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Modelling of the Military Helicopter Operation Process in Terms of Readiness

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ABSTRACT

The processes of exploitation of military objects are usually characterised by the specificity of the operation and the complexity of both the process itself and the object. This specificity may relate both to the type of tasks that these objects carry out and to the environment in which these processes take place. Complexity is usually reflected in the very structure of an object (for example, a ship, an aircraft or a helicopter) and, consequently, in its operation/maintenance system. The above mentioned features, as well as the limited access to data, naturally limits the set of publications available on this subject. In this article, the authors have presented a method of assessing the readiness of military helicopters operated by the Armed Forces of the Republic of Poland. The readiness of technical objects used in military exploitation systems is a basic indicator of equipment preparation for executing tasks. In exploitation process research, the mathematical models are usually discrete in states and continuous in time stochastic processes, in the set of which Markov models are included. The paper presents an example of using Markov processes with discrete time and with continuous time to assess the readiness of a technical object performing tasks appearing in random moments of time. At the same time, the aim of the examined system to achieve a state of balance is presented.

Keywords: Modelling; Markov chains; Markov processes; Readiness

1. INTRODUCTION

The original applications of Markov and semi-Markov processes originate from statistical and quantum physics, chemistry, spectroscopy and metrology[1-4]. In the 20th century, the scope of their application was systematically expanded in the fields of signal theory, telecommunications, computer science, operational research, reliability and readiness of technical facilities and mass service systems[2,4,5]. In the present period they are experiencing a renaissance of applications in the fields of speech and images, artificial intelligence, signal processing and filtering[6,7] as so-called hidden Markov models (HMM)[8-12] or fuzzy Markov models (FMM)[13-17].

Both Hidden (HMM) and Fuzzy (FMM) Markov models relate to discrete-time processes with unobservable states. In the former case, HMM may be used to predict the protein binding sites using the sequential marking technique[8] or as a method based on a unified algorithm structure designed to decode hidden Markov models, including the first order hidden Markov model in combination with any high-order hidden Markov model[18], or be used in e-commerce to reflect changes (and preferences) in vendor behaviour[19]. When referring to FMM applications, it should be borne in mind that fuzzy set theory is useful for uncertainty management, mainly in image processing[14] and enhancement applications[15]. A comprehensive review of the theoretical knowledge and possibilities of statistical application of HMM and FMM has been carried out in the study[20].

A separate area of application of Markov processes is the modelling of technical objects exploitation processes[21-25], as well as their implementation in the analysis and evaluation of both availability[5,26-28] and readiness[29-33].

In studies on modelling exploitation processes, the reliability of the vehicle fleet defined as a function of the damage stream[21,29], assessment of the degree and level of usability[30], readiness[22,25,32] of an object or system of exploitation[27,28], or population renewal including replacement according to age[23,34] often become the subject of tests.

From the point of analysis of applications of Markov models, neither hidden nor fuzzy states are assumed to exist when describing exploitation processes. An open Markov process with a given state space is assumed and the correctness of such assumption is verified according to the methodology presented in[35]. Moreover, HMMs are not useful in object readiness modelling as they concern multidimensional stochastic processes in discrete time[20], while the technical object exploitation process is usually defined as a one-dimensional stochastic process[23,24,35,36].

A slightly narrower set is made up of studies on military processes and technical systems[1,37,38], usually with limited publication capacity. Only a few publications directly refer to

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military helicopters. There are, for example, papers concerning the population exchange model[34], increasing the utilisation of fleet availability[39], optimisation of sea air transport costs[40], improvement of utilisation efficiency[41], as well as helicopter diagnostics based on flight data analysis[42].

This article presents their application for the analysis and evaluation of military helicopters operated by the Armed Forces of the Republic of Poland. The scarcity of publications to date concerning the readiness of military technical facilities is mainly related to limited access to information on military technologies, which understandably is subjected to restrictions resulting both from the manufacturer's trade secrets and from fixed grace periods generating profits in the armaments industry. As an example, the methodology of operational readiness assessment of USAF helicopters, published in 2014, which entered into mass exploitation more than half a century earlier, can be used[3]. Finding publications on Russian military helicopters in public literature is an even more difficult task, as knowledge in this area is still a guarded secret.

Readiness indicators make economic sense on a national scale. The need for research is most visible in the case of modernisation of exploitation systems and objects' replacement, when it is time to establish the reserves of new objects and requirements for their manufacturers in the scope of service deliveries and repairs.

Proper assessment of reserves of objects and service and repair requirements for manufacturers are not feasible without reliable knowledge of the factors determining the readiness measures of operating systems and objects prior to the modernisation and replacement. Without such knowledge, these assessments would be unreliable, resulting in significant financial losses caused by unnecessary or insufficient reserves and long service and repair downtimes. The readiness of technical objects is one of the factors influencing the economic efficiency of countries and multinational corporations, since the size of the reserve is not completely linearly dependent on the operational readiness index. If the execution of tasks with a probability of 0.95 is ensured by 10 ready objects, then at an operational readiness index of 0.9 one reserve object will be sufficient (according to the binomial distribution of the number of ready objects), while at an index of 0.45 as many as 5 reserve objects would be needed and the cost of using the system will increase from 10% to 50% (without taking into account the safety factor). For this reason, the readiness of technical objects is carefully monitored, studied, analysed and optimised.

The essence of mathematical modelling is to describe the examined phenomenon (process, system) using mathematical language. The modelling uses variables representing certain (and at the same time significant) from the point of view of the purpose properties of the examined phenomenon[22,24,38,43]. Examination of the exploitation system requires the identification of all important factors that define it. Secondary factors that unnecessarily complicate the model without significant improvement in its quality should be ignored or omitted, and as a result similar ones should be grouped[35]. A thorough examination of the process enables the isolation of a set of mutually separable operating states. It also imposes requirements on the empirical data used to build the model[44,45]. For a general analysis of the exploitation system, the necessary number of its distinguishable states should be sufficient to allow for reproduction of the characteristics of the process being tested in terms of calculating basic readiness indicators[46,47].

