

## Fuzzy Decision Aid for Technology Evaluation

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### ABSTRACT

Fuzzy set theory offers a possibility of handling the data and information involving the subjective characteristics of human nature in decision-making process, which is mostly true in the real-world. This paper presents a fuzzy decision theory method to evaluate the potential of project proposals submitted for final approval of corporate decision makers in any research and development organisation.

**Keywords:** DATE, fuzzy set, linguistic variables, national resource index, system realisability index

### 1. INTRODUCTION

The success of any project of a research and development (R&D) organisation depends upon several factors. The first and foremost among these is the infrastructure available within the organisation and the country for effectively carrying out the necessary R&D activities. A particular R&D project may have the requirements of several technologies, and if the infrastructure is not adequate to provide the necessary facilities, then the success of the concerned project becomes a far dream to realise. The second factor which is critical for the success of the project is the expertise available in the relevant technologies. One has to see, whether the human resources of the organisation or the country have mastered the relevant technologies, those are to be used in the concerned project. Though the infrastructure and expertise of the relevant technologies provide a firm base for the success of the concerned project, it is the confidence level of the project team that matters much, because they are the real-people who will exploit the available

infrastructure and expertise to transform the anticipated success to a real-one.

There are several other important factors which contribute towards the success of a project. The skills of the project team in dealing with the system, they are working for is one of those factors. A project team needs to skillfully decompose the system into various subsystems and sub-subsystems and so on hierarchically and has to identify the relative importance of the subsystems in realising the concerned system. In addition, they have to recognise the significance of a particular technology in the design and development of a particular subsystem. Last but not the least, the skills of the project team in system integration is very crucial in realising the goal as intended.

Many of the factors contributing towards the success of an R&D project, as discussed, are imprecise in nature and the imprecision may come from a variety of sources such as unquantifiable information, incomplete information, and non-obtainable

information. Therefore, while evaluating the potential of a project proposal, a decision maker (DM) has to deal with certain kind of uncertainties which are imprecise and vague in nature. It has been widely approved that, fuzzy set theory (FST) is a powerful tool to handle imprecise data and vague information. Therefore, FST can be effectively applied in by a DM for effective project evaluation and selection.

Different R&D organisations adopt different evaluation methodologies for according sanctions to various project proposals. But the inherent nature of the problem is very much the same in all the cases. Here for illustration, the technology evaluation process of Defence R&D Organisation (DRDO), Ministry of Defence, Govt of India has been taken as a case study. DRDO uses a decision-support tool known as decision aid for technology evaluation (DATE) for systematic analysis of a project for its technological content and to determine the technology gaps.

## 2. TECHNOLOGY EVALUATION PROCESS OF DRDO

The methodology incorporated into DATE-specifically addresses the system development projects of DRDO. It facilitates systematic analysis of a project for its technology content and evaluation of feasibility in the context of technological expertise and facilities available in the country.

The projects that DRDO undertakes are often multidisciplinary and require working at cutting-edge technologies. It is essential to ensure that a proposed project is capable of tapping all the critical technologies going into it. There are three types of projects undertaken in DRDO: (i) system development projects (SDPs), (ii) technology demonstration projects (TDPs), and (iii) science and technology projects (STPs). The SDPs are meant for developing major systems meeting the specific requirements of the Defence Services. These are user-driven, strongly time-bound and are executed in a mission mode. The TDPs are taken up to master the critical technologies required for system development. These are thus precursors to system development projects and are time-bound. The STPs help DRDO look

into futuristic technologies in anticipation of exploitation at a later date and are the vehicles for converting scientific knowledge to usable technology. These projects are typically less time-critical and involve relatively low-investment.

A large numbers of technologies dealt within DRDO have been bunched into a few technology groups. Core technologies have been identified in each technology group. These core technologies have been decomposed into subtechnologies. Wherever necessary, these subtechnologies have been further decomposed into sub-subtechnologies. The technologies have been grouped into the following major heads: (i) aeronautics, (ii) armaments, (iii) electronics, (iv) electrooptics, (v) engineering, (vi) materials, (vii) missiles, (viii) naval, and (ix) vehicles. It is mentioned that major system development projects, though primarily classified under a particular technology group, may require technologies called as allied technologies from other groups as well.

Different technologies coming under each technology group have been consolidated into a few core technologies, and it has been ensured that there is little or no overlap between the core technologies identified. For example, the core technologies identified under Electronics Technology group are: (i) microelectronics devices and components, (ii) microwave components and devices, (iii) mm-wave components and devices, (iv) microwave and mm-wave technology, (v) RF engineering, (vi) signal processing, (vii) communication/networking, (viii) computer architecture/hardware, (ix) software, (x) power processing, (xi) electronic materials, and (xii) information technology. The core technologies and corresponding subtechnologies of all technology groups are available<sup>1</sup>.

In the technology evaluation methodology, the DM keeps track of the national technology base wrt expertise and infrastructure, while the project team has to assess its own confidence level to exploit the available national technology resources for a given project. Thereafter, the project team decomposes the system to be developed into subsystems and assesses the significance of the different technologies for each subsystem. The realisability of the proposed system is then determined by considering the viability

of each of the subsystems and its criticality, as well as system integration capability of the team. High level of system realisability indicates high possibility of success of any project and thus ensures acceptance of the project proposal, while low-level concludes existence of the technology gaps.

## 2.1 Factors and Indices considered in DATE

In the technology evaluation process of DRDO, a series of factors and indices have been defined. Factors are parameters with user-assigned values, typically an integer between 0 and 10. Indices are derived values and are, in general, not integers; these are rounded off to one or two decimal places for convenience.

### 2.1.1 Expertise Factor

System development projects of DRDO primarily rely on the core technology-base-available in its laboratories. Often, this is supplemented by the expertise available in other R&D establishments, industry, and academic institutions in the country. Expertise factor (EF) is the parameter used for indicating the level of expertise available in DRDO, and elsewhere within the country, in a specific technology.

### 2.1.2 Infrastructure Factor

Technologies require proper infrastructure for their effective utilisation. Infrastructure factor (IF) is used to indicate the level of infrastructure available in DRDO, and elsewhere in the country, in a specific technology.

### 2.1.3 National Resource Index

National resource index (NRI) is a composite parameter indicating the level of expertise and infrastructure position in the country in a given technology. It is derived from EF and IF as

$$NRI = \frac{\alpha EF + (1 - \alpha)IF}{10}$$

where  $\alpha$  is a factor assigned a value between 0 and 1, ( $0 \leq \alpha \leq 1$ ). The value of  $\alpha$  indicates the relative importance of expertise compared to infrastructure for the technology concerned.

### 2.1.4 Confidence Factor

Confidence factor (CF) reflects the ability, as evaluated by the project team, to effectively exploit the national technological resources (expertise and infrastructure) to the extent required by the project within the prescribed time frame of the project.

