the target can be taken as impossible to be damaged. Without decoys around, the probability of detection is equal to probability of attack for the target. Obviously, it is very difficult to reduce the probability of detection to a little value such as 5 per cent in the most cases. The probability of detection is inclined to high, especially on condition that it is hard to camouflage the target. After the decoys are deployed, the demand to camouflage the target can be reduced. For example, if  $p_f = 1$ , m = n = 1, then the detected probability is < 25per cent to make the probability of damage reduce to < 5 per cent.

If a is the number ratio of the decoys to targets, that is  $a = \frac{n}{m}$ , then from Eqn (2), i.e.,

$$a = \frac{n}{m} = \frac{p_d}{p_a p_f} \left( p_d - p_a \right) \tag{3}$$

Figure 1 shows the relation between the ratio of decoys to targets and the probability of detection, the probability of attack of target on condition that the distance is far enough and the decoy is made effective, that is  $p_f = 1$ . It can be seen from Fig. 1 that the probability of attack reduces as the ratio of decoys to targets increases. The higher the ratio of decoys to targets, the lower the probability of attack. However, if the number of decoys is too large, the cost will also increase too much. How to balance the decoy number and the cost becomes an important issue.

### 3. COST-EFFECTIVENESS OF DECOY

In practical battle disposition, the question arises how many decoys are needed around a target to gain the most operational effectiveness with the least cost? In other words, what the ratio of decoys to targets should be to reach the best cost-effectiveness? It is an important issue<sup>4</sup> and can be analysed in the following manner:

The cost-effectiveness can be defined as  $\eta$  in Eqn (4).

$$\eta = \frac{\text{Cost of targets survived with decoys} - \text{Cost of targets survived without decoys}}{\text{Cost of all targets and decoys}}$$
(4)

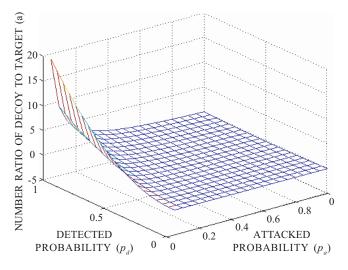


Figure 1. Relation between the ratio of decoys to targets a and the probability of detection  $p_a$ , the probability of attack  $p_a$ .

Suppose the target cost is  $c_t$  and the decoy cost is  $c_f$ , the cost ratio of decoy-to-target is  $b = \frac{c_f}{c_t}$ , then

$$\eta = \frac{m(p_d - p_a)c_t}{mc_t + nc_f}$$

$$= \frac{mnp_f p_d c_t}{(mp_d + np_f)(mc_t + nc_f)}$$

$$= \frac{ap_f p_d}{(p_d + ap_f)(1 + ab)}$$
(5)

Obviously, the cost-effectiveness  $\eta$  is related to the number ratio a of the decoys to targets. To get the maximum  $\eta$  and the value of a to make

 $\eta$  to maximum, make partial differentiation to Eqn (5)

$$\frac{\partial \eta}{\partial a} = \frac{p_f p_d \left( p_d - a^2 b p_f \right)}{\left( p_d + a p_f \right)^2 \left( 1 + a b \right)^2} \tag{6}$$

Make Eqn (6) equal to zero, that is  $\frac{\partial \eta}{\partial a} = 0$ , Then

$$a_{\text{max}} = \sqrt{\frac{p_d}{bp_f}} \tag{7}$$

$$\eta_{\text{max}} = \frac{p_d p_f}{\left(\sqrt{bp_d} + \sqrt{p_f}\right)^2} = \left(\frac{1}{\sqrt{\frac{b}{p_f}} + \sqrt{\frac{1}{p_d}}}\right)^2 \tag{8}$$

 $a_{max}$  is called the maximum ratio of decoys to targets, which makes the cost-effectiveness reach maximum  $\eta_{max}$ .

### 4. RESULTS AND DISCUSSION

From Eqn (5), the relation between the cost-effectiveness and the ratio of decoys-to-targets can be gained. Figure 2 shows it with the parameter  $p_f = 1$ ,  $p_d = 0.5$ , b = 0.005.

It can be seen from Fig. 2 that the curve ascends rapidly until  $\eta$  reaches maximum and then the curve falls slowly. It proves that the costeffectiveness increases with the number of decoys increasing at the very start and then decreasing after the number of decoys exceeds a special value. The result indicates that there is a value of a to make the cost-effectiveness highest.