Helicopters operated by the Armed Forces of the Republic of Poland are in most cases outdated both in terms of technical thought and manufacturing technology. In this article, a method for assessing the readiness of a technical object using Markov processes has been developed.

The 9-state Markov model presented in the publication, which helps to determine the readiness of a helicopter, has not been so far the subject of any open publications in the field of aircraft exploitation. Only a few studies[24,35] contain such problems. It would not be possible without reliable knowledge of the exploitative process under investigation and the authentic empirical data collected. The scientific achievement of the authors is the formulation of a complex 9-state model of the military helicopter's process defining the value of the readiness index in the specific environment of its exploitation.

2. MATERIAL AND METHODS

According to the methodology of construction and analysis of exploitation process event models, the following assumptions have been formulated to develop a mathematical description of the exploitation process of the examined helicopters[4,24,32,35]:

Assumption 1. A technical object may at any time be in only one of the possible exploitation states, the set of which creates a countable and finite process phase space.

Assumption 2. Returns ($S_i \rightarrow S_i$ passages) are prohibited during the exploitation of objects.

Assumption 3. No transient states are assumed, but inter-state transitions are time leaps.

Assumption 4. Deterministic, environmental and external factors influencing the exploitation process are known.

Assumption 5. Times of changes of object states are recorded with any accuracy.

Assumption 6. The process described is a process without memory.

The first stage of developing the mathematical model was to collect data from the actual system of exploitation. The helicopter activity records in the Armed Forces of the Republic of Poland are recorded in two ways, i.e. through the SAMANTA IT system and in a traditional paper form in the form of Current Service Cards. Then, using the MS Excel spreadsheet, raw databases were developed, which, when supplemented with variables (random, grouping and mathematical), were processed into source databases. These variables facilitate database control and processing of phase trajectory into matrix elements. The hypothesis concerning the applicability of Markov processes to the description of the exploited process should be initially verified on the basis of phase trajectory analysis.

Initial investigation of the exploitation process of complex technical objects such as helicopters resulted in the selection of 37 operating states, which were systematically aggregated functionally in terms of preparedness calculation. The 9-phase space has met both the assumptions 1-5 and the conditions for using the analytical software. The following types of exploitation states have been adopted: S_1 - diagnostics; S_2 - test execution; S_3 - supply; S_4 - ready with pilot; S_5 - ready without pilot; S_6 - hangaring; S_7 - work on the ground; S_8 - task execution; S_9 - unsuitability.

In order to develop a proper mathematical model reflecting the examined process, it is necessary to determine the correct set of transitions allowed for the object from the previous to the next state. It has been identified on the basis of technical documentation, operational knowledge of the exploitation process under consideration and a number of factual consultations with experts from air bases. Its mathematical description is the matrix of transitions allowed $S_i \rightarrow S_j$ from the previous state S_i (rows) to the next state S_j (columns). The process under investigation has possible and prohibited transitions in accordance with a directed graph shown in Fig. 1. As it is known[10,13,17] when considering the processes of exploitation of technical objects, returns to states ($S_i \rightleftarrows S_i$ transitions) are not included. Due to the fact that the process under investigation has an established organisation (exploitation strategy), not all theoretically possible inter-state transitions are allowed[22,25]. Test sample of helicopters has narrowed down the number of 72 theoretically possible passages to 30 allowed passages (Fig. 1).

A directed graph (Fig. 1) depicting the exploitation of helicopters contains nine states, forming the process phase space. Exploitation is understood as the movement of an object through the individual states. The graph shows the most (S_1, S_3, S_7) and least (S_8) communicated states.

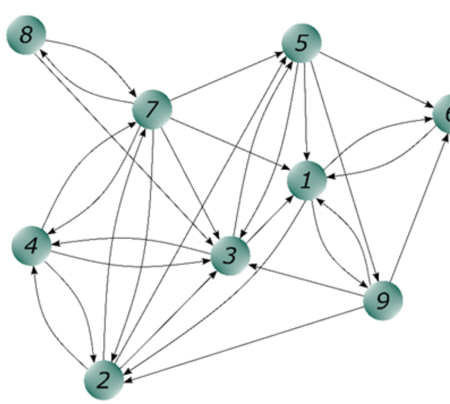

Figure 1. Graph of allowed transitions for a nine-state helicopter exploitation model.

3. THEORY OF PROCESS TESTING

The first stage in the construction of Markov process with discrete time is the estimation of the probabilities of transitions, as the $\hat{p}_{ij}$ estimator values of p_{ij} elements and the P probabilities matrix. The values of these estimators from the test sample are the frequencies w_{ij} of transitions from the state S_i to the state S_j, calculated according to Eqn (1):

$$\hat{p}_{ij} = w_{ij} = \frac{n_{ij}}{n_i} \tag{1}$$

where:

w_{ij} - frequency of transitions from the state S_i to the state S_j,

n_{ij} - number of transitions from the state S_i to the state S_j,

n_i - number of all transitions (exits) from the state S_i.