### 2.1.5 Capability Index

Capability index (CI) indicates the ability of a project team to synergise the national resources in a specific technology area, and the confidence it has in exploiting the same. It is derived from NRI and CF as

$$CI = \frac{NRI \times CF}{10}$$

### 2.1.6 Relative Significance Factor

For facilitating analysis of a system development project, the system is first decomposed into subsystems. Realisation of the different subsystems requires varying amounts of technology input. Relative significance factor (RSF) indicates the relative importance of a particular technology in the design and development of a specific subsystem under consideration.

### 2.1.7 Subsystem Viability Index

Subsystem viability index (VI) is a derived parameter based on the CI for each technology area and the RSF for that technology area for the given subsystem. VI is defined as

$$VI_{\text{subsystem}} = \frac{\sum_t CI \times RSF}{\sum_t RSF}$$

where  $\sum_t$  indicates the summation over all applicable technologies.

### 2.1.8 Relative Importance Factor

While all the subsystems of a system are necessary, certain subsystems may be more important. There may be other subsystems that are not so critical but are required only for enhancing system performance or improving versatility. This aspect

has been catered for through a parameter called relative importance factor (RIF). The RIF of each subsystem is assessed depending on its importance for realising the system.

2.1.9 System Integration Factor

A crucial phase in major system development projects is system integration. System integration factor (SIF) indicates the ability of the project team to effectively integrate all subsystems for realising the system as intended.

2.1.10 System Realisability Index

System realisability index (SRI) is a measure of the feasibility of successful completion of a system development project within the given timeframe and resources. It is derived from VI, RIF, and SIF as

$$SRI = \frac{\sum_s VI \times RIF}{\sum_s RIF} \times SIF$$

where,  $\sum_s$  indicates the summation over all the subsystems.

2.2 Interpretation of System Realisability Index in DATE

In DATE, SRI can take any value between 0 and 10 ( $0 \leq SRI \leq 10$ ). According to the value of SRI, the DMs in DRDO classify the project proposal as follows:

- If  $SRI \geq 8$ , the proposed project normally does not require any R&D. Development/supply of the subsystems, or even the whole system, may as well be left to the industry, with DRDO providing consultancy as needed.
- If  $4 \leq SRI < 8$ , then it would imply reasonable chance of success for the project from technology point of view.
- If  $SRI < 4$ , the DMs suggest the project team for further analysis to determine the possible technology gaps coming in the way of project success.

3. WEAKNESS OF DATE

In DATE, it is suggested that the user should assign the numerical values to the factors while keeping in mind the correspondence between the linguistic variables and the positive integers as indicated in Table 1. It indicates interpretation for six values, viz., 10, 8, 6, 4, 2, 0. It is, therefore, suggested that to have a finer distinction in specific cases, intermediate values (9, 7, 5, 3, and 1) too may be used.

Table 1. Typical interpretation of values assigned to factors

| Value | Typical interpretation |                     |
|-------|------------------------|---------------------|
| 10    | Excellent              | Extremely important |
| 9     |                        |                     |
| 8     | Very good              | Highly important    |
| 7     |                        |                     |
| 6     | Good                   | Important           |
| 5     |                        |                     |
| 4     | Fair                   | Necessary           |
| 3     |                        |                     |
| 2     | Poor                   | Unimportant         |
| 1     |                        |                     |
| 0     | Non, non-existent      | Non-consequential   |

The different factors considered for technology evaluations are in actuality subjective and imprecise in nature. It is well-established that the human nature is better equipped to deal with subjective data in real-world decision-making processes. The human brain readily understands linguistic expressions rather than precise numbers in practical situations, where the information is vague and imprecise in nature. Hence, rightly it is suggested to consider the linguistic variables as tabulated in Table 1 while assigning the values to the different factors. But interpreting<sup>1</sup> the crisp numbers in terms of linguistic variables is quite unrealistic, because linguistic expressions are fuzzy rather than crisp. Therefore, it is not wise to use crisp arithmetic in such cases. In other words, the FST is a marvelous tool handling the data and information that are vague, imprecise, and uncertain by nature involving the subjective characteristics of human nature in decision-making process. The FST is rich enough to interpret

linguistic variables in terms of fuzzy numbers<sup>2,3</sup> (Appendix 1). Therefore, the authors have a strong feeling that the use of fuzzy arithmetic can make DATE a better tool than before. Here, one is not suggesting any change in the basic philosophy of DATE, but suggesting an improvement in DATE using FST. Hence onwards, the proposed methodology will be referred to as fuzzy decision aid for technology evaluation (FDATE).

**4. FUZZY DECISION AID FOR TECHNOLOGY EVALUATION**

The remedies for eliminating the weakness of DATE have been presented here. The user inputs as various factors considered in DATE are fuzzy in nature, hence may be expressed in linguistic terms, and then the linguistic terms should be transformed into fuzzy numbers using appropriate conversion scale. In the proposed FDATE, the linguistic terms used to assign the factors and their corresponding triangular fuzzy numbers are given in Table 2.

**4.1 Factors and Indices considered in FDATE**

The factors and indices considered in FDATE are very much similar to those in DATE, except the fact that one will use fuzzy numbers instead of crisp numbers.

*4.1.1 Fuzzy National Resource Index*

Fuzzy national resource index ( $\tilde{NRI}$ ) is a composite parameter indicating the level of expertise and infrastructure position in the country in a given technology. It is derived from  $\tilde{EF}$  and  $\tilde{IF}$  as

$$\tilde{NRI} = \frac{\alpha\tilde{EF} + (1 - \alpha)\tilde{IF}}{10}$$

where  $\tilde{EF}$  and  $\tilde{IF}$  are the fuzzy expertise factor and fuzzy infrastructure factor, respectively.  $\alpha$  is a factor assigned a value between 0 and 1, ( $0 \leq \alpha \leq 1$ ), which indicates the relative importance of expertise compared to infrastructure for the technology concerned. However, one can assign

**Table 2. Linguistic terms and corresponding triangular fuzzy numbers**

| Linguistic term to be assigned to a factor | Triangular fuzzy number |
|--|-------------------------|
| None                                       | (0, 0, 0.1)             |
| Exceptionally low<br>Exceptionally poor    | (0.1, 0.1, 1.0)         |
| Extremely low<br>Extremely poor            | (0.1, 1.0, 2.0)         |
| Very low<br>Very poor                      | (1.0, 2.0, 3.0)         |
| Low<br>Poor                                | (2.0, 3.0, 4.0)         |
| Medium low<br>Medium poor                  | (3.0, 4.0, 5.0)         |
| Medium<br>Fair                             | (4.0, 5.0, 6.0)         |
| Medium high<br>Medium good                 | (5.0, 6.0, 7.0)         |
| High<br>Good                               | (6.0, 7.0, 8.0)         |
| Very high<br>Very good                     | (7.0, 8.0, 9.0)         |
| Extremely high<br>Extremely good           | (8.0, 9.0, 10.0)        |
| Exceptionally high<br>Exceptionally good   | (9.0, 10.0, 10.0)       |

a standardised fuzzy number to  $\alpha$ , to signify that the relative importance is also fuzzy.