Figure 3 shows the relation between b,  $p_d$  and  $a_{\max}$  when  $p_f = 1$ . Figure 4 shows the relation between b,  $p_d$ , and  $\eta_{\max}$  when  $p_f = 1$ . It can be seen from Figs 3 and 4 that in a condition when  $p_d$  and  $p_f$  are invariable, lower the cost ratio b is, higher the maximum cost-effectiveness  $\eta_{\max}$  is, and the more number of decoys are demanded. The result indicates that the maximum operational effectiveness can be further increased using large number of low-cost decoys.

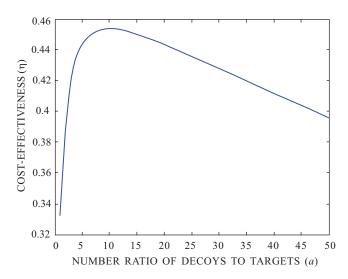


Figure 2. Relation between cost-effectiveness and ratio of decoys-to-targets.

Figure 5 shows the relation between b,  $p_f$  and  $a_{\max}$  when  $p_d = 0.5$ . Figure 6 shows the relation between b,  $p_f$  and  $\eta_{\max}$  when  $p_d = 0.5$ . It can be seen from Figs 5 and 6 that  $a_{\max}$  decreases with  $p_f$  increasing, while  $\eta_{\max}$  increases. The result indicates that better the performance of decoy is, less the ratio  $a_{\max}$  is needed to make the cost-effectiveness reach maximum, and at the same time, the maximum cost-effectiveness  $\eta_{\max}$  can be improved.

Finally, it is concluded that the following methods can be used to reduce the cost or number of decoys.

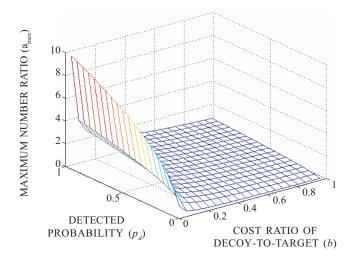


Figure 3. Relation between b,  $p_d$ , and  $a_{max}$  when  $p_f = 1.0$ .

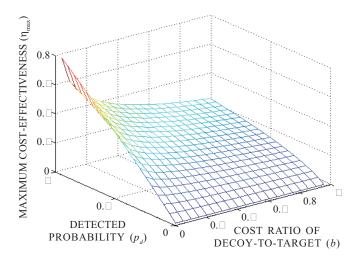


Figure 4. Relation between b,  $p_d$  and  $\eta_{max}$  when  $p_f = 1$ .

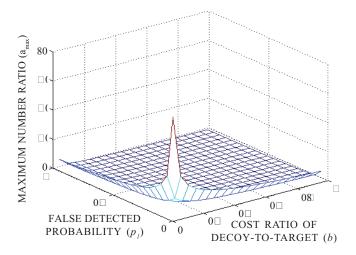


Figure 5. Relation between b,  $p_p$ , and  $a_{\text{max}}$  when  $p_d = 0.5$ .

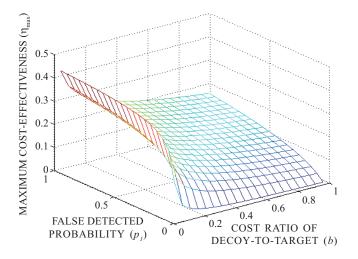


Figure 6. Relation between b,  $p_p$  and  $\eta_{max}$  when  $p_d = 0.5$ .

- 1 To reduce the high cost of standard decoys, some local material in battlefield can be used to make quantities of decoys to substitute expensive standard decoys. Large number of low-cost non-standard decoys can also have good effect if they are laid out properly.
- 2 The cost reduction is commonly very little if the limiting factor from battlefield condition induces decoys number less than  $a_{\text{max}}$  which can be seen from Fig. 2.
- 3 The ratio of decoys-to-targets can be reduced if the probability of detection of the target is reduced or the probability that the decoy is taken as target, is improved.

#### REFERENCES

- 1. Wall, Robert. Better air defenses shape gunship decoy. *Avia. Week & Space Technol.*, 2002, **156**(17), 31-32.
- 2. Decoy systems. J. Elect. Def., 2002, 1, 79-93.
- 3. Mcgahan, Robert V. A sampling of EW expendable rounds. *J. Elect. Def.*, 2002, **25**(5), 61-65.
- 4. Hershaft, A. Effectiveness of imperfect decoys. *Operations Research*, 1968, **16**(1), 10-17.