For the analysed process of exploitation, a 9x9 matrix of inter-state transitions P is created:

$$P[p_{ij}] = \begin{bmatrix} 0 & p_{12} & 0 & 0 & 0 & p_{16} & 0 & 0 & p_{19} \\ 0 & 0 & p_{23} & p_{24} & p_{25} & 0 & p_{27} & 0 & 0 \\ p_{31} & 0 & 0 & p_{34} & p_{35} & 0 & 0 & 0 & 0 \\ 0 & p_{42} & p_{43} & 0 & 0 & 0 & p_{47} & 0 & 0 \\ p_{51} & 0 & p_{53} & 0 & 0 & p_{56} & 0 & 0 & p_{59} \\ p_{61} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ p_{71} & p_{72} & p_{73} & p_{74} & p_{75} & 0 & 0 & p_{78} & 0 \\ 0 & 0 & p_{83} & 0 & 0 & 0 & p_{87} & 0 & 0 \\ p_{91} & p_{92} & p_{93} & 0 & 0 & p_{96} & 0 & 0 & 0 \end{bmatrix} \tag{2}$$

For Markov processes with discrete time, ergodic probabilities can be calculated from the transition matrix boundary P in n steps by solving a system of linear equations or an equivalent matrix equation, i.e. by passing from the continuous time t to the discrete time n, being the number of consecutive experiences of observing the vehicle at time Δt, according to relation (3):

$$\underset{j}{\wedge} p_j = \lim_{n\to\infty} p_j(n) = \sum_i p_i p_j = p_j \Leftrightarrow P^T\left[p_j\right] = \left[p_j\right], \text{ at } \sum_j p_j = 1 \tag{3}$$

where: P^T - transposed transition matrix P with $P = [p_j; i, j \in S]$ - probability boundary vector and p_{ij} - probability of transition from state i to state j.

In order to determine the boundary probabilities $p_j(n)$, the systems of equations (4) must be resolved with the condition of standardisation (5):

$$\begin{cases} 0.28481p_3 + 0.0614p_5 + p_6 + 0.00282p_7 + 0.2p_9 - p_1 = 0 \\ 0.46371p_1 + 0.04419p_4 + 0.00282p_7 + 0.08888p_9 - p_2 = 0 \\ 0.14843p_2 + 0.01105p_4 + 0.33333p_5 + 0.26836p_7 \\ \quad + 0.00546p_8 + 0.06667p_9 - p_3 = 0 \\ 0.64062p_2 + 0.27848p_3 + 0.15536p_7 - p_4 = 0 \\ 0.20312p_2 + 0.43671p_3 + 0.05367p_7 - p_5 = 0 \\ 0.41532p_1 + 0.47368p_5 + 0.64444p_9 - p_6 = 0 \\ 0.00781p_2 + 0.94475p_4 + 0.99453p_8 - p_7 = 0 \\ 0.51694p_7 - p_8 = 0 \\ 0.12096p_1 + 0.13157p_5 - p_9 = 0 \end{cases} \tag{4}$$

$$\sum_{j=1}^{9} p_j = 1 \tag{5}$$

The condition (5) is an additional equation which guarantees the exclusion of null solutions $p_j(n) = 0$. The solutions of the equation systems presented in Fig. 2 were obtained using the *Mathematica* software, ver.11. The software

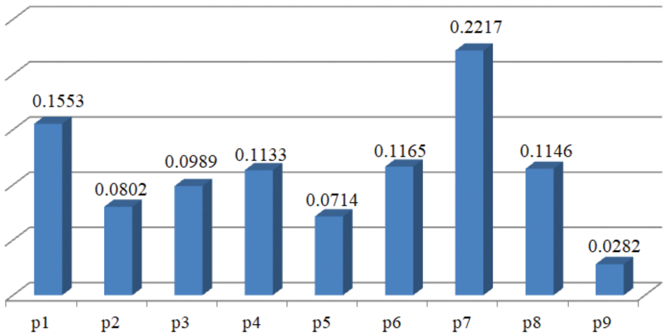

Figure 2. Boundary probabilities $p_j(n)$ of the Markov chain for the helicopter.

presents them in the form of intricate functions in a set of composite numbers which, due to their complexity, are not presented in this publication. The probability boundaries (Fig. 2) related to the discrete time do not exceed 0.23 for the process under investigation. On the one hand, this may be a sign of the imposed by instructions specific organisation of the exploitation process and the factors that stabilise is a desirable feature that indicates the existence of deterministic components that regulate such process. The highest probability of entry exists for the states of work on the ground $p_7(n)$ and diagnostics $p_1(n)$. Almost the same values (+/- 0.003) were observed for the states: hangaring , task execution $p_8(n) = 0.1146$ and ready with pilot $p_4(n) = 0.1133$. The lowest probability of entry was observed for the state of unsuitability $p_9(n) = 0.0282$.

Calculated for discrete time, the probability boundary $p_j(n)$ should be treated only in terms of quantitative measurement of the process and cannot be used as a basis for evaluating the readiness of a technical object. The results of continuous process analysis are a qualitative measure. The transition from discrete to continuous time is done using the Λ intensity matrix. The extra-diagonal intensities of transitions $\lambda_{ij} \geq 0$ for $i \neq j$ are defined as right-hand derivatives of the time transition probabilities according to the relation:

$$\lambda_{ij}\ (t_0) = d(p_{ij}) / dt \mid t = t_{0+} \tag{6}$$

The diagonal intensities $\lambda_{ii} \leq 0$ for $i = j$ are defined as the sum cofactor of the intensities of transitions from S_i for $i \neq j$ to 0:

$$\lambda_{ii} + \sum_j \lambda_{ij} = 0 \tag{7}$$

from here:

$$\lambda_{ii} = -\sum_j \lambda_{ij} \tag{8}$$

The $|\lambda_{ii}| = -\lambda_{ii}$ modules are called the intensities of the S_i state exits. For homogeneous Markov processes, the transition intensity is a constant and equals the inverse of the average ${}_{av}t_{ij}$ time the object stays in the state S_i before the state S_j:

$$\hat{\lambda}_{ij} = \frac{1}{{}_{av}t_{ij}} \tag{9}$$

while:

$${}_{av}t_{ij} = \frac{\sum_j t_{ij}}{n_i} \tag{10}$$

where: $t_{ij} = t_{k+1} - t_k$ only for ${}_lS_k = S_j$ - time spent by the object in the state S_i before the state S_j, which is equal to the value of the step-by-step variable for the object with the number l of the observation number k and ${}_{av}t_{ij} = \left(\sum_j t_{ij}\right) / n_i$ means the average time spent in the state S_i before the state S_j.