*4.1.2 Fuzzy Capability Index*

Fuzzy capability index is ( $\tilde{CI}$ ) derived from  $\tilde{NRI}$  and fuzzy confidence factor ( $\tilde{CF}$ ) as:

$$\tilde{CI} = \frac{\tilde{NRI} \times \tilde{CF}}{10}$$

#### 4.1.3 Fuzzy Subsystem Viability Index

Fuzzy subsystem viability index ( $\tilde{VI}_{\text{subsystem}}$ ) is a derived parameter based on the  $\tilde{CI}$  for each technology area and the fuzzy relative significance factor ( $R\tilde{SF}$ ) for that technology area for the given subsystem. It is defined as

$$\tilde{VI}_{\text{subsystem}} = \frac{\sum_t \tilde{CI} \times R\tilde{SF}}{\sum_t R\tilde{SF}}$$

where  $\sum_t$  indicates summation over all applicable technologies.

#### 4.1.4 Fuzzy System Realisability Index

Fuzzy system realisability index is denoted by  $S\tilde{RI}$  and derived from  $\tilde{VI}$ , fuzzy relative importance factor ( $R\tilde{IF}$ ) and fuzzy system integration factor ( $S\tilde{IF}$ ) as

$$S\tilde{RI} = \frac{\sum_s \tilde{VI} \times R\tilde{IF}}{\sum_s R\tilde{IF}} \times S\tilde{IF}$$

where  $\sum_s$  indicates the summation over all the subsystems.

#### 4.1.5 System Realisability Index

Decision makers in DRDO classify the project proposal on the basis of the system realisability of the project. For this purpose,  $S\tilde{RI}$  should be defuzzified to obtain a crisp value, on the basis of which a decision on the project proposal can be taken. Hence in case of FDATE, the system realisability index, SRI, is taken as  $DF(S\tilde{RI})$ , where  $DF(S\tilde{RI})$  denotes the defuzzified value of  $S\tilde{RI}$ . The interpretation of SRI will be the same as in DATE.

## 4.2 Aggregation of Experts' Opinion

The effectiveness of the technology evaluation process is mainly dependent on the availability of reliable data regarding the state of different technologies. Several committees of experts are

constituted to assess the expertise and infrastructure available in the country in respect of the technologies of interest to DRDO. Each committee has members drawn from DRDO, academic institutions, and the industry.

Similarly, the team proposing the project needs to access its own confidence in exploiting the nationally available technological resources for the project. They have to determine technologywise capability and potential of the project team for realisation of the system with the available resources and within the specified time frame. For these assessments, a committee of experts should be constituted in the concerned laboratory. The members of the committee should be comprised nominated members from the project team and experts in the laboratory possessing knowledge about the project, available resources, and the strengths and weaknesses of the project team.

One of the most commonly used techniques of aggregation of expert opinion is Delphi method<sup>4</sup>, which resolves the conflict in opinions through group consensus. Kaufmann and Gupta<sup>5</sup> have suggested a fuzzy version of Delphi method, called Fuzzy-Delphi method, which handles fuzzy opinion of experts. The general trend in this method is that the experts generally do not meet, and give their responses through fax/mail/telephone. The method is based on an iterative approach involving several rounds of questionnaires from the DMs and responses from experts. The process continues until either no expert changes his/her forecast or a level of general agreement exist.

Though, this method is widely accepted and adopted for group decision-making, few difficulties exist which make the method difficult for implementation. Firstly, the several rounds of questionnaires make the job of the DMs tedious. Secondly, conflict still may persist in opinions after several rounds of questionnaires. Thirdly, expert weighting, which is a must for a heterogeneous group of experts, is not considered in Fuzzy-Delphi method. When more than one expert are involved, then it is necessary to assign weights to experts according to their experience and expertise in the relevant field. For example, an expert, who is very experienced in

Kalman filtering, may not give good assessments for antenna design. Therefore, expert weighting is necessary and a good method of aggregating multiple expert opinions must consider the degree of importance of each expert in aggregating procedure. The weightage assigned to an expert will vary from technology to technology. In the following paragraphs, a method of aggregation of expert opinions has been suggested, which overcomes these difficulties.

Analytic hierarchy process (AHP) by Saaty<sup>6</sup> is theoretically well-established for assigning weightages to different alternatives (experts) wrt a criterion (technology) using a pairwise comparison technique and an appropriate scale. Suppose the degree of importance of an expert  $E_k$  ( $k = 1, 2, \dots, M$ , where  $M$  is the no. of experts) obtained using pairwise comparison technique of AHP is  $W(E_k)$ ,

where  $W(E_k) \in [0,1]$  and  $\sum_{k=1}^M W(E_k) = 1$ . If the importance of each expert is equal, then

$$W(E_k) = 1/M \quad (k = 1, 2, \dots, M).$$

Since each expert may have a different opinion according to his/her experience and expertise in the relevant field, it is necessary to aggregate experts' opinions to reach at a consensus. Ölcner and Odabasi<sup>7</sup> presented an algorithm to aggregate the linguistic opinions of homogeneous/heterogeneous group of experts which is presented below:

- (a) Suppose each expert,  $E_k$  ( $k = 1, 2, \dots, M$ ) expresses his/her opinion on a particular attribute against a specific context by a predefined set of linguistic variables. Convert the linguistic variables to positive fuzzy numbers using appropriate conversion scale. Assume that each expert,  $E_k$ 's linguistic expression has been converted to positive trapezoidal fuzzy number,

$$\tilde{R}_k = (a_k, b_k, c_k, d_k), \text{ where } 0 \leq a_k \leq b_k \leq c_k \leq d_k \leq m.$$

- (b) Translate each positive trapezoidal fuzzy number,  $\tilde{R}_k = (a_k, b_k, c_k, d_k)$ , into standardised trapezoidal fuzzy number,

$$\tilde{R}_k^* = (a_k^*, b_k^*, c_k^*, d_k^*) = \left( \frac{a_k}{m}, \frac{b_k}{m}, \frac{c_k}{m}, \frac{d_k}{m} \right)$$

It is mentioned that  $0 \leq a_k^* \leq b_k^* \leq c_k^* \leq d_k^* \leq 1$ .

- (c) Calculate the degree of agreement (or degree of similarity)  $S_{uv}(\tilde{R}_u^*, \tilde{R}_v^*)$  of the opinions between each pair of experts  $E_u$  and  $E_v$ , where  $S_{uv}(\tilde{R}_u^*, \tilde{R}_v^*) \in [0,1]$ ,  $1 \leq u, v \leq M$ , and  $u \neq v$ .