Matrix Λ (11) shows the intensities of transitions for the nine-state helicopter exploitation process. The intensities listed are expressed in the number of transitions per hour for one object. The name transition intensity was unfortunately adopted by theorists for the parameter of the Chapman-Kolmogorov equation system[35], which is not the intensity (frequency, or power of the phenomenon) in the technical sense. As a derivative of probability with respect to time, the theoretical intensity of the transition has no technical interpretation and, because of the unit of measurement, is sometimes misinterpreted as the average frequency of transitions. The elements of the intensity matrix as in (11) have no direct exploitation interpretations. They only give the information that the transition intensities of the average object had a very high dynamics in the order of 1:1000 - from the minimum $\lambda_{61} = 0.000346 / day$ for the transition from the hangaring state S_6 to the diagnostics state S_1 to the maximum $\lambda_{34} = 0.66565 / day$ for the transition from the supply state S_3 to the ready with pilot state S_4. The physical interpretation has an inverse intensity of 2941.2 minutes, meaning that the helicopter has, on average, spent so much time, i.e. slightly more than 2 days (2.04 days) in the S_6 hangaring state prior to transitioning to the S_1 diagnostics state.

$$\Lambda = \begin{bmatrix} -0.0844 & 0.0149 & 0 & 0 & 0 & 0.0043 & 0 & 0 & 0.0653 \\ 0 & -0.3913 & 0.0357 & 0.0523 & 0.0532 & 0 & 0.25 & 0 & 0 \\ 0.3741 & 0 & -0.5034 & 0.6656 & 0.4635 & 0 & 0 & 0 & 0 \\ 0 & 0.0287 & 0.0085 & -0.0355 & 0 & 0 & 0.0141 & 0 & 0 \\ 0.0057 & 0 & 0.0094 & 0 & -0.0851 & 0.0036 & 0 & 0 & 0.0664 \\ 0.0003 & 0 & 0 & 0 & 0 & -0.0003 & 0 & 0 & 0 \\ 0.1428 & 0.3333 & 0.1715 & 0.1957 & 0.1627 & 0 & -1.1527 & 0.1466 & 0 \\ 0 & 0 & 0.0076 & 0 & 0 & 0 & 0.0169 & -0.0245 & 0 \\ 0.006 & 0.0046 & 0.0084 & 0 & 0 & 0.0037 & 0 & 0 & -0.0227 \end{bmatrix} \tag{11}$$

After substituting the matrix Λ in the equation $[p_j]^T \cdot \Lambda = 0$, the following equation in the matrix form (12) was obtained for the exploitation process under investigation:

$$\begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \\ p_7 \\ p_8 \\ p_9 \end{bmatrix}^T \cdot \begin{bmatrix} -\lambda_{11} & \lambda_{12} & 0 & 0 & 0 & \lambda_{16} & 0 & 0 & \lambda_{19} \\ 0 & -\lambda_{22} & \lambda_{23} & \lambda_{24} & \lambda_{25} & 0 & \lambda_{27} & 0 & 0 \\ \lambda_{31} & 0 & -\lambda_{33} & \lambda_{34} & \lambda_{35} & 0 & 0 & 0 & 0 \\ 0 & \lambda_{42} & \lambda_{43} & -\lambda_{44} & 0 & 0 & \lambda_{47} & 0 & 0 \\ \lambda_{51} & 0 & \lambda_{53} & 0 & -\lambda_{55} & \lambda_{56} & 0 & 0 & \lambda_{59} \\ \lambda_{61} & 0 & 0 & 0 & 0 & -\lambda_{66} & 0 & 0 & 0 \\ \lambda_{71} & \lambda_{72} & \lambda_{73} & \lambda_{74} & \lambda_{75} & 0 & -\lambda_{77} & \lambda_{78} & 0 \\ 0 & 0 & \lambda_{83} & 0 & 0 & 0 & \lambda_{87} & -\lambda_{88} & 0 \\ \lambda_{91} & \lambda_{92} & \lambda_{93} & 0 & 0 & \lambda_{96} & 0 & 0 & -\lambda_{99} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{12}$$

It is a homogeneous system with an infinite number of solutions, among which there may be solutions that meet the condition of standardisation (13):

$$\sum_{j=1}^{9} p_j = 1 \tag{13}$$

The solution of the above system (12) with a restriction (13) was obtained using the *Mathematica* software, the results are shown in Fig. 3. It is shown that a helicopter spends most of its time on average in the states of hangaring $p_6(t) = 0.8854$ and much less in the states of unsuitability $p_9(t) = 0.0602$ and ready with pilot $p_4(t) = 0.00231$. In relation to the other distinct states of exploitation, the object stays on average very shortly, i.e. in the range of 0.0133 for the state of diagnostics (S_1) to slightly over 0.007 for the state of supply (S_3). In both cases, these are small values that do not affect the assessment of helicopter readiness.