The method introduced by Chen and Lin<sup>8</sup> is used for measuring the degree of similarity between trapezoidal fuzzy numbers. According to this approach, let  $\tilde{A}^* = (a_1^*, a_2^*, a_3^*, a_4^*)$  and  $\tilde{B}^* = (b_1^*, b_2^*, b_3^*, b_4^*)$  be two standardised trapezoidal fuzzy numbers, where  $0 \leq a_1^* \leq a_2^* \leq a_3^* \leq a_4^* \leq 1$  and  $0 \leq b_1^* \leq b_2^* \leq b_3^* \leq b_4^* \leq 1$ . Then the degree of similarity between the standardised trapezoidal fuzzy numbers  $\tilde{A}^*$  and  $\tilde{B}^*$  can be measured by the similarity function  $S$ , which is defined

$$\text{as } S(\tilde{A}^*, \tilde{B}^*) = 1 - \frac{1}{4} \sum_{i=1}^4 |a_i^* - b_i^*|$$

where  $S(\tilde{A}^*, \tilde{B}^*) \in [0,1]$

For standardised triangular fuzzy numbers,  $\tilde{A} = (a_1^*, a_2^*, a_3^*)$  and  $\tilde{B} = (b_1^*, b_2^*, b_3^*)$  the formula for similarity function will be:

$$S(\tilde{A}^*, \tilde{B}^*) = 1 - \frac{|a_1^* - b_1^*| + 2|a_2^* - b_2^*| + |a_3^* - b_3^*|}{4}$$

The larger the value of  $S(\tilde{A}^*, \tilde{B}^*)$ , the greater the similarity between the standardised trapezoidal/triangular fuzzy numbers  $\tilde{A}^*$  and  $\tilde{B}^*$ . It can be noted that  $S(\tilde{A}^*, \tilde{B}^*) = S(\tilde{B}^*, \tilde{A}^*)$ .

- (d) Construct the agreement matrix (AM), after all the agreement (or similarity) degrees between experts are measured:

$$AM = \begin{bmatrix} 1 & S_{12} & \dots & S_{1v} & \dots & S_{1M} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ S_{u1} & S_{u2} & \dots & S_{uv} & \dots & S_{uM} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ S_{M1} & S_{M2} & \dots & S_{Mv} & \dots & 1 \end{bmatrix}$$

where

$$S_{uv} = \begin{cases} S(\tilde{R}_u^*, \tilde{R}_v^*), & \text{if } u \neq v \\ 1, & \text{if } u = v \end{cases}$$

By the definition of  $S(\tilde{R}_u^*, \tilde{R}_v^*)$ , the diagonal elements of AM are unit.

- (e) Calculate the average degree of agreement  $AA(E_u)$  of expert,  $E_u$  ( $u = 1, 2, \dots, M$ ) using the AM of the problem, where

$$AA(E_u) = \frac{1}{M-1} \sum_{\substack{v=1 \\ v \neq u}}^M S(\tilde{R}_u^*, \tilde{R}_v^*)$$

- (f) Calculate the relative degree of agreement,  $RA(E_u)$  of expert,  $E_u$  ( $u = 1, 2, \dots, M$ ) as

$$RA(E_u) = \frac{AA(E_u)}{\sum_{u=1}^M AA(E_u)}$$

- (g) Calculate the consensus degree coefficient  $CC(E_u)$  of expert,  $E_u$  ( $u = 1, 2, \dots, M$ ), where

$$CC(E_u) = \beta \cdot W(E_u) + (1-\beta) \cdot RA(E_u)$$

where  $\beta$  ( $0 \leq \beta \leq 1$ ) is a relaxation factor of the proposed method. It shows the importance of  $W(E_u)$  over  $RA(E_u)$ . When  $\beta = 0$ , no importance has been given to the weightage of an expert and hence a homogeneous group of experts problem is considered. When  $\beta = 1$ , the consensus degree of an expert is the same as his/her weightage in the decision-making process. When  $\beta = 0.5$ , equal importance has been given to the weightage of an expert and his/her relative degree of agreement on the issue in the decision-making process.

The consensus degree coefficient of each expert is a good measure for evaluating the relative worthiness of each expert's opinion. It is the responsibility of the DM to assign an appropriate value to  $\beta$ .

- (h) Finally, the aggregation result of the fuzzy opinion is  $\tilde{R}_{AG}$  as

$$\tilde{R}_{AG} = CC(E_1) \times \tilde{R}_1 + CC(E_2) \times \tilde{R}_2 + \dots + CC(E_M) \times \tilde{R}_M$$

### 5. CASE STUDY: EVALUATION OF PROJECT PROPOSAL ON BATTLEFIELD INFORMATION SYSTEM USING FDATE

Now consider a case study to evaluate a project proposal on battlefield information system (BIS) using FDATE. The following steps illustrate the evaluation process in a systematic manner. The expert opinions considered here are hypothetical in nature for understanding.

#### Step 1. Construction of Hierarchical Structure of Technologies of Project BIS

It has already been mentioned, any SDP of DRDO is primarily classified under a particular technology group and may require some technologies from other technology groups; those can be called as allied technologies. The project BIS primarily comes under the Technology Group-Electronics, apart from that, it requires allied technologies from the Technology Group-Engineering. The relevant core technologies and subsequently the subtechnologies and sub-subtechnologies are identified for the project, BIS. A partial hierarchical diagram of technologies for project, BIS, is shown in Fig.1.

#### Step 2. Evaluation of NRI through Consensus in Expert Opinion

Suppose a panel of three experts  $E_1$ ,  $E_2$  and  $E_3$  has been constituted to evaluate the NRI of core technology signal processing, from each of its subtechnologies. It is mentioned that only those subtechnologies (or sub-subtechnologies) that have relevance for the project need to be considered while calculating NRI of the core technologies. Thus, in the present example of project BIS, assuming

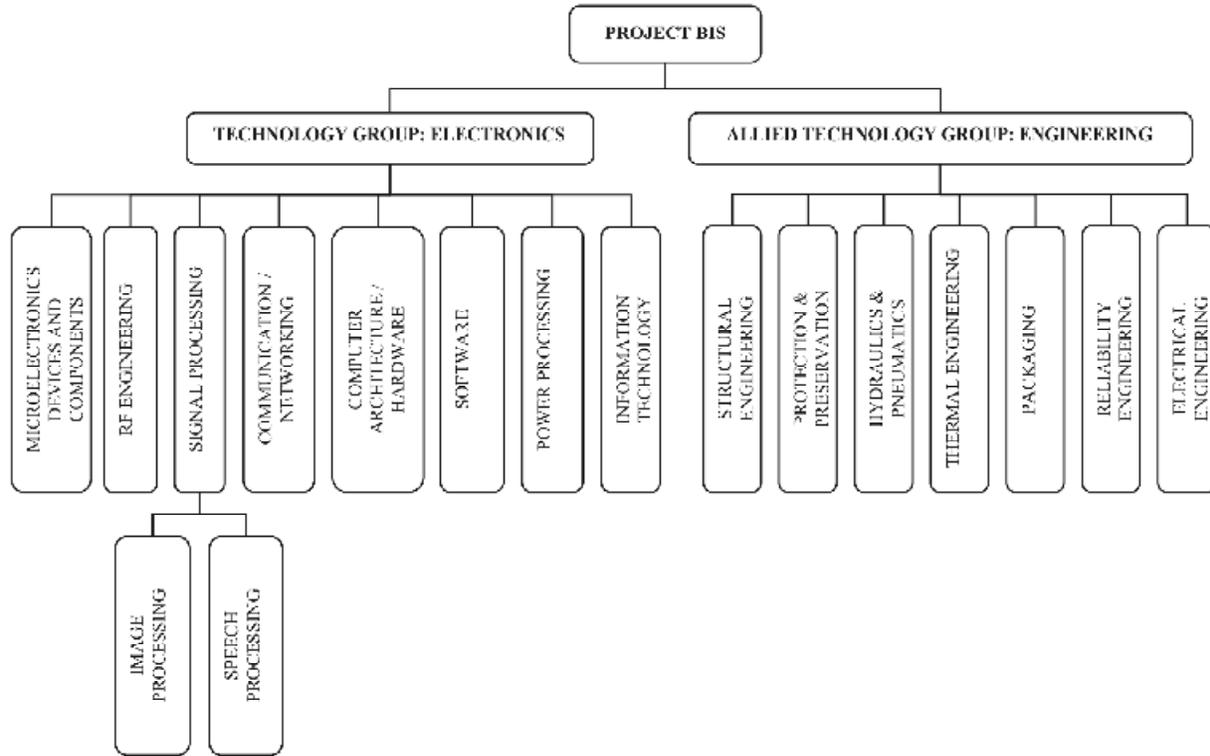


Figure 1. Partial hierarchical diagram of technologies of project battlefield information system.

that the team determines that only ‘image processing and speech processing’ are relevant, the  $\tilde{NRI}$  for signal processing is to be derived by averaging the  $\tilde{NRI}$  of only those two subtechnologies. Let the weightages of three experts determined by pairwise comparison technique, are  $W(E_1) = 0.6370$ ,  $W(E_2) = 0.1047$  and  $W(E_3) = 0.2583$ . Table 3 shows experts' opinion in linguistic terms on  $\tilde{EF}$  of

image processing and speech processing and their corresponding fuzzy numbers. Table 4 shows the aggregation of expert opinions given in Table 3 using the fuzzy technique. In Table 4 one can see that, the aggregated opinion on  $\tilde{EF}$  of image processing and speech processing are two triangular fuzzy numbers (5.1113, 6.1113, 7.1113) and (6.9232, 7.9232, 8.9232), respectively. Similarly Table 5 gives  $\tilde{IF}$

Table 3. Experts' opinion in linguistic terms on  $\tilde{EF}$  of subtechnologies of signal processing (relevant to project BIS) and their corresponding fuzzy numbers

|                | Expert opinion             | Image processing | Speech processing |
|----------------|----------------------------|------------------|-------------------|
| E <sub>1</sub> | Linguistic opinion         | Medium           | Very high         |
|                | Corresponding fuzzy number | (4.0, 5.0, 6.0)  | (7.0, 8.0, 9.0)   |
|                | Standardised fuzzy number  | (0.4, 0.5, 0.6)  | (0.7, 0.8, 0.9)   |
| E <sub>2</sub> | Linguistic opinion         | Extremely high   | Extremely high    |
|                | Corresponding fuzzy number | (8.0, 9.0, 10.0) | (8.0, 9.0, 10.0)  |
|                | Standardised fuzzy number  | (0.8, 0.9, 1.0)  | (0.8, 0.9, 1.0)   |
| E <sub>3</sub> | Linguistic opinion         | Medium high      | High              |
|                | Corresponding fuzzy number | (5.0, 6.0, 7.0)  | (6.0, 7.0, 8.0)   |
|                | Standardised fuzzy number  | (0.5, 0.6, 0.7)  | (0.6, 0.7, 0.8)   |

of image processing and speech processing as (5.1110, 6.1110, 7.1110) and (4.8106, 5.8106, 6.8106), respectively. In Table 6  $\tilde{NRI}$  of signal processing is calculated as (0.5344, 0.6344, 0.7344), which is the average of  $\tilde{NRI}$  of image processing and speech processing. The weightage of  $\tilde{EF}$  over  $\tilde{IF}$  for a particular subtechnology is given by the crisp number  $\alpha$  ( $0 \leq \alpha \leq 1$ ). The aggregated  $\alpha$  is the geometric mean of the expert opinions on  $\alpha$ . In a similar fashion  $\tilde{NRI}$  of all other core technologies and allied technologies are calculated. These are shown in the Tables 7 and 8. During the whole process of aggregation state, one has assumed  $\beta = 0.5$  to calculate  $CC(E_u)$ .

*Step 3. Evaluation of  $\tilde{SRI}$  through Consensus in Project Team*

The team proposing the project needs to access its own confidence in exploiting the nationally available technological resources for the project,

in the time span of the project and then has to determine its technologywise capability. The project team should breakdown the system to be developed into major subsystems and quantify the relative importance of each subsystem. It should also assess the significance of the different technologies for each of the subsystems. From these, the viability of development of each subsystem is to be computed. The project team should then assess its capability for the system integration. The potential for realisation of the system, with the available resources and within the specified timeframe, is computed from these parameters.

In the case study, the subsystems identified for the hypothetical BIS system are: (i) servers, (ii) workstations, (iii) communication, and (iv) shelters.

To evaluate  $\tilde{SRI}$ , each member from a committee of experts constituted by the concerned laboratory, should be given some weightage  $W(E_u)$  to signify