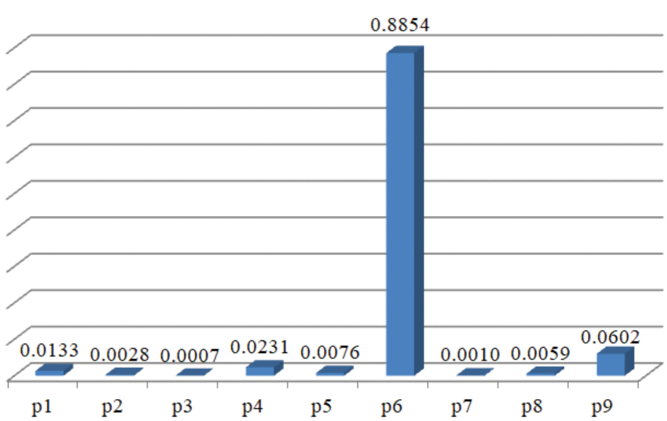

Figure 3. Boundary probabilities $p_j(t)$ of the Markov process for the helicopter.

4. RESULTS OF THE EVOLUTION OF HELICOPTER EXPLOITATION PROCESS

The systems of Chapman - Kolmogorov - Smoluchowski (CH-K-S) equations are examined and solved in order to determine the characteristic times of an object's progress to a stationary state after a given set of initial states, e.g. times of determining ergodic probabilities with a specific error. In real time, after applying the initial vector, they present, in the form of graphs, the dynamics of changes in the process from the disordered state to the equilibrium state (called stationary), for which probability values in abstract infinity asymptotically reach boundary values (ergodic). The process determination time is assumed to be approximately 2.5 times the duration of the longest state.

The systems of CH-K-S equations[2,4] have the following matrix form:

$$\frac{d}{dt} P(t) = \Lambda \cdot P(t) \wedge \left(\sum_j p_j = 1 \right) \tag{14}$$

where: $P(t)$ - a column vector of the probabilities of the process being in particular states; Λ – transition intensity matrix for the 9-state model of the process; $\sum_j p_j = 1$ condition of normalisation of the system.

For the Markov process under study, in its developed form, they take the following matrix form:

$$\begin{bmatrix} p_1(t) \\ p_2(t) \\ p_3(t) \\ p_4(t) \\ p_5(t) \\ p_6(t) \\ p_7(t) \\ p_8(t) \\ p_9(t) \end{bmatrix} \cdot \begin{bmatrix} -\lambda_{11} & \lambda_{12} & 0 & 0 & 0 & \lambda_{16} & 0 & 0 & \lambda_{19} \\ 0 & -\lambda_{22} & \lambda_{23} & \lambda_{24} & \lambda_{25} & 0 & \lambda_{27} & 0 & 0 \\ \lambda_{31} & 0 & -\lambda_{33} & \lambda_{34} & \lambda_{35} & 0 & 0 & 0 & 0 \\ 0 & \lambda_{42} & \lambda_{43} & -\lambda_{44} & 0 & 0 & \lambda_{47} & 0 & 0 \\ \lambda_{51} & 0 & \lambda_{53} & 0 & -\lambda_{55} & \lambda_{56} & 0 & 0 & \lambda_{59} \\ \lambda_{61} & 0 & 0 & 0 & 0 & -\lambda_{66} & 0 & 0 & 0 \\ \lambda_{71} & \lambda_{72} & \lambda_{73} & \lambda_{74} & \lambda_{75} & 0 & -\lambda_{77} & \lambda_{78} & 0 \\ 0 & 0 & \lambda_{83} & 0 & 0 & 0 & \lambda_{87} & -\lambda_{88} & 0 \\ \lambda_{91} & \lambda_{92} & \lambda_{93} & 0 & 0 & \lambda_{96} & 0 & 0 & -\lambda_{99} \end{bmatrix} = \begin{bmatrix} p_1'(t) \\ p_2'(t) \\ p_3'(t) \\ p_4'(t) \\ p_5'(t) \\ p_6'(t) \\ p_7'(t) \\ p_8'(t) \\ p_9'(t) \end{bmatrix} \tag{15}$$

or they may be presented in the form of an equivalent system of differential equations:

$$\left\{ \begin{array}{l} p_1'(t) = -\lambda_{12} \cdot p_2(t) - \lambda_{16} \cdot p_6(t) - \lambda_{19} \cdot p_9(t) + \lambda_{31} \cdot p_3(t) \\ \quad + \lambda_{51} \cdot p_5(t) + \lambda_{61} \cdot p_6(t) + \lambda_{71} \cdot p_7(t) + \lambda_{91} \cdot p_9(t) \\ p_2'(t) = -\lambda_{23} \cdot p_3(t) - \lambda_{24} \cdot p_4(t) - \lambda_{25} \cdot p_5(t) - \lambda_{27} \cdot p_7(t) \\ \quad + \lambda_{12} \cdot p_1(t) + \lambda_{42} \cdot p_4(t) + \lambda_{72} \cdot p_7(t) + \lambda_{92} \cdot p_9(t) \\ p_3'(t) = -\lambda_{31} \cdot p_1(t) - \lambda_{34} \cdot p_4(t) - \lambda_{35} \cdot p_5(t) + \lambda_{23} \cdot p_2(t) \\ \quad + \lambda_{43} \cdot p_4(t) + \lambda_{53} \cdot p_5(t) + \lambda_{73} \cdot p_7(t) + \lambda_{83} \cdot p_8(t) + \lambda_{93} \cdot p_9(t) \\ p_4'(t) = -\lambda_{42} \cdot p_2(t) - \lambda_{43} \cdot p_3(t) - \lambda_{47} \cdot p_7(t) \\ \quad + \lambda_{24} \cdot p_2(t) + \lambda_{34} \cdot p_3(t) + \lambda_{74} \cdot p_7(t) \\ p_5'(t) = -\lambda_{51} \cdot p_1(t) - \lambda_{53} \cdot p_3(t) - \lambda_{56} \cdot p_6(t) - \lambda_{59} \cdot p_9(t) \\ \quad + \lambda_{25} \cdot p_2(t) + \lambda_{35} \cdot p_3(t) + \lambda_{75} \cdot p_7(t) \\ p_6'(t) = -\lambda_{61} \cdot p_1(t) + \lambda_{16} \cdot p_1(t) + \lambda_{56} \cdot p_5(t) + \lambda_{96} \cdot p_9(t) \\ p_7'(t) = -\lambda_{71} \cdot p_1(t) - \lambda_{72} \cdot p_2(t) - \lambda_{73} \cdot p_3(t) - \lambda_{74} \cdot p_4(t) \\ \quad - \lambda_{75} \cdot p_5(t) - \lambda_{78} \cdot p_8(t) + \lambda_{27} \cdot p_2(t) + \lambda_{47} \cdot p_4(t) + \lambda_{87} \cdot p_8(t) \\ p_8'(t) = -\lambda_{83} \cdot p_3(t) - \lambda_{87} \cdot p_7(t) + \lambda_{78} \cdot p_7(t) \\ p_9'(t) = -\lambda_{91} \cdot p_1(t) - \lambda_{92} \cdot p_2(t) - \lambda_{93} \cdot p_3(t) \\ \quad - \lambda_{96} \cdot p_6(t) + \lambda_{19} \cdot p_1(t) + \lambda_{59} \cdot p_5(t) \end{array} \right. \tag{16}$$