**Table 4. Aggregation of experts' opinion on  $\tilde{EF}$  of subtechnologies of signal processing (relevant to project BIS) and their corresponding fuzzy numbers**

|  |                       | Image processing         | Speech processing        |
|--|-----------------------|--------------------------|--------------------------|
| Expert opinion<br>(standardised fuzzy<br>number) | $\tilde{R}_1^*$       | (0.4000, 0.5000, 0.6000) | (0.7000, 0.8000, 0.9000) |
|  | $\tilde{R}_2^*$       | (0.8000, 0.9000, 1.0000) | (0.8000, 0.9000, 1.0000) |
|  | $\tilde{R}_3^*$       | (0.5000, 0.6000, 0.7000) | (0.6000, 0.7000, 0.8000) |
| Degree of agreement                              | $S_{12}$              | 0.6000                   | 0.9000                   |
|  | $S_{13}$              | 0.9000                   | 0.9000                   |
|  | $S_{23}$              | 0.7000                   | 0.8000                   |
| Average degree of agreement                      | $AA(E_1)$             | 0.7500                   | 0.9000                   |
|  | $AA(E_2)$             | 0.6500                   | 0.8500                   |
|  | $AA(E_3)$             | 0.8000                   | 0.8500                   |
| Relative degree of agreement                     | $RA(E_1)$             | 0.3409                   | 0.3462                   |
|  | $RA(E_2)$             | 0.2955                   | 0.3269                   |
|  | $RA(E_3)$             | 0.3636                   | 0.3269                   |
| Consensus degree of agreement                    | $CC(E_1)$             | 0.4890                   | 0.4916                   |
|  | $CC(E_2)$             | 0.2001                   | 0.2158                   |
|  | $CC(E_3)$             | 0.3109                   | 0.2926                   |
| Aggregated expert opinion                        | $\tilde{R}_{AG}^{HT}$ | (5.1113, 6.1113, 7.1113) | (6.9232, 7.9232, 8.9232) |

**Table 5. Aggregation of experts' opinion on  $\tilde{IF}$  of subtechnologies of signal processing (relevant to project BIS) and their corresponding fuzzy numbers**

|  |                 | <b>Image processing</b>  | <b>Speech processing</b> |
|--|-----------------|--------------------------|--------------------------|
| Expert opinion<br>(standardised fuzzy<br>number) | $\tilde{R}_1^*$ | (0.4000, 0.5000, 0.6000) | (0.4000, 0.5000, 0.6000) |
|  | $\tilde{R}_2^*$ | (0.5000, 0.6000, 0.7000) | (0.5000, 0.6000, 0.7000) |
|  | $\tilde{R}_3^*$ | (0.8000, 0.9000, 1.0000) | (0.6000, 0.7000, 0.8000) |
| Degree of agreement                              | $S_{12}$        | 0.9000                   | 0.9000                   |
|  | $S_{13}$        | 0.6000                   | 0.8000                   |
|  | $S_{23}$        | 0.7000                   | 0.9000                   |
| Average degree of<br>agreement                   | $AA(E_1)$       | 0.7500                   | 0.8500                   |
|  | $AA(E_2)$       | 0.8000                   | 0.9000                   |
|  | $AA(E_3)$       | 0.6500                   | 0.8500                   |
| Relative degree of<br>agreement                  | $RA(E_1)$       | 0.3409                   | 0.3269                   |
|  | $RA(E_2)$       | 0.3636                   | 0.3462                   |
|  | $RA(E_3)$       | 0.2955                   | 0.3269                   |
| Consensus degree of<br>Agreement                 | $CC(E_1)$       | 0.4890                   | 0.4820                   |
|  | $CC(E_2)$       | 0.3110                   | 0.2254                   |
|  | $CC(E_3)$       | 0.2000                   | 0.2926                   |
| Aggregated expert opinion $\tilde{R}_{AG}^{HT}$  |                 | (5.1110, 6.1110, 7.1110) | (4.8106, 5.8106, 6.8106) |

**Table 6. Calculation of  $\tilde{NRI}^{**}$  of subtechnologies of core technology signal processing**

|               | <b>Image processing</b>          | <b>Speech processing</b>         |
|---------------|----------------------------------|----------------------------------|
| $\tilde{EF}$  | (5.1113, 6.1113, 7.1113)         | (6.9232, 7.9232, 8.9232)         |
| $\tilde{IF}$  | (5.1110, 6.1110, 7.1110)         | (4.8106, 5.8106, 6.8106)         |
| $\alpha$      | $(0.7*0.5*0.4)^{(1/3)} = 0.5192$ | $(0.6*0.2*0.4)^{(1/3)} = 0.3634$ |
| $\tilde{NRI}$ | (0.5111, 0.6111, 0.7111)         | (0.5578, 0.6578, 0.7578)         |

\*\*  $\tilde{NRI}$  of signal processing = (0.5344, 0.6344, 0.7344)

**Table 7.  $\tilde{NRI}$  calculated for relevant core technologies under electronics**

| <b>Technology</b>                       | <b><math>\tilde{NRI}</math></b> |
|---|---------------------------------|
| Microelectronics devices and components | (0.5993, 0.6993, 0.7993)        |
| RF engineering                          | (0.5314, 0.6314, 0.7314)        |
| Signal processing                       | (0.5344, 0.6344, 0.7344)        |
| Communication/Networking                | (0.6515, 0.7515, 0.8515)        |
| Computer architecture/Hardware          | (0.6011, 0.7011, 0.8011)        |
| Software                                | (0.7919, 0.8919, 0.9919)        |
| Power processing                        | (0.3313, 0.4313, 0.5313)        |
| Information technology                  | (0.5515, 0.6515, 0.7515)        |

his/her degree of importance in the decision-making process. Then each member will give one's assessment on  $\tilde{C}F, \tilde{R}\tilde{S}F, \tilde{R}\tilde{I}F$  and  $\tilde{S}\tilde{I}F$  using the linguistic variables from Table 2. Then, their opinions are aggregated in a similar fashion as in Step 2. Using these aggregated factors represented by fuzzy numbers and the calculated  $\tilde{N}RI$ s from Step 2, the detailed

calculations in the process of evaluating  $\tilde{S}RI$  are shown in Tables 9-12.

*Step 4. Evaluation & Interpretation of SRI*

In this step,  $\tilde{S}RI$  is defuzzified to obtain SRI, on the basis of which the DM takes a decision on the project proposal.

**Table 8.  $\tilde{N}RI$  calculated for engineering (allied technology)**

| Technology                  | $\tilde{N}RI$            |
|-----------------------------|--------------------------|
| Structural engineering      | (0.6315, 0.7315, 0.8315) |
| Protection and preservation | (0.5643, 0.6643, 0.7643) |
| Hydraulics and pneumatics   | (0.6111, 0.7111, 0.8111) |
| Thermal engineering         | (0.5913, 0.6913, 0.7913) |
| Packaging                   | (0.6516, 0.7516, 0.8516) |
| Reliability engineering     | (0.3914, 0.4914, 0.5914) |
| Electrical engineering      | (0.3317, 0.4317, 0.5317) |

**Table 9. Calculation of  $\tilde{C}I$**

| Technology                              | $\tilde{N}RI$            | $\tilde{C}F$             | $\tilde{C}I$             |
|---|--------------------------|--------------------------|--------------------------|
| Microelectronics devices and components | (0.5993, 0.6993, 0.7993) | (3.5519, 4.5519, 5.5519) | (0.2129, 0.3183, 0.4438) |
| RF engineering                          | (0.5314, 0.6314, 0.7314) | (7.7823, 8.7823, 9.7823) | (0.4136, 0.5545, 0.7155) |
| Signal processing                       | (0.5344, 0.6344, 0.7344) | (7.2125, 8.2125, 9.2125) | (0.3854, 0.5210, 0.6766) |
| Communication/Networking                | (0.6515, 0.7515, 0.8515) | (7.4555, 8.4555, 9.4555) | (0.4857, 0.6354, 0.8051) |
| Computer architecture/Hardware          | (0.6011, 0.7011, 0.8011) | (7.8619, 8.8619, 9.8619) | (0.4726, 0.6213, 0.7900) |
| Software                                | (0.7919, 0.8919, 0.9919) | (7.9988, 8.9988, 9.9988) | (0.6334, 0.8026, 0.9918) |
| Power processing                        | (0.3313, 0.4313, 0.5313) | (7.1023, 8.1023, 9.1023) | (0.2353, 0.3495, 0.4836) |
| Information technology                  | (0.5515, 0.6515, 0.7515) | (7.5432, 8.5432, 9.5432) | (0.4160, 0.5566, 0.7172) |
| *Engineering                            | —                        | —                        | (0.3784, 0.5126, 0.6669) |

\*Allied Technology,  $\tilde{C}I$  evaluated at Table 10

**Table 10. Calculation of  $\tilde{C}I$  for engineering**

| Technology                | $\tilde{N}RI$            | $\tilde{C}F$             | $\tilde{C}I$             |
|---------------------------|--------------------------|--------------------------|--------------------------|
| Structural engineering    | (0.6315, 0.7315, 0.8315) | (7.8871, 8.8871, 9.8871) | (0.4981, 0.6501, 0.8221) |
| Protection & preservation | (0.5643, 0.6643, 0.7643) | (5.9193, 6.9193, 7.9193) | (0.3340, 0.4596, 0.6053) |
| Hydraulics & pneumatics   | (0.6111, 0.7111, 0.8111) | (7.6823, 8.6823, 9.6823) | (0.4695, 0.6174, 0.7853) |
| Thermal engineering       | (0.5913, 0.6913, 0.7913) | (6.5798, 7.5798, 8.5798) | (0.3891, 0.5240, 0.6789) |
| Packaging                 | (0.6516, 0.7516, 0.8516) | (6.4983, 7.4983, 8.4983) | (0.4234, 0.5636, 0.7237) |
| Reliability engineering   | (0.3914, 0.4914, 0.5914) | (7.8991, 8.8991, 9.8991) | (0.3092, 0.4373, 0.5854) |
| Electrical engineering    | (0.3317, 0.4317, 0.5317) | (6.7934, 7.7934, 8.7934) | (0.2253, 0.3364, 0.4675) |
| Engineering               | —                        | —                        | (0.3784, 0.5126, 0.6669) |

Table 11. Relative significance factor and viability index

| Technology                              | Relative significance factors ( ) of subsystems |                          |                          |                          |
|---|---|--------------------------|--------------------------|--------------------------|
|   | Servers   | Workstations             | Communication            | Shelters                 |
| Microelectronics devices and components | (0.2129, 0.3183, 0.4438)                        | —                        | (4.3561, 5.3561, 6.3561) | —                        |
| RF engineering                          | (0.4136, 0.5545, 0.7155)                        | (4.8723, 5.8723, 6.8723) | (7.9865, 8.9865, 9.9865) | (5.4621, 6.4621, 7.4621) |
| Signal processing                       | (0.3854, 0.5210, 0.6766)                        | (4.5673, 5.5673, 6.5673) | (7.8934, 8.8934, 9.8934) | —                        |
| Communication / Networking              | (0.4857, 0.6354, 0.8051)                        | (5.6721, 6.6721, 7.6721) | (7.9835, 8.9835, 9.9835) | —                        |
| Computer architecture/ Hardware         | (0.4726, 0.6213, 0.7900)                        | (7.8564, 8.8564, 9.8664) | (5.7629, 6.7629, 7.7629) | —                        |
| Software                                | (0.6334, 0.8026, 0.9918)                        | (7.7986, 8.7986, 9.7986) | (6.5329, 7.5329, 8.5329) | —                        |
| Power processing                        | (0.2353, 0.3495, 0.4836)                        | —                        | —                        | (2.5612, 3.5612, 4.5612) |
| Information technology                  | (0.4160, 0.5566, 0.7172)                        | (7.6843, 8.6843, 9.6843) | (5.3186, 6.3186, 7.3186) | —                        |
| Engineering                             | (0.3784, 0.5126, 0.6669)                        | (1.2512, 2.2512, 3.2512) | (6.3405, 7.3405, 8.3405) | (7.8932, 8.8932, 9.8932) |
| Viability Index ( )                     | (0.3718, 0.6466, 1.0926)                        | (0.3546, 0.6082, 1.0098) | (0.3318, 0.5743, 0.9603) | (0.3318, 0.5743, 0.9603) |

Table 12. System realisability index

| Technology                              | Subsystems                |                           |                           |                           |
|---|---------------------------|---------------------------|---------------------------|---------------------------|
|   | Servers                   | Workstations              | Communication             | Shelters                  |
| Microelectronics devices and components | (0.3718, 0.6466, 1.0926)  | (0.3546, 0.6082, 1.0098)  | (0.3318, 0.5743, 0.9603)  | (0.3318, 0.5743, 0.9603)  |
| RF engineering                          | (7.8814, 8.8814, 9.8814)  | (7.1125, 8.1125, 9.1125)  | (7.2531, 8.2531, 9.2531)  | (3.5638, 4.5638, 5.5638)  |
| Signal processing                       | (7.5849, 8.5849, 9.5849)  | (7.5849, 8.5849, 9.5849)  | (7.5849, 8.5849, 9.5849)  | (7.5849, 8.5849, 9.5849)  |
| Communication / Networking              | (2.0283, 5.1944, 11.3842) | (2.0283, 5.1944, 11.3842) | (2.0283, 5.1944, 11.3842) | (2.0283, 5.1944, 11.3842) |
| SRI                                     | 5.9503                    |                           |                           |                           |

For the present project proposal of BIS, one has obtained  $SRI = 5.9503$ . Hence, according to the interpretations suggested above, there is a reasonable chance of success of the project BIS.

## 6. CONCLUSION

This paper has presented a fuzzy decision tool for DMs to evaluate the project proposals of R&D organisations. To make the model more realistic, accurate and reliable, one has taken input data in terms of linguistic variables, which then converted to fuzzy numbers to apply fuzzy arithmetic for computational purposes. For illustration, one has considered the existing decision-support tool DATE of DRDO. Few weaknesses in DATE have been pointed out and the possible remedies using fuzzy set theoretic approach has been suggested. In the process, a fuzzy decision-support tool FDATE is suggested. The authors' opinion FDATE can be applied to the project evaluation of any R&D organisation with a little modification in the terminologies only.