The correct analytical solution of the Ch-K-S equation system with restriction and normalisation condition was determined using the Mathematica Markov Continuous module. It is assumed that at initial $t = 0$ time the $X(t)$ process is in the state S_1. The obtained probabilities of observing the states $S_1 - S_9$ are in practice complex functions (they are not solutions according to the classical method). When analysing the dynamics of the helicopter's exploitation process, it is important to examine the characteristic times after which the object will reach the state of equilibrium. Such examination is possible with the *Mathematica* ver.11 software. For the analysed process of exploitation, the initial distribution vector in the following form $p_j = [1,0,0,0,0,0,0,0,0]$ was assumed. The evolution of changes of process in the period of time [0,1440] minutes is presented in Fig. 4 - 12.

As shown by the graphs presented in Figs. 4 - 12, the process under investigation is characterised by a significant dynamic of changes in the initial phase for the distribution vector $p_j = [1,0,0,0,0,0,0,0,0]$.

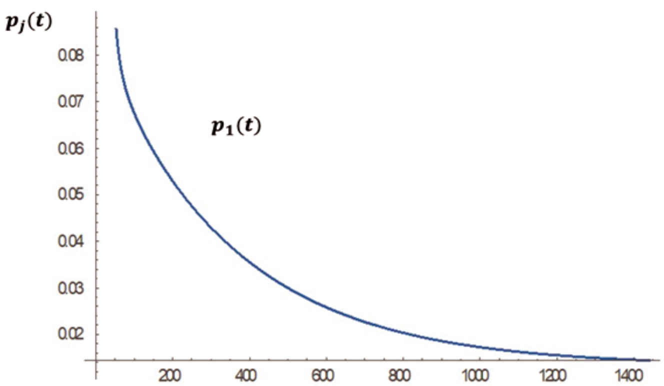

Figure 4. Evolution of changes in probability of helicopter being in state S_1 in the time interval [0, 1440] minutes.

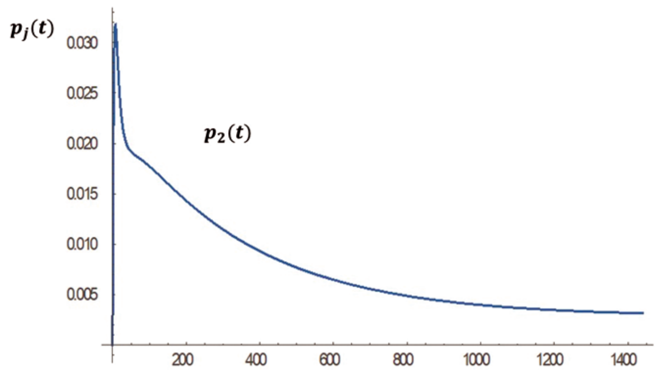

Figure 5. Evolution of changes in probability of helicopter being in state S_2 in the time interval [0, 1440] minutes.

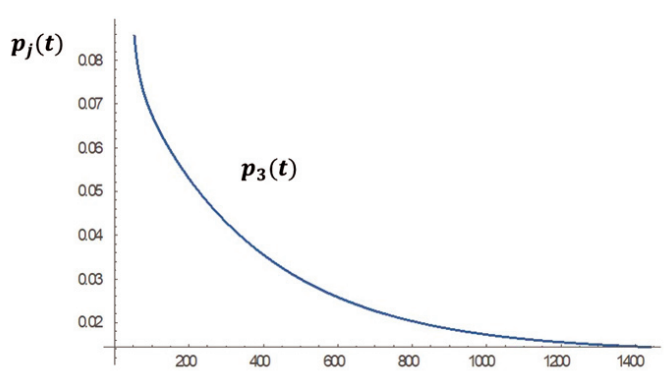

Figure 6. Evolution of changes in probability of helicopter being in state S_3 in the time interval [0, 1440] minutes.

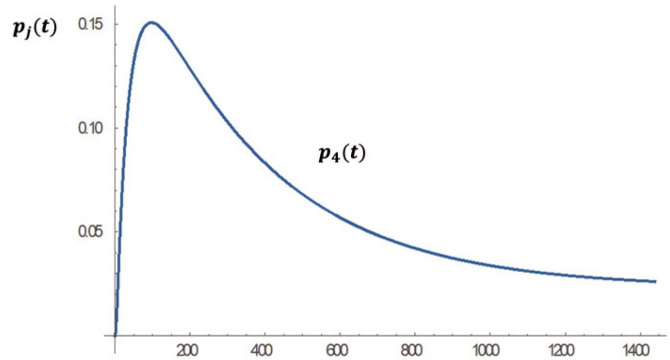

Figure 7. Evolution of changes in probability of helicopter being in state S_4 in the time interval [0, 1440] minutes.

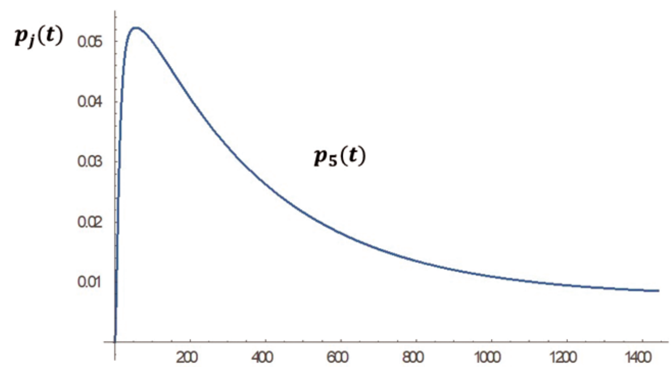

Figure 8. Evolution of changes in probability of helicopter being in state S_5 in the time interval [0, 1440] minutes.

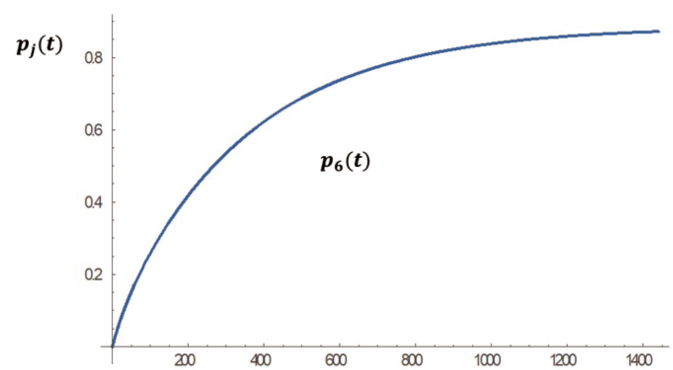

Figure 9. Evolution of changes in probability of helicopter being in state S_6 in the time interval [0, 1440] minutes.

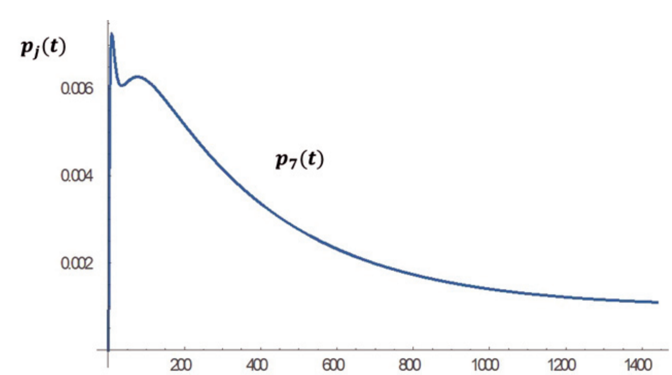

Figure 10. Evolution of changes in probability of helicopter being1 in state S_7 in the time interval [0, 1440] minutes.

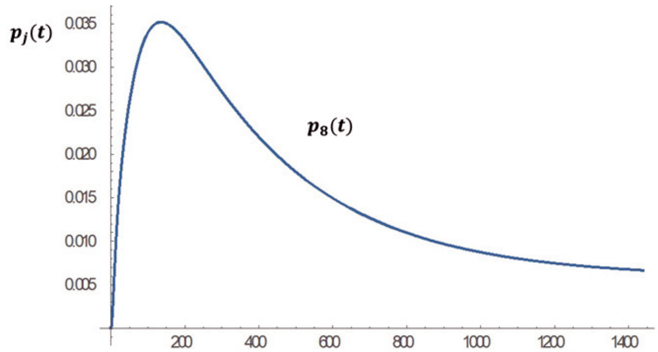

Figure 11. Evolution of changes in probability of helicopter being in state S_8 in the time interval [0, 1440] minutes.

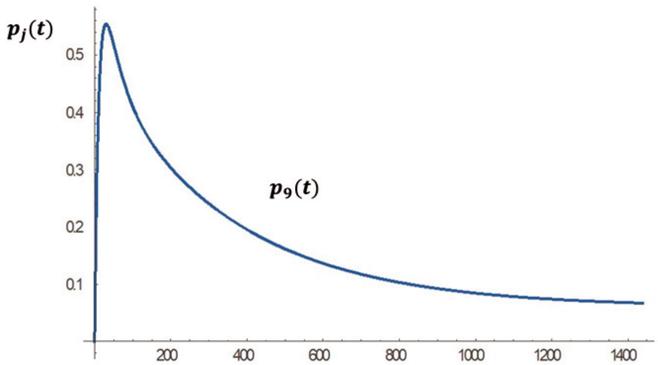

Figure 12. Evolution of changes in probability of helicopter being in state S_9 in the time interval [0, 1440] minutes.

In Figures 4-12 two characteristic intervals can be observed:

(i) The interval (0,1000 minutes) for which the evolution of the probabilities for each operating state is visible after the application of any initial state vector. The common characteristic in this interval is a sharp decrease (or a sharp increase in the case of p_6) of probabilities in the initial stage of the process applied by the extortion vector $p_j = [1,0,0,0,0,0,0,0,0]$. The abrupt change of probability value is connected to the lack of influence of factors stabilising the process after applying any initial distribution vector. Such trend is characteristic for most exploitation processes started at random times from any exploitation state.

(ii) The interval (1000,1440 minutes) for which the rate of change of the values of individual probabilities systematically decreases (or increases as in relation to p_6) compared to the interval (0,1000 minutes).