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### Basic Concepts of Fuzzy Set Theory

*Definition 1.* Let  $X$  be a nonempty set, a universe of discourse. A fuzzy set  $\tilde{A}$  in  $X$  is characterised by its membership function:

$$\mu_{\tilde{A}} : X \rightarrow [0,1]$$

and  $\mu_{\tilde{A}}(x)$  is interpreted as the degree of membership of element  $x$  in fuzzy set  $\tilde{A}$  for each  $x \in X$ .

*Definition 2.* A fuzzy set  $\tilde{A}$  in the universe of discourse  $X$  is normal, if

$$\sup_x \mu_{\tilde{A}}(x) = 1$$

*Definition 3.* A fuzzy set  $\tilde{A}$  in the universe of discourse  $X$  is convex, if

$$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min \{ \mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2) \}$$

$$\forall x_1, x_2 \in X \text{ and } \forall \lambda \in [0,1]$$

*Definition 4.* Let  $\tilde{A}$  be a fuzzy set in the universe of discourse  $X$ . For any number  $\alpha \in [0, 1]$ , the  $\alpha$ -cut,  $\tilde{A}^\alpha$ , and the strong  $\alpha$ -cut,  $\tilde{A}^{\alpha+}$  are defined as

$$\tilde{A}^\alpha = \{x \in X : \mu_{\tilde{A}}(x) \geq \alpha\}$$

$$\tilde{A}^{\alpha+} = \{x \in X : \mu_{\tilde{A}}(x) > \alpha\}.$$

Strong 0-cut,  $\tilde{A}^{0+}$  is called as the support of  $\tilde{A}$  and is denoted by  $\text{supp}(\tilde{A})$ .

*Definition 5.* Let the universe of discourse be the set of real numbers,  $R$ .

A fuzzy number is a fuzzy set  $\tilde{A}$  on  $R$  possessing the following properties:

- $\tilde{A}$  is a normal fuzzy set.
- $\tilde{A}^\alpha$  is a closed interval for every  $\alpha \in [0,1]$ .
- $\text{Supp}(\tilde{A})$  is bounded.

If  $\mu_{\tilde{A}}(x)$  is positive for each  $x \in X$ , then  $\tilde{A}$  is called a positive fuzzy number.

*Definition 6.* A fuzzy number  $\tilde{A}$  is called a triangular fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x \in (-\infty, a_1) \\ (x - a_1)/(a_2 - a_1), & x \in [a_1, a_2] \\ (x - a_3)/(a_2 - a_3), & x \in [a_2, a_3] \\ 0, & x \in [a_3, \infty) \end{cases}$$

It is denoted by the triplet  $(a_1, a_2, a_3)$ .

*Definition 7.* Let  $\tilde{A}=(a_1, a_2, a_3)$  and  $\tilde{B}=(b_1, b_2, b_3)$  be two positive triangular fuzzy numbers. Then their sum, difference, product and division, respectively defined as

$$\begin{aligned} \tilde{A} + \tilde{B} &= (a_1 + b_1, a_2 + b_2, a_3 + b_3) \\ \tilde{A} - \tilde{B} &= (a_1 - b_3, a_2 - b_2, a_3 - b_1) \\ \tilde{A} \times \tilde{B} &= (a_1 \times b_1, a_2 \times b_2, a_3 \times b_3) \\ \tilde{A} \div \tilde{B} &= (a_1 \div b_3, a_2 \div b_2, a_3 \div b_1) \end{aligned}$$

*Definition 8.* For a triangular fuzzy number,  $\tilde{A}=(a_1, a_2, a_3)$  and a real number  $k$ , the scalar multiple of  $\tilde{A}$  by  $k$ , denoted by  $k\tilde{A}$  is a triangular fuzzy number given by the triplet  $(ka_1, ka_2, ka_3)$  if  $k$  is positive or by  $(ka_3, ka_2, ka_1)$ , if  $k$  is negative.

*Definition 9.* For a triangular fuzzy number,  $\tilde{A}=(a_1, a_2, a_3)$ , its defuzzification value is defined to be  $DF(\tilde{A}) = (a_1 + 2a_2 + a_3)/4$ .

*Definition 10.* A fuzzy number is called a trapezoidal fuzzy number, if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x \in (-\infty, a_1) \\ (x - a_1)/(a_2 - a_1) & x \in [a_1, a_2] \\ 1, & x \in [a_2, a_3] \\ (x - a_4)/(a_3 - a_4) & x \in [a_3, a_4] \\ 0, & x \in (a_4, \infty) \end{cases}$$

It is denoted by the quadruplet  $(a_1, a_2, a_3, a_4)$ . If  $a_2 = a_3$ ,  $\tilde{A}$  becomes a triangular fuzzy number.

*Definition 11.* Let  $\tilde{A}=(a_1, a_2, a_3, a_4)$  and  $\tilde{B}=(b_1, b_2, b_3, b_4)$  be two positive trapezoidal fuzzy numbers. Then their sum, difference, product and division, respectively, are defined as

$$\begin{aligned} \tilde{A} + \tilde{B} &= (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4) \\ \tilde{A} - \tilde{B} &= (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1) \\ \tilde{A} \times \tilde{B} &= (a_1 \times b_1, a_2 \times b_2, a_3 \times b_3, a_4 \times b_4) \\ \tilde{A} \div \tilde{B} &= (a_1 \div b_4, a_2 \div b_3, a_3 \div b_2, a_4 \div b_1) \end{aligned}$$

*Definition 12.* For a trapezoidal fuzzy number,  $\tilde{A}=(a_1, a_2, a_3, a_4)$  and a real number  $k$ , the scalar

multiple of  $\tilde{A}$  by  $k$  denoted by  $k\tilde{A}$  is a trapezoidal fuzzy number given by the quadruplet  $(ka_1, ka_2, ka_3, ka_4)$  if  $k$  is positive or by  $(ka_4, ka_3, ka_2, ka_1)$  if  $k$  is negative.

*Definition 13.* For a trapezoidal fuzzy number  $\tilde{A}=(a_1, a_2, a_3, a_4)$ , its defuzzification value is defined to be  $DF(\tilde{A})=(a_1+a_2+a_3+a_4)/4$ .

*Definition 14.* A linguistic variable is a variable, whose states are fuzzy numbers assigned to relevant linguistic terms. A linguistic variable is characterised by a quintuple  $(x, T(x), U, G, \tilde{M})$ , where (i)  $x$  is the name of the value, (ii)  $U$  is the universe of discourse, which is associated with the base variable  $u$ , (iii)  $T(x)$  denotes the term  $x$ , i.e., the set of the name of linguistic value of  $x$ , with each value being a fuzzy variable denoted generically by  $x$  and ranging over  $U$ , (iv)  $G$  is a syntactic rule for generating the name  $X$ , of values  $x$ . A particular  $X$ , that is name generated by  $G$ , is called a term, and (v)  $M$  is a semantic rule for associating with each  $X$  its meaning,  $\tilde{M}(x)$  which is a fuzzy subset of  $U$ .