It can be concluded that the existence of such two characteristic intervals for the exploitation process without an absorbent (terminal, end) state reflects the common occurrence in a random process triggered by any initial distribution vector, for which the initial disruption stage (disturbance state), the relative stabilisation stage and the total stabilisation stage can be observed. In practice, the achievement of the state of equilibrium varies over time for each probability value. However, after 4320 minutes from the moment of extortion, all probabilities reach the boundary values (total stabilisation stage). Achieving the third stage proves the correctness and orderliness of the process designed in accordance with specified rules (regulations) and the existence of natural factors that stabilise such a process.

5. RESULTS AND DISCUSSION

The boundary probabilities of the 9-state helicopter model in the discrete time domain $p_j(n)$ and continuous time $p_j(t)$ vary considerably (Fig. 2 and Fig.3). However, they cannot be compared because they have a different substantive interpretation. In regard to discrete time, it should be seen only in quantitative terms, i.e. as the limit in the infinite number of entries to the *i*-th operating state against the background of transitions to all possible states. The $p_j(t)$ values allow for qualitative conclusions in relation to the readiness measures. The difference in the interpretation of the obtained research results is due to the fact that each process considered in discrete time has infinite number of realisations in continuous time. Summarising the results of the helicopter's boundary probabilities related to the Markov process in a discrete time $p_j(n)$, it should be concluded that the highest probabilities of entry were observed for the states respectively: work on the ground $p_7 = 0.2217$, diagnostics $p_1 = 0.1553$, hangaring $p_6 = 0.1165$, task execution $p_8 = 0.1146$ and ready with pilot $p_4 = 0.1133$, and the lowest for states: unsuitability $p_9 = 0.0282$, ready without pilot $p_5 = 0.0714$, test execution $p_2 = 0.0802$ and supply $p_3 = 0.0989$, respectively. For the continuous time period, the highest probability of staying was observed for two out of nine states, i.e. hangaring and unsuitability $p_6 = 0.8854$ and $p_9 = 0.0602$, respectively. Bearing the above in mind, it can be concluded that the tested helicopter was, on average, hangared for over 88% of its exploitation time during the two-year research period. In addition, it was ready with pilot more than 2.3% of the time and more than 1.3% of the time in diagnostics. This could be a manifestation of the general reduction of flight time caused by the budgetary restrictions of the Ministry of Defence, while maintaining a certain state of airworthiness and readiness of the military equipment.

The probabilities of being in the remaining states for the continuous time are relatively short-lived and constitute in total about 5% of the exploitation time during the two-year research period and therefore do not have a fundamental influence on the readiness of the examined object. The calculated functional readiness index of the helicopter at the air base, understood as: $\sum_{j=4}^{8} p_j(t)$, is 0.9223. It is therefore high, which indicates a correctly planned and executed exploitation process from the point of view of technical availability of the equipment used at the air base.

The results of tests of the 9-state model of the exploitation process of helicopters equipped with the Armed Forces of the Republic of Poland discussed above cover the period of so-called planned exploitation in which any malfunctions were reflected by the S_9 unsuitability state, taking up to 960 minutes (two working shifts). In this case, any faults were rectified at the location of the helicopter's permanent location, i.e. at the air base. However, detailed analysis of phase trajectories revealed two other types of characteristic trajectory sections:

(i) Periods of exploitation breaks during which the object was not recorded in the exploitation records of the air base but was directed to repair facilities for scheduled repair,

(ii) Periods of random renewals extended in the base with a realisation time of over 960 minutes.

The subject of subsequent publications will be to take into account the above discussed preliminary conclusions from the analysis of phase trajectories and to develop a model covering these random-deterministic conditions of the complex process of helicopter exploitation.

6. CONCLUSIONS

To sum up, this publication proposes a method for calculating the readiness of helicopters used by the Armed Forces of the Republic of Poland. The author's own 9-state descriptive model of exploitation process was developed for use with complex military objects using the Markov theory.

As already mentioned in the introduction, Markov processes are now widely used in literature in relation to physics, chemistry, telecommunications, operational research, spectroscopy, metrology, logistics, and many others. On the other hand, there is a much narrower collection of publications in the field of Markov theory applications relating to the construction and operation of technical facilities and even narrower in the field of military helicopters, which are the subject of this work. Among the advantages of the proposed method we can count the universality through the possibility of its application to the whole family of Russian and Polish-Russian-made helicopters. This method is dedicated to the majority of helicopter types manufactured by the USSR and now by Russia with an uniform operating strategy. It enables reliable analysis and evaluation of basic reliability and readiness indicators, such as: technical, functional, task-oriented, operational, potential or initial readiness. A characteristic feature of Markov processes which is the basis for their application is the fact that it forces designers, analysts or users to get to know the process in depth and to analyse it in detail. The authors contribution to this work is based on the reliability of the empirical tests carried out on military helicopters, the priority of its publication and its practical suitability for aviation. The practical suitability criterion is met because the proposed model reliably reflects the process under consideration, which is confirmed by the calculated value of the readiness index of 0.9223 for the planned exploitation process. The proposed 9-state model also has the following drawbacks:

- The impact of atmospheric conditions (weather conditions) on helicopter exploitations is not taken into account; the weather conditions are not subject to strict exploitation records despite the fact that helicopters are, after all, weather-sensitive objects;
- Planned repairs carried out in repair facilities, which temporarily exclude the given object from the exploitation records, are not taken into account (usually it is the fourth quarter of a given calendar year in a two-year cycle);
- Also excluded from the model are so-called extended renewals carried out at the air base with an operating time of over 24 hours.

The above mentioned limitations have influenced the high value of the calculated readiness rate, therefore the aim of the next publications will be to propose a complex macro-model for the description of exploitation consisting of three main sub processes, taking into account both the repairs carried out in the repair facilities as well as the mentioned extended renewal.

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